INTRODUCTION

1.1 Introduction and Motivation:

Einstein believed that Mach's principle should pay a major role in obtaining a meaningful spacetime geometry, though it was investigated later that his field equations presented some solutions which were not Machian. When Einstein introduced the cosmological constant $\Lambda$ into his field equations to obtain a static solution, he was guided by Mach's principle, which argued that the distribution of matter determined the precise geometrical nature of spacetime and hence forbade the notion of empty Universe. He believed that the presence of matter was essential for a meaningful spacetime geometry. However, he had to discount his idea when de-Sitter obtained a cosmological model with cosmological constant $\Lambda$ and no matter at all which had both static and dynamic nature of the Universe. Later he also dismissed, $\Lambda$ when it was found that the Universe was expanding. However, one notices that if a dynamic $\Lambda(t)$ is introduced into Einstein's field equations, no
solution is possible in the absence of matter. It is obvious from the divergence of the field equations

\begin{equation}
\left[ R^{ij} - \frac{1}{2} R g^{ij} \right]_{ij} = 0
\end{equation}

\begin{equation}
= -8\pi G \left[ T^{ij} - \frac{\Lambda(t)}{8\pi G} g^{ij} \right]_{ij}.
\end{equation}

It is obvious that a solution with a dynamic \( \Lambda \) is possible only if

\begin{equation}
T^{ij} \neq 0 \quad \text{and} \quad T_{;j}^{ij} \neq 0.
\end{equation}

In the absence of matter or even if the matter is conserved, \( \Lambda \) has got to remain a constant. Hence, the empty spacetime may not be obtained as a solution of general relativity with a dynamic \( \Lambda (t) \). This way of introducing \( \Lambda \) into Einstein's equations gives it a status of a source term. Now \( \Lambda / 8\pi G \) represents the energy density of emptiness i.e. vacuum and therefore invites particle physics to interact with general relativity through \( \Lambda \). It is to be noted that the only possible covariant form for the energy momentum tensor of the quantum vacuum is
\[(1.3) \quad T_{ij} = -\rho_v g_{ij}, \]

which is equivalent to the cosmological constant. It behaves like a perfect fluid with the energy density

\[(1.4) \quad \rho_v = \Lambda/8\pi G, \]

an isotropic pressure

\[(1.5) \quad p_v = -\rho_v = -\Lambda/8\pi G. \]

The conserved quantity is now the sum of matter and vacuum and not the two separately as is clear from equation (1.1). The value of vacuum energy at the Planck epoch comes out as \( \approx 10^{76} \text{GeV}^4 \), which is 123 orders of magnitude larger than its value predicted by the Friedmann equation

\[(1.6) \quad \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \Lambda/3, \]

which gives

\[(1.7) \quad \Lambda_0 \approx H_0^2 \]

or, equivalently

\[(1.8) \quad \rho_{vo} \approx 10^{-47} \text{ GeV}^4, \]
as shown by Weinberg (1989), Sahni and Starobinsky (2000), where the subscript 0 denotes the value at present epoch. The predicted value for $\Lambda$ is also coincident with the recent observations of type Ia supernovae by Perlmutter et al (1999), Riess et al (1998), and the anisotropy measurements of the cosmic microwave radiation by Jaffe et al (2001), Poyke et al (2001), Netterfield et al (2001), taken together with the complimentary observational constraints on matter density as described by Turner (2001). All show that the present constituent of the Universe is dominated by some weird kind of energy with negative pressure, commonly known as dark energy. The simplest candidate for dark energy is the cosmological constant, though plagued with this so called the cosmological constant problem. The problem arises due to the incompatibility of general relativity and particle physics.

In fact, the dynamical $\Lambda$ was invoked in an attempt to solve this problem. The rationale behind this process is that $\Lambda$ was large during the early epochs and it decayed as the Universe evolved, reducing to a small value at the present epoch. There is another phenomenological approach to solve this problem, which has become very popular since recent observations suggested the
existence of nonzero $\Lambda$. This invokes a slowly rolling down scalar field $\phi$, commonly known as quintessence with an appropriate potential $V(\phi)$ to explain the observations. It is to be noted that the quintessence fields also acquire negative pressure during the matter dominated phase and behave like dynamical $\Lambda$ with

\begin{equation}
\Lambda_{\text{effective}} = 8\pi G \rho_\phi.
\end{equation}

They are in general fundamentally different from the dynamical $\Lambda$. In the former case, quintessence and matter fields are assumed to be conserved separately through the assumption of minimal coupling of the scalar field with the matter fields. In the latter case, the conserved quantity is $\left[T^{ij} + T^i_v\right]$, which implies that there is a continuous creation of matter from the decaying $\Lambda$ as it is obvious from the equation

\begin{equation}
\rho = Ca^{-3(1+w)} - a^{-3(1+w)} \frac{\dot{a}}{8\pi G} \int \dot{\Lambda}(t) dt + a^{3(1+w)} dt,
\end{equation}

