Chapter - I

Introduction
1.1 INTRODUCTION

Technological change, technical efficiency and scale economies / diseconomies constitute three major technological characteristics of a firm. A study that aims at evaluating the performance of a firm should therefore analyse technological change, technical efficiency and productivity growth together. The effect of technological change on productivity could be evaluated in terms of changes in the inputs that are used in firm production. For a given level of output and input price ratio, a labour-saving technological change results in a higher capital-labour ratio; a capital-saving technological change in a lower capital-labour ratio and a neutral technological change in an unchanged capital-labour ratio respectively. However, this definition is suitable only in the short-run, that is when output levels are assumed to remain as constants. In the long run, the technological change may be either embodied or dis-embodied. It is said to be dis-embodied if the technological change occurs not in the inputs but outside of them. The resulting shift in the production function could be represented by including a time variable 't' along with inputs, say labour (L) and capital (K). Hence, \( Q = f(L, K, t) \). This implies that the same input vector produces different levels of output 'Q' at different time periods. The technological change is called embodied, when such changes are embodied in inputs of different vintages. For instance, capital services provided by old and new machines, could show different results. Similarly, new technically sound staff and the old-technical labour produce different
results. In such circumstances technological change is defined in three alternative ways as suggested by Hicks (1956), Harrod (1948) and Solow (1957) respectively. Hicksian Technological change is defined as labour-saving, capital-saving or neutral if it rises, lowers or remains unchanged with the marginal product of capital relative to the marginal product of labour for a given capital-labour ratio. Hence, a Hicks neutral technological change shifts the isoquant inwards which makes the slope of the isoquant unchanged along a ray through the origin. Hicks neutrality implies that the technological change is equally capital and labour augmenting. In the Harrod definition of technological change, capital-output ratio is given. Harrod neutrality implies that the technological change in labour augmenting. Similarly, Solow defines the technological change for a given labour-output ratio. Solow neutrality implies that technological change is found to be capital augmenting.

Productivity is the relationship between the output (outputs) generated by a production unit and the inputs provided for this purpose. It is a measure of an efficient use of resources in the production of goods because the improving productivity means production of more goods and services with the same resources or maintaining the same level of production with less resources. A firm exhibiting productivity growth is more likely to show a better financial performance. There is no single accepted notation to measure productivity or productivity growth. Generally it is measured in a production function frame work. In this
context productivity is considered as the degree of efficiency exhibited in the process of turning inputs into outputs. Since several inputs are required to produce a product and productivity could be defined in the partial and total form. The partial factor productivity measures the ratio of the quantity of the output 'Q' to the amount used of one single input 'X'. The partial factor productivity can be written as Q/X. The Total Factor Productivity describes the ratio of output to a combination of all the inputs used. It is written as $A(t) = \frac{Q}{\phi(w,x)}$, where w is the vector of input weights.

In 1950, the Organization for European Economic Cooperation (OEEC) has offered a more formal definition of productivity as "the productivity is the quotient obtained by dividing the output by one of the factors concerning the production". Like this, it is possible to focus on the productivity of capital, investment or raw materials based on the fact that whether output is considered in relation to capital, investment or raw materials etc.

The term "Productivity" is often confused with the term "Production". Many people are of the opinion that the greater the production, the greater the productivity. But this is not necessarily be true. Production is concerned with the activity of producing goods, but the productivity is concerned with an efficient utilization of resources (inputs) in producing goods (output). In quantitative terms, production is the
quantity of output produced, while productivity is the ratio of output produced to inputs used.

The basic tool for an efficiency measurement is PRODUCTION FUNCTION. A production function is a relationship that explains how inputs are combined to obtain a maximum possible output. This assumption assumes that the firms under consideration are technically efficient. However, in an industry, for example, all the firms may not be technically efficient because the firms may vary in the adoption of latest technology. These firms may vary with respect to managerial efficiency also. Thus, there is a need to the model for inefficiency into the production and therefore into the production function.

Any production function which violates the property with which each input combination maximum output is associated could be addressed as technically inefficient.

