Slip Flow and Magneto-NANOFLOWD over an Exponentially Stretching Permeable Sheet with Heat Generation/Absorption

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Abstract: The present study analyzes the steady boundary layer slip flow of magneto-nanofluid due to an exponentially permeable stretching sheet with heat generation/absorption. In this paper, the effects of Brownian motion and thermophoresis on heat transfer and nanoparticle volume fraction are considered. Using shooting technique along with fourth-order Runge-Kutta method the transformed equations are solved. The study reveals that the governing parameters, namely, the magnetic parameter, wall mass suction parameter, Prandtl number, the Lewis number, slip parameter, heat generation/absorption parameter, Brownian motion parameter, and thermophoresis parameter, have major effects on the flow field, the heat transfer, and the nanoparticle volume fraction as well as skin friction, local Nusselt number and local Sherwood number has been discussed in detail.

Keywords: MHD, nanoparticle, slip flow, heat transfer, heat generation/absorption.

I. Introduction

Among the tasks facing by the engineer is the development of ultrahigh-performance cooling in many industrial technologies. This is where nanotechnology takes important part for further development of high performance, compact, cost-effective liquid cooling systems. Other than that, nanofluids have effective applications in many industries such as electronics, transportation, biomedical and many more [1]. Nanotechnology has been an ongoing topic of discussion in public health as some of the researchers claimed that nanoparticles could present possible dangers in health and environment [2]. Hamid et al.[3] studied the problem of two-dimensional laminar Marangoni-driven boundary layer flow in nanofluids with the effects of radiation. Three different types of nanoparticles, namely Cu, Al₂O₃, and TiO₂ are considered.

During the last many years, the study of boundary layer flow and heat transfer over a stretching surface has achieved a lot of success because of its large number of applications in industry and technology. Few of these applications are materials manufactured by polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning etc. After the pioneering work by Sakiadis [4], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces [5-10]. Nadeem and Lee [11] investigated the steady boundary layer flow of nanofluid over an exponential stretching surface is investigated analytically.

Magnetohydrodynamics (MHD) boundary-layer flow of nanofluid and heat transfer over a linearly stretched surface have received a lot of attention in the field of several industrial, scientific, and engineering applications in recent years. Nanofluids have many applications in the industries since materials of nanometer size have unique chemical and physical properties. With regard to the sundry applications of nanofluids, the cooling applications of nanofluids include silicon mirror cooling, electronics cooling, vehicle cooling, transformer cooling, etc. This study is more important in industries such as hot rolling, melt spinning, extrusion, glass fiber production, wire drawing, and manufacture of plastic and rubber sheets, polymer sheet and filaments, etc. Khan et al. [12] studied the Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects using finite difference method. Liu [13] investigated the flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to a transverse magnetic field. Khan et al. [14] studied the viscoelastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work.

An effect of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet was investigated by Cortell [15]. Bhattacharyya [16] analyzes the boundary layer flow and heat transfer caused due to an exponentially shrinking sheet and Bhattacharyya and Pop [17] show the effect of external magnetic field on the flow over an exponentially shrinking sheet. Recently, Bhattacharyya and Layek [18] investigated the magnetohydrodynamic boundary layer flow of nanofluid over an exponentially stretching permeable sheet.

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The transient oscillatory MHD convection past a flat plate adjacent to a porous medium with heat generation effects was solved in [19] and transient MHD natural convection with viscous heating was considered by Zueco [20]. Recently the effects of variable suction and thermophoresis on steady MHD flow over a permeable inclined plate was analyzed by Alam et al. [21] while the heat and mass transfer of thermophoretic hydromagnetic flow with lateral mass flux, heat source, Reddy et al.[22] investigated the thermo diffusion and chemical reaction effects on unsteady free mhd convection flow past a vertical porous plate in slip-flow regime using perturbation technique. However, the interactions of magneto-nanofluid due to an exponentially permeable stretching sheet in the presence of heat generation/absorption and slip effects. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runge-Kutta method along with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, nanoparticle volume friction, local Nusselt number and local Sherwood number are shown in figures and analyzed in detail.

II. Mathematical Formulation

Consider the steady boundary layer flow of nanofluid over an exponentially stretching sheet in presence of a transverse magnetic field. The governing equations of motion and the energy equation may be written in usual notation as [18, 23, 24]

Continuity equation
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
\[\text{(2.1)}\]

Momentum equation
\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - \frac{\sigma B^2}{\rho_f} u
\]
\[\text{(2.2)}\]

Energy equation
\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\rho_f}{\rho_c} \nu \frac{\partial^2 T}{\partial y^2} + \left(\frac{\rho c}{\rho_f} - \frac{\rho c}{\rho_p}\right) \frac{\partial N}{\partial y} + \frac{D_B}{T_x} \left(\frac{\partial T}{\partial y}\right)^2 + q(T - T_\infty)
\]
\[\text{(2.3)}\]

Volumetric species equation
\[
\frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_B \frac{\partial^2 N}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}
\]  
\[\text{(2.4)}\]

The boundary conditions are
\[
\begin{align*}
u &= U_w + \frac{L}{\partial y} \frac{\partial u}{\partial y}, & v &= v_w, & T &= T_w = T_0 e^{\frac{x}{L}}, & N &= N_w = N_0 + N_\infty e^{\frac{x}{L}} \\
\Rightarrow u &\to 0, & T &\to T_\infty, & N &\to N_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]  
\[\text{(2.5)}\]

Where \(u\) and \(v\) are the velocity components in \(x\) and \(y\) directions, respectively, \(V\) is kinematic viscosity and \(\alpha\) is thermal conductivity. \(\rho_f\) is density of the base fluid, \(\left(\rho c\right)_f\) and \(\left(\rho c\right)_p\) are heat capacities of base fluid and nanoparticles, respectively. \(T\) is temperature, \(q\) is heat generation/absorption rate constant, \(D_B\) is Brownian diffusion coefficient, \(N\) is nanoparticle volumetric fraction, \(D_T\) is thermophoretic diffusion coefficient and \(T_\infty\) is the ambient fluid temperature. \(T_w\) is the variable temperature at the sheet with \(T_0\) being a constant which measures the rate of temperature increase along the sheet, \(N_w\) is the variable wall nanoparticle volume fraction with \(N_0\) being a constant, and \(N_\infty\) is constant nanoparticle volume fraction in free stream.
Here the variable magnetic field $B(x)$ is taken the form [17, 25]

$$B(x) = B_0 e^{\frac{x}{L}}$$

(2.6)

Where $B_0$ is the constant.

The stretching velocity $U_w$ is given by

$$U_w(x) = c e^{\frac{x}{L}}$$

(2.7)

where $c > 0$ is stretching constant. A physical model with the coordinate system of the problem is sketched in Fig. A.

Here $v_w$ is the variable wall mass transfer velocity and is given by

$$v_w(x) = v_0 e^{\frac{x}{L}}$$

(2.8)

where $v_0$ is a constant with $v_0 < 0$ for mass suction and $v_0 > 0$ for mass injection.