(1.11) $C = \text{constant}$,
which follows from eq. (1.1) and suggesting that there is a positive contribution to $\rho$ from the decaying $\Lambda$ and $\dot{\Lambda} < 0$. Here

\begin{equation}
(1.12) \quad p = \rho w,
\end{equation}

as the equation of state of the matter field. Obviously the quintessence models need not be consistent with Mach's principle. It may also be noted that for a given pair of $a(t)$ and $\rho(t)$ it is always possible to obtain $V(\phi)$ which explains the observations, as presented by Padmanabhan (2001). This result is irrespective of what the future observations reveal about the given $a(t)$ and $\rho(t)$, and makes these models trivial. The powerful and true solution of the cosmological constant $\Lambda$ should be provided by a full theory of quantum cosmology, which is not available at the present moment. However, some argument have been made, based on the quantum gravitational uncertainty principle and the discrete structure of spacetime at Planck length, which made it possible to relate with the cosmological constant $\Lambda$ with the microstructure of spacetime as shown by Padmanabhan (2001), Sorkin (1997).
By assuming that $\Lambda$ is a stochastic variable arising from the quantum fluctuations and other arguments it has been shown that the uncertainty in the value of $\Lambda$ may be written as

\begin{equation}
\Delta \Lambda = \frac{1}{\sqrt{V_4}},
\end{equation}

where $V_4$ be the four volume of the Universe. If the radius of Universe

\begin{equation}
a \approx ct \approx CH^{-1},
\end{equation}

then one obtains

\begin{equation}
\Delta \Lambda \approx H^2 (c = 1),
\end{equation}

which matches exactly with the present observations.

There are also other two ways which provide $\Lambda \alpha H^2$ as follows:

(i) It is known that $\Lambda$ produces a force of repulsion between two bodies which increases in proportion to the distance between them. This force experienced by a test particle at a scale of the whole Universe is $C\Lambda H^{-1}$. If this repulsive force roughly balances the
The gravitational attraction \(4\pi Gc \rho/3H\) of the Universe on the test particle, one obtains

\[
\Lambda \approx H^2, \tag{1.16}
\]

provided

\[
\sqrt{\ell m} = 8\pi G\rho/3H^2 \approx 1. \tag{1.17}
\]

(ii) From the dimensional considerations, it is always possible to write \(\Lambda\) in terms of Planck energy density times a dimensionless quantity as shown by Chen and Wu (1990), Carvalho and Lima (1992)

\[
\Lambda \approx 8\pi G \rho\rho_c \left[ \frac{t_{\rho_i}}{t_H} \right]^{\alpha} \alpha t_{p_i}^{-2} \left[ \frac{t_{p_i}}{t_H} \right]^{\alpha}, \tag{1.18}
\]

where

\[
t_{\rho_i} = \left( G\hbar/c^5 \right)^{\frac{1}{2}}, \tag{1.19}
\]

\[
t_H = H^{-1}, \tag{1.20}
\]

are the Planck and Hubble times respectively and

\[
\rho_{\rho_i} = c^5/G^2\hbar, \tag{1.21}
\]
as the Planck energy density. For $\alpha = 2$, one obtains the value of $\Lambda$ at the present epoch

(1.22) \[ \Lambda \approx H^2. \]

If one writes this law as $\Lambda = nH^2$, where $n$ as a constant parameter, the dynamics of the resulting model may be obtained as

(1.23) \[ \rho \alpha \Lambda \alpha H^2 \alpha t^{-2}, \]

(1.24) \[ a \alpha t^{2/(3-n)(1+w)}], \quad n > 3, \]

where $k = 0$. The cases $n \geq 3$ where $\rho \leq 0$ are either unphysical or not compatible with $\Lambda = \Lambda(t)$. It is to be noted that the ansatz $\Lambda = nH^2$ is equivalent to assume that

(1.25) \[ \Omega_\Lambda \equiv \Lambda / 3H^2 = n/3. \]

Hence

(1.26) \[ \Omega_m = 1 - n/3. \]

in a flat model. Hence,

(1.27) \[ \rho_v / \rho_m \rho_v = n/3. \]

The deceleration parameter reads in this model
\[ q = \frac{(3-n)(1+w)}{2} - 1, \]

which is also constant and implies \( q \gtrless 0 \) in according as \( n \gtrless (1+3)/(1+w) \).