1.2 MEANING OF THE CONCEPT OF EFFICIENCY :-

The core of any economic activity, whether it is consumption or production or any other property, is to strive for a maximum possible efficiency. Productive efficiency has been defined by Farrell (1957) in terms of two main concepts: "Technical efficiency" and "Factor price efficiency" which may have any of the following meanings,
I. We can say that a machine or organization is technically efficient, if it is adequate to meet the demands put before it, or it stands up to the claims made from it.

II. The technical efficiency could be assessed based on some quantitative standards of performance such as degree to which a domestic heating appliance converts the potential heat contained in a unit of fuel into units of actual heat.

III. Technical efficiency could mean that doing a job in the cheapest possible way, that is, production of a given level of output from the lowest possible combination of inputs.

The second element of the productive efficiency namely, the factor price efficiency measures the skill in achieving the best combination of the inputs by taking into account their relative prices. This is very important because one input is substituted for another during the process of the production. To have a clear picture on the two elements of productive efficiency, let us consider the following diagram.

![Figure (1.2.1)](attachment:image_url)
Here, the curve II' indicates an isoquant which shows the most efficient combination of the two factors $X_1$ and $X_2$ that are used to produce a given level of output of a commodity. "Most efficient", means the minimum combination of factors could be required according to the 'best practice' production function for the commodity. In practice, a firm may deviate from the II' curve and thus causing the inefficiency in the factors that are used.

Let us take $P$ as the actual situation where the firm uses $OD$ and $OC$ quantities of the two factors $X_1$ and $X_2$ respectively to produce that specified level of output. The technical efficiency of the firm at $P$ in practice to the 'best practice', frontier II' can be measured by the ratio $OQ/OP$.

$AB$ is the isoquant line in the diagram indicating the combination of the two factors that can be purchased from a given amount of money and factor prices.

Then, the factor price efficiency for the firm could be measured from the ratio $OQ_1 / OQ$. This is because any combination of the two factors beyond $AB$ line will not be possible when the amount of total resources and factor prices are treated as fixed values. Then, the productive efficiency is given by

$\text{Productive Efficiency} = \frac{OQ}{OP} \times \frac{OQ_1}{OQ} = \frac{OQ_1}{OP}$
The nearer this ratio moves to the unity, the higher will be the productive efficiency. At point 'R' the productive efficiency could be found in maximum and this is a familiar tendency in the isoquant analysis.

A system is said to be economically efficient if it is technically efficient and if it succeeds in rationing out its scarce resources, and the scarce product of these resources, in a most desirable way.

The study of change in output as a consequence of change in the factors of production forms the subject matter of return to scale. The return to scale may be constant, increasing or decreasing. If we increase all factors of production in a given proportion and the output increases in the same proportion, then the return to scale is said to be a constant. Therefore, if a doubling or trebling of all factors could result in doubling or trebling of outputs, the return to scale are found to be constant. But, if the increase in all factors could lead to more than proportionate in the output, then return to scale is said to be increasing. Thus, if all factors are doubled and output is increased by more than two folds, then the return to scale is said to be increasing, on the otherhand, if the increase in all factors could lead to less than proportionate increase in the output, return to scale is said to be decreasing. In otherwords, for a homogeneous production function is represented as $u = f(L, K)$.

Suppose

$$f(\lambda L, \lambda K) = \lambda^e f(L, K), \lambda > 1$$
Then

$\theta > 1$ indicates increasing return to scale

$\theta < 1$ indicates decreasing return to scale

$\theta = 1$ indicates constant return to scale

1.3 MEASUREMENT OF THE EFFICIENCY LEVELS:

Once we succeed in defining the efficiency conditions, the next logical step is to measure them. The technical efficiency could be measured through some physical indicators such as capital-output ratio, capital labour ratio, or actual cost-standard cost ratio etc. The overall efficiency of the firm, whether we take productive efficiency or economic efficiency, might be difficult to measure them precisely. Three methods are generally used for this purpose. One is the use of some type of optimization model such as linear programming, the second is the use of the ratios like total Productivity and the third is the use of econometric methods.