The continuity equation (2.1) is satisfied by the Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

(2.9)

where $\psi(x, y)$ is the stream function.

In order to transform the equations (2.2) to (2.5) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\psi = \sqrt{2\nu L c f(\eta)} e^{\frac{x}{2L}}, \eta = y \sqrt{\frac{c}{2\nu L}} e^{\frac{x}{2L}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}$$

$$Pr = \frac{\nu}{\alpha}, Le = \frac{\nu}{D_B}, M = \frac{2L \sigma B_0^2}{\rho c}, \beta = \frac{2qL}{c}, q = q_0 e^{x/L}$$

$$Nb = \frac{D_B (\rho c)_p (N_w - N_\infty)}{v (\rho c)_f}, Ni = \frac{D_T (\rho c)_p (T_w - T_\infty)}{T_\infty (\rho c)_f}$$

(2.10)
where \( f(\eta) \) is the dimensionless stream function, \( \theta \) - the dimensionless temperature, \( \phi \) - the dimensionless nanoparticle volume fraction, \( \eta \) - the similarity variable, \( M \) - the magnetic parameter, \( \text{Le} \) - the Lewis number, \( Nb \) - the Brownian motion parameter, \( \text{Nt} \) - the thermophoresis parameter, \( \text{Pr} \) - the Prandtl number.

In view of the equation (2.10), the equations (2.2) to (2.6) transform into

\[
\begin{align*}
\theta'' + \text{Pr}(f\theta' - f'\theta + Nb\theta'\phi' + Nt\theta'^2 + \beta) &= 0 \\
\phi'' + \text{Le}(f\phi' - f'\phi) + \frac{Nt}{Nb} \theta'' &= 0
\end{align*}
\]

The transformed boundary conditions can be written as

\[
\begin{align*}
f &= S, \quad f' = 1 + \gamma f''(0), \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0 \\
f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty
\end{align*}
\]

where \( S = -v_0/\sqrt{\nu c/2L} \) is the wall mass transfer parameter. \( S > 0(v_0 < 0) \) corresponds to mass suction and \( S < 0(v_0 > 0) \) corresponds to mass injection and \( \gamma = \frac{cL}{\sqrt{2\nu}} \) is the slip parameter.

Physical quantities of interest are Local skin friction coefficient \( C_f \), Local Nusselt number \( Nu \) and Local Sherwood number \( Sh \), defined as \[24\]

\[
C_f = \frac{\nu}{U^2 \text{Re}_{x}^{2/3}} \left( \frac{\partial u}{\partial y} \right)_{\eta=0}, \quad Nu = \frac{-x}{T_w - T_{\infty}} \left( \frac{\partial T}{\partial y} \right)_{\eta=0}, \quad Sh = \frac{-x}{(N_w - N_{\infty})} \left( \frac{\partial N}{\partial y} \right)_{\eta=0}
\]

or by introducing the transformations (2.10), we have

\[
\sqrt{2\text{Re}_s} C_f = f''(0), \quad \frac{Nu}{\sqrt{2\text{Re}_s}} = -\frac{x}{2L} \theta'(0), \quad \frac{Sh}{\sqrt{2\text{Re}_s}} = -\frac{x}{2L} \phi'(0)
\]

Where \( \text{Re}_s = \frac{U_w x}{\nu} \) is the local Reynolds number.

### III. Solution Of The Problem

The highly nonlinear coupled ODEs (2.11-2.13) along with the boundary conditions (2.14) form a two-point boundary value problem (BVP) and those are solved using shooting method \[26\text{–}27\]. The following first-order system is set:

\[
\begin{align*}
f' &= p, \quad p' = q \\
q' &= 2p^2 - fq + Mp \\
\theta' &= r \\
r' &= pr(fr - p\theta + Nb rz + Nt r'^2 + \beta \theta) \\
\phi' &= z \\
z' &= -le(fz - p\phi) - \frac{Nt}{Nb} r'
\end{align*}
\]

with the boundary conditions

\[
f(0) = S, \quad p(0) = 1 + \gamma q(0), \quad \theta(0) = 1, \quad \phi(0) = 1
\]

The set of nonlinear first-order ordinary differential equations (3.1) with boundary conditions (3.2) have been solved by shooting method using the fourth-order Runge-Kutta algorithm with a systematic guessing of \( q(0) \), that is, \( f''(0), \quad r(0), \) that is, \( \theta'(0), \) and \( z(0), \) that is, \( \phi'(0). \) The step size is taken as \( \Delta \eta = 0.01 \) and the suitable finite value of \( \eta \to \infty, \eta_{e}, \) is taken as 30 in all cases. The guess values \( f''(0), \quad \theta'(0), \) and \( \phi'(0) \) are adjusted using “secant method” to give better approximation for the solution. An asymptotic convergence criterion of \( 10^{-7} \) level for the boundary conditions \( f'(\eta_{e}), \quad \theta(\eta_{e}), \phi(\eta_{e}) \) is taken in the computation.
IV. Results And Discussion

In order to get a clear insight of the physical problem, the velocity, temperature and nanoparticle volume friction have been discussed by assigning numerical values to the governing parameters encountered in the problem. Numerical computations are shown from figs.1-13.

The velocity, temperature and nanoparticle volume friction are plotted in Fig. 1(a)-(c) for different values of the magnetic field parameter ($M$). As is now well known, the velocity of the fluid decreases with increases in the magnetic field parameter due to an increase in the Lorentz drag force that opposes the fluid motion. Also noticed that temperature and nanoparticle volume friction increases with an increasing the magnetic parameter. Figs. 2(a)-(c) shows the effect of the mass suction parameter on the velocity, temperature and mass volume fraction profiles. It is observed that the velocity, temperature and concentration of the fluid decrease with an increase the mass suction parameter. Due to mass suction, the fluid is brought closer to the sheet and it thins velocity boundary layer thickness as well as the thermal and nanoparticle volume boundary layer thicknesses. Opposite effect is found for mass injection case; that is, the fluid is taken away from the sheet. Consequently, the velocity, thermal, and nanoparticle volume boundary layer thicknesses become broader. Figs. 3(a)-(c) shows the effect of the slip parameter on the velocity, temperature and mass volume fraction profiles. It is observed that the velocity of the fluid decreases with an increase the slip parameter and temperature of the fluid as well as mass volume fraction of the fluid increases the slip parameter.

The effect of thermophoresis parameter on temperature and mass volume friction is shown in figs. 4(a) & (b). An increase in $Nt$ the temperature of the fluid is increases as well as mass volume friction. The effect of Brownian motion parameter $Nb$ on the dimensionless temperature and the dimensionless nanoparticle volume fraction is plotted in Figure 5(a) & 5(b). The figure reveals that the temperature of the fluid increases and the nanoparticle volume fraction decreases with increasing values of $Nb$. In nanofluid system, due to the presence of nanoparticles, the Brownian motion takes place and for the increase in $Nb$ the Brownian motion is affected and consequently the heat transfer characteristic of the fluid changes. Also, when the value of $Nb$ increases, the nanoparticle volume boundary layer thickness decreases. The effect of heat generation/absorption parameter on the dimensionless temperature and the dimensionless nanoparticle volume fraction is plotted in Figure 6(a) & 6(b). The figure reveals that the temperature and the nanoparticle volume fraction with increasing values of heat generation/absorption. The effect of the Prandtl number ($Pr$) on temperature is shown in fig.7. Since the Prandtl number is the ratio of momentum diffusivity to the nanofluid thermal diffusivity. It is noticed that temperature of the fluid decreases with increases the Prandtl number. From fig. 8 show that the effect of Lewis number ($Le$) on temperature. Since Lewis number is the ratio of nanoparticle thermal diffusivity to Brownian diffusivity. It is observed that the temperature of the fluid decreases with an increase in the Lewis number.