Observations of the luminosity type Ia supernovae as described by Perlmutter et al (1999), Riess et al (1998) supported by the discovery of cosmic microwave background angular temperature fluctuations on degree scales by Benoit et al (2003) and measurements of the power spectrum of galaxy clustering as given by Pereival et al (2002) convincingly present that our Universe is approximately spatially flat, with \( \approx 30\% \) of its critical energy density in non-relativistic matter (cold dark matter and baryons) and the remaining \( \lesssim 70\% \) in a smooth component having a large negative pressure (dark energy). Dark energy is obviously the most abundant form of matter in the Universe, in terms of the effective energy density, yet both its nature and its cosmological origin remain enigmatic at present. It is clear that the observational features of dark energy are tantalizingly very close to that of cosmological constant \( \Lambda \), yet dark energy need not be \( \Lambda \) exactly as

In order to obtain consequences both for the current Universe and its ultimate fate, is that the dark energy may be decaying. The possibility that dark energy could be unstable is in fact suggested by the remarkable qualitative analogy between the presence of dark energy to day and the features of a different type of dark energy-the inflation field-postulated in the inflationary scenario of the early Universe. This analogy works in two ways. On one way, that the form of matter having a large negative pressure dominates the universe to day makes it not unnatural that a similar form of matter having \( w < 0 \) could have dominated the Universe in the distant past. On the other way, since the dark energy in the early Universe i.e. the inflation was unstable decayed aeons ago, one might be tempted to understand whether the nature of dark energy observed to day will be any different.

Decaying dark energy gives numerous interesting possibilities including the fact that the current epoch of cosmic acceleration could be a transient which ends after the dark energy density has dropped to sufficiently small values. For example, a
decaying dark energy universe may not possess horizons which are characteristics of $\Lambda$ cold dark matter cosmology, as well as tracker-driven quintessence model as described by Sahni (2002). Within the context of quintessence models another interesting, though more speculative, possibility is given by potentials which are not constrained to be positive but which may become negative for certain values of $\phi$ as presented by Felder et al (2002). These potentials are either bounded from below, in which case $V(\phi)$ has one or more minima at which $V(\phi) < 0$.

One of the central themes of modern cosmology is observational evidence for accelerating Universe for the past few years. In the framework of the standard cosmology, the explanation of these observations, requires an exotic form of energy which violates the strong energy condition. A variety of scalar field models have been presented for this purpose including quintessence by Frieman et al (1995), Ferreira and Joyce (1995), Brax and Martin (2000), Sahni et al (2000), and recently tachyonic scalar fields by Garougi (2000), Bergshoeff et al (2000), Wasserman (2003), Padmanabhan and Choudhury (2002). Other approach to the cosmic acceleration is advocated by Babak and Grishchuck
The issues related to the cosmological constant and the dark energy were presented by Carroll (2001), Peebles and Ratra (2003), Caldwell (2002). By selecting an appropriate potential $V(\phi)$ and with suitable parameters of the model one may account for the current acceleration of the Universe with $\Omega_\phi = 0.7$ and $\Omega_m = 0.3$. However, all these models lead to the equation of state parameter $\omega_\phi \geq -1$. The recent observation favours the value of this parameter less than -1.

A scalar field with negative kinetic energy called the phantom field is proposed by Caldwell, Carroll and others to realise the possibility of late time acceleration with $\omega_\phi < -1$. Such a field has a very unusual dynamics as it violates null dominant energy condition. The models with equation of state parameter less than -1 are known to face the problem of future curvature singularity which can, however, be overcome in specific models of phantom field. Inspite of the fact that the field theory of phantom fields does encounter the problem of stability which one could try to bypass by assuming them to be effective field by Carroll et al (2003), it is nevertheless interesting to study their cosmological implications.
Phantom fields have glorious lineage in Hoyle's Steady State Theory. In adherence to the Perfect Cosmological Principle a creation field (C-field) was for the first time introduced by Hoyle (1948) to reconcile with homogeneous density by creation of new matter in the voids caused by the expansion of the Universe. It was further refined and reformulated by Hoyle and Narlikar (1964, 1972) theory of gravitation for details of the C-field cosmology. The C-field appeared on the right hand side of the Einstein equation. What was conserved was the sum of the stress tensors of matter and C-field and neither was conserved separately. The C-field violated the weak energy condition. In this context, it may also be noted that wormholes do require violation of energy conditions, in particular of the average null energy condition as shown by Visser (1995). Hence, phantom fields though very exotic are not entirely new and out of place.