In the first method, a firm has to specify in quantitative terms the objective function and also the constraints that are met with to achieve and then apply the standard mathematical tools to solve the problem. In the case of second method, the firms may set some target for total factor productivity or profitability for themselves. If they achieve that one, then they may be called as efficient, otherwise, they are not. The total factor productivity is the ratio of the gross revenue divided by the total cost of Production, Profitability is the return on the capital invested in the business.
The choice of the indicators for the efficiency depends on the goals of the firm. The use of econometric methods for measuring the efficiency is most elegant and scientific in nature. Based on economic reasoning, models are specified to measure technical efficiencies of the firms. The quantitative estimation of parameters and other properties of the models provide reliable estimates of the efficiencies for the firms.

The estimation of technical efficiency is normally carried out by using the production function. The basic step used in this procedure is to estimate the production frontier underlying a sample of firms in the industry. Once the frontiers are estimated, efficiency of industrial units can be measured on the basis of actual shortfall in output.

Two types of frontiers are used for this purpose, (i) deterministic and (ii) stochastic: in a deterministic framework it is assumed that all firms shall have a common production frontier and variations in firm's performance with respect to it is attributed to variation in firm's efficiency. It is normally specified as \( y = f(x)e^{-t} \) for an empirical estimation. Here \( y \) is output, \( x \) is vector of inputs and \( u \) is an error term. The core \( f(x) \) of production function is common to all firms. The estimation of individual firm's efficiency in such model depends on the assumption of some kind of distribution pattern for the error term. Afriat (1972), Aigner and Chu (1968), Forsund and Lennart (1974), Richmond (1974) and Schmidt (1976)
have provided methodologies for estimation of the frontier production functions.

The approach of using stochastic production frontiers has been suggested by a number of economists such as Aigner et al. (1977), Schmidt and Lovell (1980), Stevenson and Greene (1980). The stochastic production function that was used by them is written in a general form as \( y = f(x)e^\nu \), here \( y \) is output, \( x \) is vector of inputs, \( f(x)e^\nu \) is the stochastic production frontier with \( \nu \) as normally distributed random disturbance term, \( e^\nu \) is one sided error component which captures technical inefficiency relative to the stochastic production frontier, \( u \geq 0 \). The economic measure of the \( u \) component is that each firm's production must lie either on or below the production frontier \( f(x)e^\nu \). Any downward deviation from the frontier is due to a technical inefficiency of the firm.

In the productive efficiency theory in Economics, we come across efficiency measures such as (a) pure technical, (b) scale, (c) overall technical, (d) allocative and (e) overall productive efficiency measures. These are obtained for both inputs and outputs.

Let us consider two input sets \( L(u) \) and \( L^k(u) \) as shown in the figure.
Let us assume that the former and the latter are consistent, with constant and non-constant (increasing or decreasing) return to scale. Then, input productive efficiency measures are given below.

(a) Pure Technical Efficiency

\[ PTE = \frac{OQ}{OP} \]

(b) Scale Efficiency

\[ SE = \frac{OR}{OQ} \]

(c) Overall Technical Efficiency

\[ OTE = \frac{OR}{OP} \]

(d) Allocative Efficiency

\[ AE = \frac{OS}{OR} \]
(e) **Overall Productive Efficiency**

\[
\text{OPE} = \frac{OS}{OP}
\]

Then, the producer who operates at \( P \) is said to be productive inefficient.

Analogous to the input technical efficiency measures, the output technical efficiency measures could also be defined as follows.

![Graph](image)

**Figure (1.3.2)**

Then,

(a) *(OUTPUT)* Pure Technical Efficiency is given by

\[
PTE = \frac{OP}{OQ}
\]

(b) *(OUTPUT)* Overall Technical Efficiency is given by

\[
OTE = \frac{OR}{OQ}
\]
(c) (OUTPUT) Scale Efficiency is given by

\[
SE = \frac{OR}{OP} = \left( \frac{OR}{OQ} \right) \left( \frac{OP}{OQ} \right)
\]

\[
\therefore SE = \frac{OTE}{PTE}
\Rightarrow OTE = (SE)(PTE)
\]

Thus, the output overall technical efficiency could be expressed as the product of pure technical efficiency and scale efficiency.