Fig.9 shows the effects of $S$, $\gamma$, and $M$ on skin friction. From fig.9 it is seen that the skin friction increases with an increase $M$ or $S$ and decrease with an increase $\gamma$. The variations of $S$, $\gamma$ and $M$ on reduced Nusselt number is shown in fig.10. It is observed that the reduced Nusselt number increases with an increase the parameter $S$ and decrease with an increasing the parameters $\gamma$ or $M$. The effect of $S$, $\gamma$ and $M$ on Sherwood number is shown in fig.11. It is found that the Sherwood number enhances with a decrease in the parameters $\gamma$ or $M$ whereas increases with $S$. The variations of $Nt$ and $Nb$ on reduced Nusselt number is shown in fig.12. It is observed that the reduced Nusselt number decrease with an increasing the parameters $Nt$ or $Nb$. The effect of $Nt$ and $Nb$ on Sherwood number is shown in fig.13. It is found that the Sherwood number enhances with a increase in the parameter $Nb$ whereas decreases with $Nt$. Table 1 is shows to compare our results for the viscous case in the absence of the parameter $M$, $\gamma$ and $S$. These results are found to be in good agreement.

V. Conclusions

In this paper numerically investigated the nanoparticle effect on magnetohydrodynamic boundary layer flow of Williamson fluid over a stretching surface with slip effect. The important findings of the paper are:

- The velocity of the fluid decreases with an increase of the magnetic field.
- The fluid temperature increases with the influence of magnetic field parameter or slip parameter or heat generation/absorption parameter.
- The nanoparticle volume friction enhances the magnetic field or thermophoresis parameter or Brownian parameter.
- Due to mass suction, the velocity boundary layer thickness as well as the thermal and nanoparticle volume boundary layer thicknesses becomes thinner, whereas mass injection makes those thicker.
- The skin friction coefficient and local Nusselt number and local Sherwood number enhances the mass suction parameter.
- The skin friction coefficient and local Nusselt number and local Sherwood number reduces the slip parameter.
- The local Nusselt number and local Sherwood number reduces the Brownian parameter.
Fig. 1 (a) Velocity for different values of $M$

Fig. 1 (b) Temperature for different values of $M$

Fig. 1 (c) Nan particle volume fraction for different values of $M$
Fig. 2 (a) Velocity for different values of $S$

Fig. 2 (b) Temperature for different values of $S$

Fig. 2 (c) Nanoparticle volume fraction for different values of $S$
Fig. 3(a) Velocity for different values of $\gamma$

Fig. 3(b) Temperature for different values of $\gamma$

Fig. 3(c) Nanoparticle volume fraction for different values of $\gamma$
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Fig. 4 (a) Temperature for different values of $N_t$

Fig. 4 (b) Nanoparticle volume fraction for different values of $N_t$

Fig. 5 (a) Temperature for different values of $N_b$
Fig. 5 (b) Concentration for different values of Nb

Fig. 6 (a) Temperature for different values of \( \beta \)

Fig. 6 (b) Concentration for different values of \( \beta \)
Fig. 7 Temperature for different values of $Pr$

Fig. 8 Nanoparticle volume fraction for different values of $Le$

Fig. 9 Effect of $S$, $\gamma$ and $M$ on the reduced skin friction
Fig. 10 Effect of $S$, $\gamma$ and $M$ on the reduced Nusselt number

Fig. 11 Effect of $S$, $\gamma$ and $M$ on the reduced Sherwood number

Fig. 12 Effect of $Nb$ and $Nt$ on the reduced Nusselt number
Fig. 13 Effect of $Nb$ and $Nt$ on the reduced Sherwood number

Table 1 Comparison for $-f''(0)$ and $f(\infty)$ for $M=S=\gamma=0$ and $\beta=0$

<table>
<thead>
<tr>
<th></th>
<th>Present Study</th>
<th>Magyari and Keller [28]</th>
<th>Bhattacharyya and Layek [18]</th>
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<tr>
<td>$-f''(0)$</td>
<td>1.28181</td>
<td>1.281808</td>
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<td>$f(\infty)$</td>
<td>0.905641</td>
<td>0.905639</td>
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REFERENCES


Slip Flow And Magneto-Nanofluid Over An Exponentially Stretching Permeable Sheet With Heat


Heat source/sink effects of heat and mass transfer of magneto-nanofluids over a nonlinear stretching sheet

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ABSTRACT

This study investigates theoretically the problem of free convection boundary layer flow of nanofluids over a nonlinear stretching sheet in the presence of MHD and heat source/sink. The governing partial differential equations are transformed to a system of ordinary differential equations and solved numerically using fourth order Runge-Kutta method along with shooting technique. The effects of the magnetic parameter, the Prandtl number, Lewis number, power law velocity parameter, the Brownian motion parameter, thermophoresis parameter, heat source/sink parameter on the fluid properties as well as on the heat and mass transfer coefficients are determined and shown graphically.

Keywords: nanofluid, MHD, heat source/sink, Brownian motion, thermophoresis, nonlinear stretching sheet.

INTRODUCTION

The study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many applications in the industry since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. For example, a small amount (<1% volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil is reported to increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively [1,2]. Conventional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such enhancement. However, problems of rheology and stability are amplified at high concentrations, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Other perplexing results in this rapidly evolving field include a surprisingly strong temperature dependence of the thermal conductivity [3] and a three-fold higher critical heat flux compared with the base fluids [4,5]. These enhanced thermal properties are not merely of academic interest. If confirmed and found consistent, they would make nanofluids promising for applications in thermal management. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology. Bachok et al. [6] studied boundary layer flow of a nanofluid past a moving plate in a uniform free stream. In another investigation, Bachok et al. [7] discussed heat transfer over a permeable stretching sheet for the unsteady boundary layer flow of a nanofluid. Kandasamy et al. [8] studied scaling group transformations for an electrically conducting nanofluid flow over a vertical stretching sheet. Tsou et al. [9] reported both analytical and experimental results for the flow and heat transfer in the boundary layer on a continuously
moving surface. Gorla et al. [10] studied the natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a nanofluid.