1.2 Decaying Dark Energy:

The decaying dark energy model (DDE) is analogous to inflationary models in that it undergoes quasi-homogeneous decay. Let us model decaying dark energy by a scalar field \( \phi \) which couples minimally to gravity and has one or more maxima in its
potential. Very near any one such local maxima, the field may be generically given by the potential

\[ V(\phi) = V_0 - \frac{m^2 \phi^2}{2} \]  

(1.29)

where,

\[ m^2 \equiv \left| \frac{d^2 V}{d\phi^2} \right|_{\phi=0}. \]  

(1.30)

One may assume the modulus of the tachyonic rest mass \( m \ll H_0 \) where \( H_0 \) be the Hubble constant but for the slow-roll condition

\[ m_0 \ll H_0, \]  

(1.31)

results in the absence of any noticeable observational effects for ultra-light scalars. It is obvious from observations that physical features of dark energy are very near to those of a cosmological constant at present. The equation of motion then implies that

\[ m \ll H_0, \]  

(1.31)

suggesting that all characteristic time-scales will be of the order of \( H_0^{-1} \) or greater. Hence, in this model the initial value \( \phi_0 \) of the
scalar field $\phi$ and its change during the whole process of dark energy decay are expected of the order of $G^{-1}$, which follows from the Einstein equation

\[(1.32) \quad m^2\phi^2 - \rho_{crit} \leq H_0^2/G.\]

However, since cosmology is now a precision science, one may hope to obtain much better quantitative results, beyond these simple qualitative estimates, using current observational data. It is important to mention, in principle, one may also consider other decay mechanism, for example one may use bubble like decay via quantum tunneling in the case when

\[(1.33) \quad V'' > 0,\]

or strongly inhomogeneous classical decay in the case of tachyonic mass large compared to $H_0$. However, these possibilities must be confronted with observational data other than the high-$z$ supernova data.

Now the behaviour of the scalar field as it rolls down it potential in a spatially flat Universe may obtained by the field equations

\[(1.34) \quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,\]
(1.35) \[ H^2 = \frac{8\pi G}{3} (\rho_m + \rho_\phi) \]

where

(1.36) \[ V(\phi) = V_0 - \frac{m^2 \phi^2}{2}, \]

and

(1.37) \[ \rho_\phi = V(\phi) + \frac{1}{2} \phi^2. \]

At, early times i.e.

(1.38) \[ H_0 t << 1, \]

the Universe was matter-dominated. Let us assume

(1.39) \[ \rho_m >> \rho_\phi, \]

one may obtain the exact solution for \( \phi(t) \) during the matter-dominated regime

(1.40) \[ \phi(t) = \phi_0 \frac{\sin h m t}{m t}. \]

At very early times

(1.41) \[ mt << 1, \]
the scalar field experiences very large damping and it ensures that it remains very close to its initial value at $\phi_0$. It is to be noted that

\begin{equation}
\phi_0 \lesssim \sqrt{3/8\pi G},
\end{equation}
in order that dark energy may have sufficiently large negative pressure during the present epoch. Since the point

\begin{equation}
\phi = 0, \quad \dot{\phi} = 0,
\end{equation}
be a saddle point for homogeneous solutions of the scalar field equation with the potential

\begin{equation}
V(\phi) = V_0 - \frac{m^2 \phi^2}{2}.
\end{equation}

There exists a set of non-zero measure of generic solutions. It is sufficient for this purpose that a large initial kinetic energy-if at all it existed-is redshifted by $z \sim 3$ so that the field settles on the trajectory described by eq. (1.40) by that redshift. Since the kinetic energy of the scalar field decays as $\alpha a^{-6}$ during the regime when

\begin{equation}
\dot{\phi}^2 \gg m^2 \phi^2.
\end{equation}

Of course, one may with good reason ask as to whether our set of initial conditions on $\{\phi, \dot{\phi}\}$ is not too small. The answer of
this question depends entirely on the unknown behaviour of present
dark energy at large temperatures and curvatures when the form of
potential given by

\[(1.46) \quad V(\phi) = V_0 - \frac{m^2 \phi^2}{2},\]

need not be valid. However, it should be noted that, generally
speaking, the possibility of such initial conditions will be strongly
enhanced if there are many maxima in the dark energy potential. It
occurs, for example, in the dark energy model with

\[(1.47) \quad V(\phi) = V_0 \cos(\phi/f) + V_1,\]

as presented by Frieman, Hill, Stebbins and Waga (1995),
may numerically integrate the eqs. (1.34) - (1.35) with the potential
by eq. (1.46) and obtain the scale factor \(a(t)\) and the Hubble
parameter

\[(1.48) \quad H(z) = \dot{a}/a,\]

as functions of the initial field displacement \(\phi_0\) for specific values
of \(m\) and \(\Omega_{\text{de}}\). The model is confronted against high redshift type
Ia supernova observations through its luminosity distance
which is employed to place constraints on the free parameters of the model, namely $m$ and $\phi_0$. $V_0$ is not free parameter since it is obtained by $m, \phi_0, \Omega_{om}$ and the current value of the Hubble parameter $H_0$.