(d) (OUTPUT) Overall Productive Efficiency is given by

\[
OPE = \frac{OP}{OS}
\]

(e) (OUTPUT) Allocative Efficiency is given by

\[
AE = \frac{OR}{OS}
\]

Thus, the Overall Productive Efficiency measures could now be expressed as the product of allocative and overall technical efficiency measures.

\[
OPE = OTE.AE = PTE.SE.AE.
\]

13
A production process can be inefficient in two ways only one of which can be detected by an estimated production frontier. It can be technically inefficient in the sense that it fails to produce maximum output from a given input bundle; technical inefficiency results in an equi-proportionate over utilization of all inputs. It can also be allocatively inefficient in the sense that the marginal revenue product of an input might not be equal to the marginal cost of that input. Allocative inefficiency results in utilization of inputs in the wrong proportions, given input prices.

1.4 DATA ENVELOPMENT ANALYSIS:

Data Envelopment Analysis determines a frontier production or cost function which envelops all the observations of inputs and outputs of production units that compete with each other in a competitive production environment. The approach is primarily determined by Charnes, Cooper and Rhodes (1978). They called each production unit as 'decision making unit'. These units produce multiple outputs combining multiple inputs, which are common to all decision making units.

If 'n' inputs are combined to produce 'm' outputs, then the weighted sums of inputs and outputs are given by,

$$\sum_{j=1}^{n} \alpha_i x_{ij}, \sum_{i=1}^{m} \beta_i y_{i0}$$

then, the composite output per unit composite input foregone is given by
\[
Z = \frac{\sum_{i=1}^{n} \beta_{j} x_{i0}}{\sum_{i=1}^{n} \alpha_{i} x_{i}} 
\]

...(1.4.1)

Here, the expression Z is used as an objective function. Suppose ‘N’
production units that are in competition, then we can define N ratios as

\[
\frac{\sum_{i=1}^{n} \beta_{j} x_{i}}{\sum_{i=1}^{n} \alpha_{i} x_{i}}, \; k = 1, 2, \ldots, N
\]

...(1.4.2)

Here, each of these ratios is constrained to be not larger than unity. So that
we have

\[
\frac{\sum_{i=1}^{n} \beta_{j} x_{i}}{\sum_{i=1}^{n} \alpha_{i} x_{i}} \leq 1, \; K = 1, 2, \ldots, N
\]

...(1.4.3)

and \(\beta_{j}, \alpha_{i} > 0\)

By combining the objective function and the N constraints, we can
obtain the following fractional programming problem.

\[
\text{Max } Z = \frac{\sum_{i=1}^{m} \beta_{j} x_{i0}}{\sum_{i=1}^{n} \alpha_{i} x_{i}}
\]

subject to \(\frac{\sum_{i=1}^{n} \beta_{j} x_{i}}{\sum_{i=1}^{n} \alpha_{i} x_{i}} \leq 1, \; K = 1, 2, \ldots, N\)

...(1.4.4)

and \(\beta_{j}, \alpha_{i} > 0\)

Evidently
Max \( Z \leq 1 \)

From the theory of linear fractional programming the above problem could be expressed as a linear programming problem as shown below.

\[
\text{Max } Z = \frac{\sum_{j=1}^{n} \beta_j \mu_j \rho}{\sum_{i=1}^{n} \alpha_i x_{i0}} 
\]

\( \ldots \) (1.4.5)

subject to

\[
\sum_{j} \beta_j \mu_j \rho - \sum_{i} \alpha_i x_{i0} \leq 0, \quad K = 1, 2, \ldots, N 
\]

\[
\sum_{i} \alpha_i x_{i0} = 1 
\]

and \( \beta_j > 0, \alpha_i > 0 \)

The fractional programming problem can also be expressed as

\[
\text{Min } \Pi = \frac{\sum_{i} \alpha_i x_{i0}}{\sum_{j} \beta_j \mu_j \rho} 
\]

\( \ldots \) (1.4.6)

subject to

\[
\sum_{i} \alpha_i x_{i0} \geq 1, \quad K = 1, 2, \ldots, N 
\]

\[
\sum_{j} \beta_j \mu_j \rho 
\]

and \( \beta_j > 0, \alpha_i > 0 \)

The above problem can also be written as

\[
\text{Min } \Pi = \frac{\sum_{i} \alpha_i x_{i0}}{\sum_{i} \beta_j \mu_j \rho} 
\]

\( \ldots \) (1.4.7)

subject to

\[
\sum_{i} \alpha_i x_{i0} - \sum_{j} \beta_j \mu_j \rho \geq 0 
\]
\[ \sum_{i=1}^{n} \beta_{j} z_{j,i} = 1 \]

\[ \beta_{j}, \alpha_{i} > 0 \]

Evidently, \( \min \Pi \geq 1 \)

Note: (1) For a typical decision making unit

\[ \max Z = 1 \] for the problem (1.4.5). Then, the production unit under consideration is technically efficient.