The magnetohydrodynamic (MHD) flow has attracted a great interest to many researchers during the last several decades owing to the effect of magnetic field on the boundary layer flow control and applications in many engineering and physical aspects such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. Flow characteristics and heat transfer over a stretching sheet have been studied extensively by many researchers in the recent past because of many engineering applications of stretching sheet in manufacturing processes such as hot rolling, wire drawing, drawing of plastic films and artificial fibers, metal extrusion, crystal growing, continuous casting, glass fiber production and paper production. Metallurgy lies in the purification of molten metal’s from non-metallic inclusions by applying magnetic field is other application of MHD. Chen [11] studied the effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power law stretched sheet with surface heat flux. The magnetohydrodynamic (MHD) forced convection boundary layer flow of nanofluid over a horizontal stretching plate was investigated by Nourazar et al. [12] using homotopy perturbation method (HPM). Matin et al. [13] studied entropy analysis in mixed convection MHD flow of nanofluid over a nonlinear stretching sheet. Hayat et al. [14] used homotopy analysis method (HAM) to investigate the MHD boundary layer flow and examine the heat transfer analysis for the permeable stretching sheet under varied conditions. In this continuation, Abbas and Hayat [15] studied the stagnation point flow with slip effects in heat transfer analysis for the boundary layer flow over a non-linear stretching sheet. Aman and Ishak [16] presented the hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet with prescribed surface heat flux.

The boundary layer flows of non-Newtonian fluids over a stretching sheet with heat and mass transfer are important in several areas such as extrusion process, glass fiber, paper production, hot rolling, wire drawing, electronic chips, crystal growing, plastic manufactures, and application of paints, food processing and movement of biological fluids (Hayat and Qasim, [17]). Bhargava et al. [18] discussed heat and mass transfer of boundary layer flow over a non-linear stretching sheet under the effects of different physical parameters. Khan and Pop [19] investigated the laminar fluid flow of a nanofluid from the stretching flat surface by incorporating the effects of Brownian motion, thermophoresis and reported to be the pioneer for this study of stretching sheet in nanofluid. Pal and Mondal [20] examined the heat and mass transfer over a stretching sheet by considering the effects of buoyancy and solutal buoyancy parameters. Anwar et al. [21] studied the conjugate effects of heat and mass transfer of nanofluids over a nonlinear stretching sheet.

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Vajravelu and Hadjinicalaoou [22] have studied on hydrodynamic convective heat transfer from a stretching surface with heat generation/absorption. Molla et al. [23] studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation are studied by Samad et al. [24]. Recently, Rahman et al. [25] investigated the thermophoresis effect on MHD forced convection on a fluid over a continuous linear stretching sheet in presence of heat generation and Power-Law wall temperature.

However, the interaction of conjugate effects of nanofluids over a nonlinear stretching sheet, has received little attention. Hence, the present study an attempt is made to analyze the steady conjugate effects of heat and mass transfer of nanofluid flow over a nonlinear stretching sheet in the presence of MHD and heat source/sink. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runge-Kutta method along with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, concentration, skin friction, reduced Nusselt number and reduced Sherwood number are shown in figures and analyzed in detail.

2. MATHEMATICAL FORMULATION

The steady two-dimensional boundary layer flow of a nanofluid past a stretching surface is considered under the assumptions that the external pressure on the plate in x-direction is having diluted nanoparticles. The stretching velocity is assumed to be \( U_0(x) = U_0 x^m \), where \( U_0 \) is the uniform velocity and \( m (m \geq 0) \) is a constant parameter and \( x \) is the coordinate measured along the stretching surface. The flow takes place above the stretching surface at \( y \geq 0 \). Here, \( y \) is the coordinate axis measured normal to the stretching surface. A uniform stress leading to equal and opposite forces is applied along the \( x \)-axis so that the sheet is stretched, keeping the origin fixed. A uniform magnetic field of strength \( B(x) \) is applied normal to the Sheet. It is assumed that at the stretching surface,
the temperature $T$ and the nanoparticle fraction $C$ take constant values $T_w$ and $C_w$ whereas the ambient values of temperature $T_\infty$ and the nanoparticle fraction $C_\infty$ are attained as $y$ tends to infinity. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, governing the flow field are given by

Continuity equation
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(2.1)

Momentum equation
\[
\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \rho_f \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \sigma B^2 u \\
+ \left[ (1 - C_\infty) \rho_{fc} \beta_f (T - T_\infty) - \left( \rho_f - \rho_{fc} \right) \beta_C (C - C_\infty) \right] g
\]  

(2.2)

Energy equation
\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + D_L \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{q}{\rho P C_p} (T - T_\infty)
\]  

(2.3)

Species equation
\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + D_L \frac{\partial^2 T}{\partial y^2}
\]  

(2.4)

The boundary conditions for the velocity, temperature and concentration fields are
\[
u = U_w(x) = U_0 x^m, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0
\]  
\[
u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad \hat{y} \to \infty
\]  

(2.5)

where $u$ and $v$ are the velocity components along the $x$ and $y$ axes, respectively, $g$ is the acceleration due to gravity, $\mu$ is the viscosity, $\rho_f$ is the density of the base fluid, $\rho_{fc}$ is the density of the nanoparticle, $\beta_f$ is the coefficient of the volumetric thermal expansion, $\beta_C$ is the coefficient of the volumetric concentration expansion, $\left( \rho c \right)_f$ and $\left( \rho c \right)_p$ are the heat capacity of the fluid and the effective heat capacity of the nanoparticle material respectively, $\nu = \mu / \rho_f$ is the kinematic viscosity of the fluid, $k$ is the thermal conductivity, $B(x) = B_0 x^{m-1}$ is the magnetic field of constant strength, where $B_0$ is constant, $\alpha = k / \left( \rho c \right)_f$ is the thermal diffusivity parameter, $D_B$ is the Brownian diffusion coefficient, $\tau$ is a parameter defined by $\left( \rho c \right)_f / \left( \rho c \right)_p, q = q_0 x^{m-1}$ is the heat source/sink parameter, where $q_0$ is any constant.

The continuity equation (2.1) is satisfied by the Cauchy Riemann equations
\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}
\]  

(2.6)

where $\psi(x, y)$ is the stream function.

In order to transform the equations (2.2) to (2.5) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.
\[
\psi = \sqrt{\frac{2vU(x) \eta}{m+1}}, \eta = \sqrt{m(m+1)U(x) \eta}, \theta(\eta) = \frac{T - T_w}{T_w - T_m}, \phi(\eta) = \frac{C - C_w}{C_m - C_w}
\]

\[
M = \frac{\sigma B^2}{\rho U_0}, \lambda = \frac{Gm}{Re^{1/2}}, \delta = \frac{Pr}{\alpha}, Le = \frac{v}{D_n}, \quad Nb = \frac{\tau B^2(C_w - C_m)}{v}, \quad Nt = \frac{\tau D_v(T_w - T_m)}{T_w}, \quad Re = \frac{U_1(x) \eta}{v}, \quad Q = \frac{q_0}{U_0 \rho_f c_p}
\]

\[
Gr = \frac{(1 - C_w)(\rho_f - \rho)}{\nu^2 Re^{1/2}}, \quad Gm = \frac{\left(\frac{\rho_f - \rho}{\rho_f}\right) g n T_w - T_m}{\nu^2 Re^{1/2}}
\]

where \( f(\eta) \) is the dimensionless stream function, \( \theta \) - the dimensionless temperature, \( \phi \) - the nanoparticle volume friction, \( \eta \) - the similarity variable, \( Nb \) - the Brownian motion parameter, \( Nt \) - the thermophoresis parameter, \( Pr \) - the Prandtl number, \( Q \) - heat source/sink parameter, \( \lambda \) - thermal buoyancy parameter, \( \delta \) - solutal buoyancy parameter, \( Re \) - the local Reynolds number based on the stretching velocity, \( M \) - the magnetic parameter, \( Le \) - the Lewis number, \( Gr \) - local thermal Grashof number, \( Gm \) - local solutal Grashof number. Here, \( \beta_f \) and \( \beta_C \) are proportional to \( x^{-3} \), that is \( \beta_f = n x^{-3}, \beta_C = n_1 x^{-3} \), where \( n \) and \( n_1 \) are the constant of proportionality (Makinde and Olenrewaju [26]).