After determining the permitted range of parameters values, one may proceed to obtain the future evolution of the Universe. As presented by Starobinsky (2000), reliable future predictions are only possible for finite intervals of time. Previous predictions had a depth of approximately 20 Gyrs. It is entirely reasonable to use the form (1.46) up to the point when $V_0 = m^2 \phi^2$. It occurs when the potential has declined to half its maximum value. One may also consider a potential which may become negative with the form

\begin{equation}
(1.50) \quad V = V_0 \cos \phi / f ,
\end{equation}

where

\begin{equation}
(1.51) \quad f = \frac{\sqrt{V_0}}{m} ,
\end{equation}
with the value of \( f \) such that the potential (1.50) coincides with (1.46) at small \( \phi \). Of course, the hypothesis that \( V \) may acquire negative values is speculative since it does not follow from any of the current observational data. However, such potentials often arise in supergravity and M-theory models. For this potential one may obtain very easily (i) the minimal extent of the current acceleration epoch, and (ii) the time elapsed before the Universe collapses. In fact the collapse of the Universe is a generic feature of flat cosmological models with negative potentials. Hence, the possibility that the dark energy responsible for the acceleration of the Universe decays with time.

### 1.3 The Steady State Theory:

In 1948, three astronomers Hermann Bondi, Thomas Gold and Fred Hoyle proposed an entirely new approach to cosmology. Around the same time George Gamow was initiating detailed studies of the physical properties of the Universe close to the big bang epoch. Such model now known as the steady state model and does not possess a singular big bang type epoch. In fact it does not possess either a beginning or an end on the cosmic time axis. Let us
enquire about the motivation to propose the steady state cosmology. In 1948 the measured value of $T_0$ was

$$ T_0 = H_0^{-1} \approx 1.8 \times 10^9 \text{ years}. $$

Hence, the age of earth in view of a standard Friedmann model could not exceed $T_0$. Therefore a primafacie case came in picture for doubting the conclusion that Universe began \(~1\) to 1.8 billion years ago. How matter and radiation we observe around us came into existence? The assumption that the laws of physics remained unchanged is not a verifiable fact. The problem of singularity and matter creation still remain with the standard models. Hoyle approach is based on the problem of matter creation. The cosmological principle provides that at any instant i.e. at any given cosmic time t, all fundamental observers observe the same large-scale properties of the Universe, but it does follow far enough. But the cosmological principle does not allow the assumption that the laws remain unchanged with time. Hence, Bondi and Gold presented the perfect cosmological principle. It states that in addition to the symmetries implicit in cosmological principle, the Universe is unchanging in the large scale with time.
Hence, the geometrical and physical properties of the hypersurfaces \( t = \text{constant} \) do not change with time.

The constancy of Hubble parameter \( H \) gives

\[
H = \frac{\dot{a}}{a} = \text{constant} = H_0
\]

or

\[
a = \exp(H_0 t).
\]

Again the curvature of a \( t = \text{constant} \) hypersurface reads as \( k/a^2 \), and it is different for different times unless \( k = 0 \). Hence, one obtains unique metric in view of the perfect cosmological principle

\[
ds^2 = c^2 dt^2 - e^2 H_0 t \\
\left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right].
\]

It is an important to note that we have obtained the metric (1.55) of the steady state Universe without solving any field equations, as to obtain \( a(t) \) and \( k \) in standard cosmology. Bondi and Gold obtained this result as an example of the deductive power of the perfect cosmological principle. There are two more deductions
from perfect cosmological principle i.e., expansion of the Universe and the creation of matter.

The line element (1.55) is completely characterised by $H_0$. Hence, one may put $H_0 = 0, H_0 < 0$, and $H_0 > 0$, all are consistent with perfect cosmological principle. In view of local thermodynamic conditions, one may deduce that $H_0 > 0$. If we take $H_0 < 0$ we would have a contracting Universe. Hence, the Universe must expand, i.e., $H_0 > 0$.