In view of the equations (2.6) and (2.7), the equations (2.2) to (2.5)) transform into

\[
f'''' + f f'' - \frac{2m}{m+1} f'' + \frac{2}{m+1}(\lambda \theta - \delta \phi - Mf') = 0
\]

\[
\frac{1}{Pr} \theta'' + f \theta' + Nb \theta' \phi' + Nt \theta'' + \frac{2}{m+1}Q \theta = 0
\]

\[
\phi'' + Lef \phi' + \frac{Nt}{Nb} \theta'' = 0
\]

The transformed boundary conditions can be written as

\[
f = 0, f' = 1, \theta = 1, \phi = 1 \quad \text{at} \quad \eta = 0
\]

\[
f' \to 0, \theta \to 0, \phi \to 0 \quad \text{as} \quad \eta \to \infty
\]

The skin-friction, Nusselt number and Sherwood number for the present problem of nanofluid are defined as

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2}, Nu = \frac{q_w x}{k(T_w - T_m)}, Sh = \frac{q_m x}{k(C_w - C_m)}
\]

where

\[
q_w = -k \frac{\partial T}{\partial y}, q_m = -D_B \frac{\partial C}{\partial y}, \tau_w = \mu \frac{\partial u}{\partial y} \quad \text{at} \quad y=0
\]

The associated expressions of dimensionless skin-friction – \( f''(0) \), reduced Nusselt number – \( \theta'(0) \) and reduced Sherwood number – \( \phi'(0) \) defined as
\[ C_{fx} = \frac{C_f}{2} \sqrt{\frac{2}{m+1}} \text{Re}_x, \quad Nu_x = \frac{Nu}{\sqrt{\frac{m+1}{2}} \text{Re}_x}, \quad Sh_x = \frac{Sh}{\sqrt{\frac{m+1}{2}} \text{Re}_x} \]  

(2.13)

3 SOLUTION OF THE PROBLEM

The set of non-linear coupled differential Eqs. (2.8)-(2.10) subject to the boundary conditions Eq. (2.11) constitute a two-point boundary value problem. In order to solve these equations numerically we follow most efficient numerical shooting technique with fifth-order Runge-Kutta-integration scheme. In this method it is most important to choose the appropriate finite values of \( \eta \rightarrow \infty \). To select \( \eta_{\infty} \) we begin with some initial guess value and solve the problem with some particular set of parameters to obtain \( f'', \theta' \) and \( \phi' \). The solution process is repeated with another large value of \( \eta_{\infty} \) until two successive values of \( f'', \theta' \) and \( \phi' \) differ only after desired digit signifying the limit of the boundary along \( \eta \). The last value of \( \eta_{\infty} \) is chosen as appropriate value of the limit \( \eta \rightarrow \infty \) for that particular set of parameters. The three ordinary differential Eqs. (2.8)-(2.10) were first formulated as a set of seven first-order simultaneous equations of seven unknowns following the method of superposition \([27]\). Thus, we set

\[
\begin{align*}
y_1 &= f, \quad y_2 = f', \quad y_3 = f'', \quad y_4 = \theta, \quad y_5 = \theta', \quad y_6 = \phi, \quad y_7 = \phi' \\
y_1'(0) &= y_2(0) = 1 \\
y_3'(0) &= \delta_1 \\
y_4' &= y_5(0) = 1 \\
y_5' &= -Pr \left[ y_1 y_5 + Nb y_5 y_7 + Nt y_5^2 + \frac{2}{m+1} Q y_4 \right] \\
y_6' &= y_7(0) = 1 \\
y_7'(0) &= \delta_3 \\
\end{align*}
\]

Eqs. (2.8)-(2.11) then reduced into a system of ordinary differential equations, i.e., where \( \delta_1, \delta_2 \) and \( \delta_3 \) are determined such that it satisfies \( y_2(\infty) \rightarrow 0, \quad y_4(\infty) \rightarrow 0 \) and \( y_6(\infty) \rightarrow 0 \). The shooting method is used to guess \( \delta_1, \delta_2 \) and \( \delta_3 \) until the boundary conditions \( y_2(\infty) \rightarrow 0, \quad y_4(\infty) \rightarrow 0 \) and \( y_6(\infty) \rightarrow 0 \) are satisfied. Then the resulting differential equations can be integrated by fifth-order Runge-Kutta integration scheme. The above procedure is repeated until we get the results up to the desired degree of accuracy, \( 10^{-6} \).

RESULTS AND DISCUSSION

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the governing parameters encountered in the problem. Numerical computations are shown from figs.1-11.

The velocity for different values of the magnetic field parameter \((M)\) is shown in fig.1. As is now well known, the velocity decreases with increases in the magnetic field parameter due to an increase in the Lorentz drag force that opposes the fluid motion. Figs. 2(a) to 2(c) are shows the effect of the thermal buoyancy parameter on the velocity, temperature and concentration profiles. The thermal buoyancy parameter signifies the relative effect of the thermal

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buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal buoyancy parameter, i.e. free convection effects. It is noticed that the thermal buoyancy parameter influence the velocity field almost in the boundary layer when compared to far away from the plate. It is seen that as the thermal buoyancy parameter increases, the velocity field increases. Also noticed that temperature and concentration profiles decreases with an increasing the thermal buoyancy parameter. Figs. 3(a) to 3(c) are shows the effect of the solutal buoyancy parameter on the velocity, temperature and concentration profiles. It is seen that as the solutal buoyancy parameter increases, the velocity field decreases. Also observed that temperature and concentration profiles increases with an increasing the solutal buoyancy parameter.
Fig. 2(b) Temperature for different values of $\lambda$

Fig. 2(c) Concentration for different values of $\lambda$
Fig. 3(a) Velocity for different values of $\delta$

$\lambda = m = 1$, $M = Q = 0.5$, $Pr = 2$, $Le = 4$, $Nt = Nb = 0.1$

- $\delta = 1$
- $\delta = 2$
- $\delta = 3$
- $\delta = 4$

Fig. 3(b) Temperature for different values of $\delta$

$\lambda = m = 1$, $M = Q = 0.5$, $Pr = Le = 4$, $Nt = Nb = 0.1$

- $\delta = 0.5$
- $\delta = 1.0$
- $\delta = 1.5$
- $\delta = 1.7$
Fig. 3(c) Concentration for different values of $\delta$

\[\lambda = m = 1, \quad M = Q = 0.5, \quad Pr = Le = 4, \quad Nt = Nb = 0.1\]