The second case is for creation of matter. Let us consider a proper three volume $V$ bounded by fixed $(r, \theta, \phi)$ coordinates which increases with time

$$V \propto \exp 3H_0 t,$$

(1.56)

$$\frac{\dot{V}}{V} = 3H_0.$$  (1.57)

In view of steady state postulate the density of the Universe must be constant at $\rho = \rho_0$. Hence, the amount of matter within volume $V$ must increase in mass $M$

$$M = V \rho_0.$$  (1.58)
or

\begin{equation}
\dot{M} = 3H_0 V \rho_0.
\end{equation}

Hence,

\begin{equation}
Q = 3H_0 \rho_0
\end{equation}

where \( Q \) denotes the rate of creation of matter per unit volume. In e.g.s. units one obtains

\begin{equation}
Q = 2 \times 10^{-46} \left( \frac{\rho_0}{\rho_c} \right) h_0^3 \text{gcm}^{-3} \text{s}^{-1}
\end{equation}

where

\begin{equation}
H_0 = h_0 \times 100 \text{Kms}^{-1} \text{Mpc}^{-1},
\end{equation}

\begin{equation}
0.5 < h_0 \leq 1,
\end{equation}

\begin{equation}
\rho_c = 2 \times 10^{-29} h_0^2 \text{gcm}^{-3}.
\end{equation}

The value of \( Q \) is very small which shows that there is a very slow but continuous creation of matter going on, in contrast to the explosive creation at \( r = 0 \) of the standard models.

1.4 **The Creation Field:**
The above deductive approach is though more attractive but it has its limitations. For example, there is no quantitative relation between $H_0$ and the mean density $\rho_0$ as one has Friedmann cosmologies. Also we do not have any physical theory for such an important phenomenon as the slow but continuous creation of matter. The law of conservation of matter and energy being violated during the process of matter creation. Bondi and Gold appreciated the fact that questions like these could be answered through a dynamical theory rather than their deductive approach. Hence, they attached a more importance to test the perfect cosmological principle by observations than to present a dynamical theory to obtain $H_0$, $\rho_0$, and so on quantitatively. On the other hand Hoyle argued for a field theory to account for the phenomenon of primary creation of matter. In early 1960's Hoyle and Narlikar by using the formulation of Pryce, obtained a field theory for creation i.e. C-field theory.

Hoyle and Narlikar introduced a scalar field with zero mass and zero charge, showing that C-field theory involves more terms to the standard Einstein-Hilbert action to represent the phenomenon
of the creation of matter, by denoting the field by $c$ and its
derivative with respect to the spacetime coordinate $x^i$ by $C_i$.

\begin{equation}
A = \frac{C^3}{16\pi G} \int R (-R)^{\frac{1}{2}} d^4 x = \sum_{a} m_a c \int dS_a
\end{equation}

\begin{equation}
- \frac{1}{2c} f \int C_i C^i (-g)^{\frac{1}{2}} d^4 x + \sum_{a} \int C_i da^i.
\end{equation}

It is to be noted that the last term in the above equation is
path-independent to create difference between two interactions. Let
us consider the world line $a$ of particle between end points $A_1$ and
$A_2$, one obtains

\begin{equation}
\int_{A_1}^{A_2} C_i da^i = C(A_2) - C(A_1).
\end{equation}

A theory that presents creation or annihilation of matter per
second must have world lines with finite beginnings or ends or
both. The C-field interaction term picks out precisely these end
points of particle world lines. If we vary the world line of $a$ and by
considering the change in the action $A$ in a volume containing the
point $A_1$ where the world line begins then we get $A_1$

\begin{equation}
m_a c \frac{da^i}{dS_a} g_{ik} - C_k = 0,
\end{equation}
The above equation gives that overall energy and momentum are conserved at the creation point. The 4-momentum of the created particle is compensated by the 4-momentum of the C-field. In order to obtain this balance the C-field must have negative energy. The equation of motion of a be still that of a geodesic

\[ m_a \left[ \frac{d^2 a^i}{dS_a^2} + \Gamma^i_{kl} \frac{da^k}{dS_a} \frac{da^l}{dS_a} \right] = 0. \]  

(1.68)

The constant \( f \) denotes as coupling constant. The variation of \( C \) provides the source equation of the form

\[ C^k; k = cf^{-1} n, \]  

(1.69)

where \( \bar{n} \) be the number of net creation events per unit proper 4-volume. In order to evaluate \( \bar{n} \), one uses a + sign to the point \( A_1 \) where the world line begins and a - sign to the point \( A_2 \) where the world line ends. The eq. (1.69) gives the relation between C-field and the creation/annihilation events. At last, the variation of \( g_{ik} \) gives the modified Einstein field equations

\[ R^{ik} - \frac{1}{2} g^{ik} R = -\frac{8\pi G}{c^4} \left( T^{ik}_{(m)} + T^{ik}_{(C)} \right), \]  

(1.70)

where \( T^{ik}_{(m)} \) denotes the matter tensor while
Again for $f > 0$, we get

\[(1.72) \quad T^{00}_{(C)} < 0.\]

Hence, the C-field has negative energy density that gives a repulsive gravitational effect. It is this repulsive force that drives the expansion of the Universe. In view of the Robertson-Walker metric with the assumption that a typical particle created by C-field with mass $m$, one obtains

\[(1.73) \quad \dot{C} = mc^2,\]

\[(1.74) \quad mf \left( \ddot{C} + 3 \frac{\dot{a}}{a} \dot{C} \right) = \left( \dot{\rho} + \frac{\dot{a}}{a} \rho \right) c^2,\]

\[(1.75) \quad 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2 + kc^2}{a^2} = \frac{4\pi G f}{c^4} \dot{C}^2,\]

\[(1.76) \quad 3 \left( \frac{\dot{a}^2 + kc^2}{a^2} \right) = 8\pi G \left( \rho - \frac{f}{2c^4} \dot{C}^2 \right).\]

One may easily verify the steady state solutions from above equations i.e.

\[(1.77) \quad k = 0,\]
gives

\[ a = eH_0 t, \]

\[ \rho = \rho_0 = \frac{3H_0^2}{4\pi G} = fm^2. \]

Although the C-field was introduced primarily to account for the continuous creation of matter, but Narlikar (1973) presented that it also describes explosive matter creation such as is required in the big bang cosmology.

### 1.5 Cold dark matter Halos:

Numerical simulations and analytic evaluations both provide that the density profiles of dark matter halos may contain very interesting and useful informations regarding the cosmological parameters of the Universe and the power spectrum of initial density fluctuations. The secondary infall models proposed by Gunn and Gott (1972) proved that gravitational collapse could lead to the formation of virialised systems with almost isothermal density profiles. Fillmore and Goldreich (1984), Bert-schinger (1985) obtained a cosmological significance for the observation that the rotation curves of galactic disks are flat. Hoffman and
Shaham (1985) and Hoffman (1988) observed that the structure of halos should also depend on the value of density parameter $\Omega$ and the power spectrum of initial density fluctuations. These authors studied scale-free spectra

\begin{equation}
P(k) \propto k^n \quad (-3 < n < 4),
\end{equation}

and obtained that halo density profile should steepen for larger values of $n$ and lower values of $\Omega$. For

\begin{equation}
-2 < n < -1 \quad \text{and} \quad \Omega \leq 1,
\end{equation}

approximately flat circular velocity curves were obtained. These observations were confirmed by N-body experiments by Frenk et al. (1985, 1988), Quinn, Salmon and Zurek (1986), Efstathiou et al. (1988b), Zurek, Quinn and Salmon (1988), Blumenthal et al. (1984), Davis et al. (1985) obtained the cold dark matter scenario for $n_{\text{eff}} \leq -1.5$.

It has become customary to model virialized halos by isothermal spheres characterised by two parameters: a velocity dispersion and a core radius. For galaxies, the halo velocity dispersion or its circular velocity, defined by
is often assumed to be directly proportional to the characteristic velocity of the observed galaxy. Such assumption allows observations to be compared directly to the results of cosmological N-body simulations or of analytic models for galaxy formation, as presented by Frenk et al (1988), White and Frenk (1991), Cole (1991), Lacey et al (1993), Kauffmann, White and Guideroni (1993), Cole et al (1994). However this simple hypothesis does not seem to be supported by observation. If more massive halos were indeed associated with faster rotating disks and so with brighter galaxies, a correlation would be expected between the luminosity of binary galaxies and the relative velocity of their components. Similarly, there should be a correlation between the velocity of a satellite galaxy relative to its primary and the rotation velocity of the primary's disk. No such correlations are apparent in existing data as shown by White et al (1983), Zaritsky et al (1993). A possible explanation may come from the work of Persic and Salucci (1991), who argue that halo circular velocity is only weakly related to disk rotation speed.
Observational estimates of the core radii of dark halos have also led to conflicting results. Large core radii have been advocated inorder to accommodate the contribution of the luminous component to the rotation curves of disk galaxies as presented by Barnes and White (1984), White (1984), Blumenthal et al (1986), Flores et al (1993), in order to account for the shape of the rotation curves of dwarf galaxies as shown by Flores and Primack (1994), Moore (1994), and in order to reconcile X-ray cluster cooling flow models with observations by Fabian and Nulsen (1991). On the other hand, the giant are produced by gravitational lensing of background galaxies by galaxy clusters require cluster core radii to be small as suggested by Soucail and Mellier (1994).