1. $\delta = 0.5$
2. $\delta = 1.0$
3. $\delta = 1.2$
4. $\delta = 1.5$

$\eta \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$\phi \quad 1.0 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0$

Fig. 4(a) Temperature for different values of $Nt$

\[\delta = \lambda = m = 1, \quad M = Q = 0.5, \quad Pr = Le = 4, \quad Nb = 0.1\]

1. $Nt = 0.1$
2. $Nt = 0.2$
3. $Nt = 0.25$
4. $Nt = 0.3$

$\theta \quad 1.0 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0$

$\eta \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$
Fig. 4(b) Concentration for different values of $N_t$

$\phi$ = $\delta = \lambda = m = 1$, $M = Q = 0.5$, $Pr = Le = 4$, $Nb = 0.1$

- $N_t = 0.1$
- $N_t = 0.2$
- $N_t = 0.25$
- $N_t = 0.3$

Fig. 5(a) Temperature for different values of $Nb$

$\theta$ = $\delta = \lambda = m = 1$, $M = Q = 0.5$, $Pr = Le = 4$, $N_t = 0.1$

- $Nb = 0.1$
- $Nb = 0.2$
- $Nb = 0.25$
- $Nb = 0.3$
Fig. 5(b) Concentration for different values of $Nb$

Fig. 6 Temperature for different values of $Pr$
Fig. 7 Temperature for different values of $Q$

\[ \delta = \lambda = m = 1, \ M = 0.5, \ Pr = Le = 4, \ Nt = Nb = 0.1 \]

Fig. 8 Concentration for different values of $Le$

\[ \delta = \lambda = m = 1, \ M = 0.5, \ Pr = 4, \ Nt = Nb = Q = 0.1 \]
Fig. 9 Effect of $N_t$, $N_b$, $Q$ and $m$ on the skin friction

Fig. 10 Effect of $N_t$, $N_b$, $Q$ and $m$ on the reduced Nusselt number
Fig. 11 Effect of $N_t$, $N_b$, $Q$ and $m$ on the reduced Sherwood number

The effect of thermophoresis parameter on temperature and concentration fields is shown in figs. 4(a) and 4(b). The thermophoretic force generated by the temperature gradient creates a fast flow away from the stretching surface. In this way more fluid is heated away from the surface, and consequently, as $N_t$ increases, the temperature within the boundary layer increases. The fast flow from the stretching sheet carries with it thermophoretic force leading to an increase in the concentration boundary layer thickness. The effect of the Brownian motion of the fluid on the temperature and concentration is shown in Figs. 5(a) and 5(b). As expected, the increased Brownian motion of the fluid carries with it heat and the thickness of the thermal boundary layer increases. An increase in the Brownian motion of the fluid leads to a decrease in the concentration profiles. Fig. 6 show the temperature profiles for several values of the Prandtl number ($Pr$). The temperature profiles decrease as the Prandtl number increases since, for high Prandtl numbers, the flow is governed by momentum and viscous diffusion rather than thermal diffusion. Fig. 7 shows the effect of the heat source/sink parameter ($Q$) on the temperature profiles. The temperature profiles significantly increase as the heat source/sink parameter increases. The various values of Lewis number ($Le$) on concentration is plotted in fig. 8. The concentration profiles significantly contract as the Lewis number increases.

Fig. 9 shows the effects of the thermophoresis parameter, Brownian motion parameter, non linear stretching parameter ($m$) and the magnetic parameter on the wall skin friction. It can be seen that the skin friction coefficient increases when the thermophoresis parameter, non linear stretching parameter and magnetic parameter increases. However, increasing the Brownian motion parameter leads to a decrease in the skin friction coefficient. Figs. 10 show the effects of thermophoresis parameter, Brownian motion parameter, non linear stretching parameter and the magnetic parameter on the reduced Nusselt number. We note a decrease in the reduced Nusselt number when $N_t$ or $N_b$ or $m$ or $M$ increases. Fig. 11 depicts the effects of the thermophoresis parameter, Brownian motion parameter, non linear stretching parameter and the magnetic parameter on the reduced Sherwood number $-\phi'(0)$. We observe an increase in the $-\phi'(0)$ when $N_b$ increases and decrease when $N_t$ or $m$ or $M$ increases.

The variations of skin friction coefficient, reduced Nusselt number and reduced Sherwood number for different values of $\lambda$, $\delta$, $Pr$, $Q$ and $Le$ are shown in Table 2. It is observed that $C_f$ is an increasing function of $\delta$ and $Pr$, while with increasing values of $\lambda$, $Q$ and $Le$. However, it is found that $-\theta'(0)$ decreases for large values of $\delta$ and $Q$ whereas increases for increasing values of $\lambda$, $Pr$ and $Le$. It is further observed from this table that $-\phi'(0)$ is increasing function of $\lambda$, $\delta$, $Q$ and $Le$ whereas $-\phi'(0)$ decreases with increasing values of $Pr$. Table 1 shows the comparison of reduced Nusselt number and reduced Sherwood number of the previous published data.
Table 1: Comparison of reduced Nusselt number and reduced Sherwood number for Pr=10, Le=10, m=1, δ=λ=M=Q=0

<table>
<thead>
<tr>
<th>Nb</th>
<th>Pr</th>
<th>Le</th>
<th>δ</th>
<th>m</th>
<th>Q</th>
<th>M</th>
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</tr>
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<td>1</td>
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<td>0.826524</td>
</tr>
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</table>

Table 2: Variation of $C_{fx}$, $-\theta'(0)$ & $-\phi'(0)$ at the wall with Nb, Pr, Le, δ, m, Q and M

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<tr>
<th>Nb</th>
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<th>m</th>
<th>Q</th>
<th>M</th>
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CONCLUSION

In the present numerical study, we have observed that:

- The fluid velocity decreases when increasing the magnetic parameter.
- The thermal and mass boundary layer thickness increases with the thermophoresis parameter.
- Increasing the Lewis number and solutal buoyancy parameter reduces the heat and mass transfer coefficient.
- Increasing the Brownian motion parameter and heat source/sink parameter enhances the thermal boundary layer thickness.
- Non linear stretching parameter enhances the skin-friction.

REFERENCES

MHD and Mixed Convection Flow of Maxwell Fluid on Heat Transfer near a Stagnation Point Flow

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Abstract: The effects of thermal radiation and heat transfer of a Maxwell fluid near a mixed convection stagnation point flow over a moving surface in the presence of MHD has been studied. The governing differential equations are transformed into a set of coupled non-linear ordinary differential equations and then solved with a numerical technique using appropriate boundary conditions for various physical parameters. The numerical solution for the governing non-linear boundary value problem is based on applying the fourth-order Runge–Kutta method coupled with the shooting technique using appropriate boundary conditions for various physical parameters. The effects of various parameters like the viscosity parameter, radiation parameter, mixed convection parameter, Deborah number, magnetic parameter and Prandtl number on the velocity and temperature profiles as well as on the local skin-friction coefficient and the local Nusselt number are presented and discussed.

Keywords: Mixed convection flow, Thermal radiation, Maxwell fluid, heat transfer, MHD, Convective condition.

I. Introduction

The fluids like soups, shampoos, tomato paste, condensed milk, sugar solution, apple source, mud etc. cannot be described by the Newton's law of viscosity. Such fluids are known as the non-Newtonian fluids. The non-Newtonian fluids in view of their diverse rheological properties cannot be examined through one constitutive relationship between shear stress and rate of strain. Many models of non-Newtonian fluids exist. Maxwell model is one subclass of rate type fluids. This fluid model is especially useful for polymers of low molecular weight. In view of its simplicity, this fluid model has acquired special status amongst the recent workers in the field. For instance Wang and Tan [1] discussed the flow of Maxwell fluid in a porous medium. Exact solution of Helical flows of Maxwell fluid with shear stress on the boundary is addressed by Jamil and Fetecau [2]. Zierep and Fetecau [3] studied Rayleigh-Stokes problem using Maxwell fluid. Exact solution is constructed here. Numerical solution for stagnation point flow of Maxwell fluid was computed by Sadeghy et al. [4]. Megahed [5] studied the variable fluid properties and variable heat flux effects on the flow and heat transfer in a non-Newtonian Maxwell fluid over an unsteady stretching sheet with slip velocity. An exact solution for the stretching/shrinking wall problem in a viscous fluid was provided by Yao et al. [6].

The mixed convection flow occurs in several industrial and technical applications which include nuclear reactors cooled during emergency shutdown, electronics devices cooled by fans, heat exchangers placed in a low velocity environment, and solar central receivers exposed to wind currents. In the study of fluid over heated or cooled surfaces, it is customary to neglect the effect of the buoyancy forces when the flow is horizontal. However for vertical or inclined surfaces, the buoyancy force modifies the flow field and hence the heat transfer rate. Therefore, it is not possible to neglect the effect of buoyancy forces for vertical or inclined heated or cooled surfaces. In recent years, much attention has been paid to develop efficient energy systems. Many recent studies have been focused on the problem of magnetic field effect on laminar mixed convection boundary layer flow over a vertical non-linear stretching sheet [7-9].

The radiative effects have important applications in physics and engineering particularly in space technology and high temperature processes [16]. Effects of radiation have been studied by Abdul Hakeem and Sathiyanathan [17], Seddeek and Abdelmeguid [18], Mamun Molla and Anwar Hossain [19], Hayat et al. [20] studied the mixed convection radiative flow of maxwell fluid near a stagnation point with convective condition. Soid et al. [21] studied the magnetohydrodynamics boundary layer flows over a stretching surface with radiation effect and embedded in porous medium.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al [22] has investigated...
the Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Moalem [23] studied the effect of temperature dependent heat sources taking place in electrically heating the heat transfer within a porous medium. Vajravelu and Nayfeh [24] reported on the hydro magnetic convection at a cone and a wedge in the presence of temperature dependent heat generation or absorption effects. Swati Mukhopadhyay [25] analyzes the heat transfer analysis of the unsteady flow of a Maxwell fluid over a stretching surface in the presence of a heat source/sink.

The present study contains an analysis of the effects of mixed convective flow of a Maxwell fluid over a stretching sheet by taking MHD into account. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, fourth order Runge-Kutta method along with shooting technique has been used for solving it. The results for velocity and temperature functions are carried out for the wide range of important parameters namely, magnetic parameter, viscosity parameter, thermal conductivity parameter and radiation parameter. The skin friction and rate of heat transfer have also been computed.

II. Mathematical Formulation

Let us consider the two-dimensional mixed convection stagnation point flow of an incompressible and radiative Maxwell fluid near a stretched surface. The flow is in the region \( y > 0 \) and is subjected to a non-uniform magnetic field applied normally to the surface, \( B_0 \) is the initial strength of the magnetic field. It is assumed that the magnetic Reynolds number is very small and as there is no electric field, the electric field due to polarization of charges is neglected. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by

**Continuity equation**
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(2.1)

**Linear momentum equation**
\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left[ \nu \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] = \nu \frac{du}{dx} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g \beta_f (T - T_\infty)
\]  
(2.2)

**Energy equation**
\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( \kappa \frac{\partial T}{\partial y} \right) - \frac{\partial q}{\partial y}
\]  
(2.3)

The boundary conditions for the velocity, temperature and concentration fields are
\[
u = u_0(x) = cx, v = 0, -k \frac{\partial T}{\partial y} = h(T_f - T) \text{ at } y = 0
\]
\[u = u_0(x) = ax, T \rightarrow T_\infty \text{ as } y \rightarrow \infty
\]  
(2.4)

where \( u \) and \( v \) are the velocity components along \( x \) and \( y \) directions, respectively, \( \rho \) is the fluid density, \( T \) is the temperature of the fluid, \( \lambda \) is the relaxation time, \( c_p \) is the specific heat at constant pressure, \( \mu \) is the fluid viscosity, \( k \) is the fluid thermal conductivity, \( T_\infty \) is the free stream temperature and \( T_f \) is the convective fluid temperature.

By using the Rosseland approximation the radiative heat flux \( q_r \) is given by
\[
q_r = -4\sigma \frac{T^4}{3k} \frac{\partial T^4}{\partial y}
\]  
(2.5)

Where \( \sigma \) is the Stefan-Boltzmann constant and \( k \) is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are significantly small, then equation [2.5] can be linearised by expanding \( T^4 \) into the Taylor series about \( T_\infty \), which after neglect higher order terms takes the form:
\[
T^4 \approx 4T_\infty^3T - 3T_\infty^4
\]  
(2.6)

In view of equations (2.5) and (2.6), eqn. (2.3) reduces to
\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k \rho c_p} \frac{\partial T}{\partial y}
\]  
(2.7)
The continuity equation (2.1) is satisfied by the Cauchy Riemann equations
\[ u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \] (2.8)
where \( \psi(x, y) \) is the stream function.

In order to transform equations (2.2) and (2.7) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

\[
\theta(\eta) = \frac{T - T_w}{T_f - T_w}, \quad \alpha = \frac{a}{c}, \quad \beta = \lambda \dot{c}, \quad R = \frac{4\sigma \gamma T_f^3}{k^2 \sigma^2 \rho}, \quad \lambda = \frac{Gr_f}{Re_z^2} \]
\[
M = \frac{\sigma R_{\infty}^2}{\rho c}, \quad Gr_f = \frac{2 \beta (T_f - T_w) \sigma}{v^3}, \quad Pr = \frac{v}{\alpha}
\]

where \( f(\eta) \) is the dimensionless stream function, \( \theta \) - dimensionless temperature, \( \eta \) - similarity variable, \( M \) - Magnetic parameter, \( a \) and \( c \) are constants, \( \beta \) - Deborah number, \( \lambda \) - mixed convection parameter, \( \alpha \) - ratio of rate constant, \( Gr_f \) - Grashof number, \( Re_z \) - Reynolds number, \( R \) - radiation parameter, \( Pr \) - Prandtl number.