The very existence of a core i.e. a central region where the density is approximately constant has been challenged by recent high-resolution numerical work. N-body simulations of the formation of galactic halos and galaxy-cluster halos provide no firm evidence for the existence of a core beyond that imposed by numerical limitations as shown by Dubinski and Carlberg (1991). Warren et al (1992), Navarro, Frenk and White (1995c). These studies indicate that dark matter halos are not well approximated by isothermal spheres but rather have gently changing logarithmic
slopes as in the model proposed by Hernquist (1990) for elliptical galaxies

\[ \rho(r) \propto \frac{1}{r(1+r/r_s)^3}, \]

or that proposed by Navarro et al (1995c) for X-ray cluster halos

\[ \rho(r) \propto \frac{1}{r(1+r/r_G)^2}. \]

These profiles are singular although the potential and mass converge near the centre and possess a well defined scale in which the profile changes shape, the scale radius \( r_s \). Near the scale radius the profiles are almost isothermal, so these results are quite consistent with the lower resolution simulations. The possibility that dark matter profiles may diverge like \( r^{-1} \) near the centre has important consequences for the observational issues. Navarro, Frenk and White (1996) using N-body simulations investigated the structure of dark halos in the standard cold dark matter cosmology. Halos are excised from simulations of cosmologically representative regions and are resimulated individually at high resolution. They studied objects with masses ranging from those of dwarf galaxy halos to those of rich galaxy clusters. The spherically
averaged density profiles of all halos may be fitted over two decades in radius by scaling a simple universal profile. The characteristic over density of a halo, or equivalently its concentration, correlates strongly with halo mass in a way that reflects the mass dependence of the epoch of halo formation. Halo profiles are approximately isothermal over a large range in radii but are significantly shallower than $r^{-2}$ near the centre and steeper than $r^{-2}$ near the virial radius. Matching the observed rotation curves of disk galaxies requires disk mass-to-light ratios to increase systematically with luminosity. Further, it suggests that the halos of bright galaxies depend only weakly on galaxy luminosity and have circular velocities significantly lower than the disk rotation speed. This may explain why luminosity and dynamics are uncorrelated in observed samples of binary galaxies and of satellite/spiral systems.

For galaxy clusters, halo models are consistent both with the presence of giant arcs and with the observed structure of the intracluster medium, and they suggest a simple explanation for the disparate estimates of cluster core radii. Their results also highlight two shortcomings of the cold dark matter model. Cold dark matter halos are too concentrated to be consistent with the halo parameters inferred for dwarf irregulars, and the predicted abundance of galaxy
halos is larger than the observed abundance of galaxies. The first problem may imply that the core structure of dwarf galaxies was altered by the galaxy formation process, and second problem may imply that galaxies failed to form or remain undetected, in many dark halos.

The main conclusions, on the structure of cold dark matter halos by Navarro, Frenk and White (1996) are as follows:

The main conclusions, on the structure of cold dark matter halos by Navarro, Frenk and White (1996) are as follow:

(i) The density profiles of cold dark matter halos of all masses may be well fitted by an appropriate scaling of a universal profile with no free shape parameters. This profile is shallower than isothermal near the centre of a halo and steeper than isothermal in its outer regions.

(ii) The characteristic overdensities of halos, or equivalently their concentrations, correlate with halo mass in a way that may be interpreted as reflecting the different formation redshifts of halos of differing mass.
(iii) The observed rotation curves of disk galaxies are computable with this halo structure provided that the mass-to-light ratio of the disk increases with luminosity. This implies that the halos of bright spirals have masses that correlate only weakly with their luminosity and may explain why luminosity and dynamics appear uncorrelated in samples of binary galaxies and satellite/spiral pairs. Disks with rotation velocity in the range 200-300 Km s\(^{-1}\) are predicted to have halos with a typical mass of \(1.8 \times 10^{12} M_\odot\) within 300 Kpc. This agrees well with the masses inferred from the dynamics of observed satellite galaxy samples.

(iv) Cold dark matter halos seem too centrally concentrated to be consistent with observations of the rotation curves of dwarf irregulars as shown by Moore (1994), Flores and Primack (1994). This may imply that the central regions of dwarf galaxy halos were substantially altered during galaxy formation, for example by sudden baryonic outflows occurring after a burst of star formation as presented by Navarro et al (1995a).
(v) The fact that bright galaxies are surrounded by halos with mean circular velocity lower than the observed disk rotation velocity exacerbates the discrepancy between the number of galaxy halos predicted in an $\Omega = 1$ Universe and the observed number of galaxies.

(vi) The predicted structure of galaxy clusters is consistent both with X-ray observations and with the presence of giant gravitationally lensed arcs. Previous discrepant estimates of the core radius based on these two kinds of data probably result from force-fitting of an inappropriate potential structure.

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