In view of Equations (2.8) - (2.9), the Equations (2.2) and (2.7) transform into

\[
f'' + \frac{ff' - f^2 + \alpha^2}{f^3} - \frac{M}{f} \left( f^2 f'' - 2ff' f' \right) + \lambda \theta = 0
\] (2.10)

\[
\left( 1 + \frac{4}{3} R \right) \theta'' + Pr \ f \theta' = 0
\] (2.11)

The corresponding boundary conditions are:

\[
f'(0) = 0, \quad f'(\infty) = 1, \quad \theta'(0) = -\gamma (1 - \theta(0))
\]
\[
f'(\infty) = \alpha \dot{c}, \quad \theta'(\infty) = 0
\] (2.12)
where the primes denote differentiation with respect to \( \eta \)

The physical quantities of interest are the skin friction coefficient \( C_f \), the local Nusselt number \( Nu \) which are defined as

\[ C_f = -2 \operatorname{Re}_z^{-1/2} f''(0), \quad Nu = -\operatorname{Re}_z^{1/2} \theta'(0) \] (2.13)

III. Solution of the problem

The set of coupled non-linear governing boundary layer equations (2.10) and (2.11) together with the boundary conditions (2.12) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (2.10) and (2.11) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al.[22]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size \( \Delta \eta = 0.05 \) is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient and Nusselt number, which are respectively proportional to \( f''(0) \) and \( -\theta'(0) \), are also sorted out and their numerical values are presented in a tabular form.

IV. Results and Discussion

The governing equations (2.10) - (2.11) subject to the boundary conditions (2.12) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem. The effects of various parameters on velocity profiles in the boundary layer are depicted in Figs. 1-7. The effects of various parameters on temperature profiles in the boundary layer are depicted in Figs. 8-13.

Fig. 1 shows the variation of the velocity with the viscosity parameter \( \alpha \). It is noticed that the velocity thickness increases with an increase in the viscosity parameter. Fig. 2 shows the variation of the velocity with the Deborah number \( \beta \). It is noticed that the velocity thickness decreases with an increase in the Deborah number. Fig. 3 illustrates the effect of the convective parameter \( \gamma \) on the velocity field. It is seen that as the convective parameter increases, the velocity field increases. Fig. 4 shows the dimensionless velocity profiles for different values of magnetic parameter \( M \). It is seen that, as expected, the velocity increases with an increase of
magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness increases with an increase in the magnetic parameter. Fig. 5 illustrates the effect of the mixed convection parameter \( \lambda \) on the velocity field. It is seen that as the mixed convection parameter increases, the velocity field increases. Fig. 6 shows the variation of the thermal boundary-layer with the Prandtl number \( (Pr) \). It is noticed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 7 shows the variation of the velocity with the radiation parameter \( (R) \). It is noticed that the velocity thickness increases with an increase in the radiation parameter.

Fig. 8 depicts the thermal boundary-layer with the viscosity parameter. It is noticed that the thermal boundary layer thickness decreases with an increase in the viscosity parameter. Fig. 9 depicts the thermal boundary-layer with the convective parameter. It is noticed that the thermal boundary layer thickness increases with an increase in the convective parameter. Fig. 10 shows the variation of the thermal boundary-layer with the Prandtl parameter. It is observed that the thermal boundary layer thickness decreases with an increase in the Prandtl number. Fig. 11 illustrates the effect of the mixed convection parameter on the temperature. It is noticed that as the mixed convection parameter increases, the temperature decreases. Fig. 12 shows the variation of the thermal boundary-layer with the Prandtl number. It is noticed that the thermal boundary layer thickness increases with an increase in the Prandtl number. Fig. 13 shows the variation of the thermal boundary-layer with the radiation parameter. It is observed that the thermal boundary layer thickness increases with an increase in the radiation parameter.

Table 1 shows the variation of the skin friction and Nusselt number with for different values of \( \alpha, \beta, \gamma, M, \lambda, R \) and \( Pr \). It is noticed that the skin friction increases where as Nusselt number decrease with an increase in the Deborah number or Magnetic parameter or radiation parameter. It is found that the skin friction decreases where as Nusselt number increase with an increase in the viscosity parameter or convective parameter or mixed convective parameter. It is observed that both the skin friction and Nusselt number increases with an increase in the Prandtl number. The correctness of the present numerical method is checked with the results obtained by Pop et al. [27], Mahapatra and Gupta [28] and Hayat et al. [20] for the values of Skin friction coefficient in the limiting condition. Thus, it is seen from Table 2.

V. Conclusions

The effects of thermal radiation and heat transfer of a Maxwell fluid near a mixed convection stagnation point flow over a moving surface in the presence of MHD has been studied. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. The velocity decreases as well as temperature increases with an increase in the magnetic parameter.
2. The velocity and temperature decreases with an increase in the Prandtl parameter.
3. The skin friction reduces the Prandtl number and increases with the Magnetic parameter or radiation parameter.
4. The Nusselt number reduces the magnetic parameter or radiation parameter and increases with the viscosity parameter.
Fig. 2 Velocity profiles for different values of $\beta$

Fig. 3 Velocity profiles for different values of $\gamma$

Fig. 4 Velocity profiles for different values of $M$
Fig. 5 Velocity profiles for different values of $\lambda$

Fig. 6 Velocity for different values of $Pr$

Fig. 7 Velocity profiles for different values of $R$
Fig. 8 Temperature for different values of $\alpha$

Fig. 9 Temperature for different values of $\gamma$

Fig. 10 Temperature for different values of $M$
Fig.11 Temperature for different values of $\lambda$

Fig.12 Temperature for different values of $Pr$

Fig.13 Temperature for different values of $R$

Table 1 Numerical values of $-f''(0), -\theta'(0)$ at the sheet for different values of $\alpha, \beta, \gamma, M, \lambda, R$ and $Pr$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$M$</th>
<th>$\lambda$</th>
<th>$R$</th>
<th>$Pr$</th>
<th>$-f''(0)$</th>
<th>$-\theta'(0)$</th>
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<tr>
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<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
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<td>1.47521</td>
<td>0.289624</td>
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<td>0.5</td>
<td>0.5</td>
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<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.27783</td>
<td>0.289892</td>
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Mhd And Mixed Convection Flow Of Maxwell Fluid On Heat Transfer Near A Stagnation Point Flow

Table 2 Numerical values of \( f^{(0)}(\bar{y}) \) at the sheet for different values of \( \lambda \). Comparison of the present results with that of Pop et al. [23], Mahapatra and Gupta [24] and Hayet et al.[20]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>Present study</th>
<th>Pop et al. [23]</th>
<th>Mahapatra and Gupta [24]</th>
<th>Hayet et al.[20]</th>
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<tr>
<td>0.1</td>
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</tr>
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<td>4.7290</td>
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<td>4.72954</td>
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References


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