(2) If \( \min \Pi = 1 \), the production unit of problem (1.4.7) is technically efficient, otherwise inefficient.

(3) \( \min \Pi = \max Z \)

**Dual problem:**

The dual of the linear programming problem (1.4.7) is

\[ \max \phi = \theta + \varepsilon \sum_{j=1}^{m} S_{j} + \varepsilon \sum_{i=1}^{n} S_{i}^{*} \]

subject to

\[ \sum_{i} \lambda_{i} x_{i} + S_{i}^{*} = y_{i} \] \[ \ldots (1.4.8) \]

\[ \sum_{i} \lambda_{i} y_{i} - \theta y_{j} - S_{j}^{*} = 0, \quad j = 1, 2, \ldots, m \]

\[ \lambda_{i} \geq 0, \quad S_{i}^{*} \geq 0, \quad S_{j}^{*} \geq 0 \]

Here

(i) \( \lambda_{i} \) are some intensity parameters

(ii) \( S_{i}^{*} \) are non-negative slack variables

(iii) \( S_{j}^{*} \) are non-negative surplus variables
(iv) \( \text{Max} \phi \geq 1 \)

(v) \( \text{Max} \phi = 1 \) implies that the production unit under consideration is technically efficient, otherwise inefficient.

1.5 SOME EFFICIENCY CONDITIONS IN THE THEORY OF PRODUCTION:

The Efficiency conditions that are discussed in this section involve some basic terms such as marginal product, marginal cost, marginal revenue and marginal revenue product. A brief explanation of these terms is needed here in order to understand the efficiency conditions in which they appear.

The marginal product of a factor of production is defined as the increment in total output of a commodity when one more extra unit of the factor is employed in production of that commodity, the quantities of other factors remaining the same, for a continuous production function, the first order partial derivatives of the factors of production would give us their marginal products. In the same way, we can define the marginal cost of the commodity. It is the increment in total cost of production by producing one more extra unit of the commodity. Similarly, the marginal revenue is the increment in the total revenue by products and their selling. The product of marginal revenue and marginal product of factor give us the marginal revenue product of that factor.
1.6 OBJECTIVES OF THE PRESENT STUDY

The present study aims at constructing and solving linear programming problems to estimate the technical, scale, allocative and overall productive efficiencies for certain selected manufacturing sectors from different States of India.

DATA:

The supportive data for the present study have been collected from Annual Survey of Industries (ASI) for the year 1999-2000. Further, fourteen major States of India along with 'All India' comprehensive data (1999-2000) are chosen as Decision Making Units (DMU's). These decision making units are total manufacturing sectors of (1) All India, (2) Andhra Pradesh, (3) Bihar, (4) Gujarat, (5) Haryana, (6) Karnataka, (7) Kerala, (8) Madhya Pradesh, (9) Maharashtra, (10) Orissa, (11) Punjab, (12) Rajasthan, (13) Tamil Nadu, (14) Uttar Pradesh and (15) West Bengal.

The variables of the study are

(i) Value Added
(ii) Number of persons employed
(iii) Fixed Capital
and (iv) Wages and Salaries Including Employer's Contribution

These variables are explained as follows:

(i) The value added is arrived at by deducting total input and depreciation from total output.
(ii) The number of persons employed includes all persons employed directly or through any agency and engaged in any manufacturing process or in any other kind of work connected with the manufacturing process. The number of workers or employees is an average number obtained by dividing mandays worked for the number of days, the factory has worked during the reference year.

(iii) Fixed Capital represents the depreciated value of fixed assets owned by the factory as on the closing day of the accounting year. Fixed assets are those, which have normal productive life of more than one year. Fixed capital includes land including leasehold land, buildings, plant and machinery, furniture and fixtures, transport equipment, water system and road ways and other fixed assets such as hospitals, schools etc., used for benefit of factory personnel.

(iv) Wages and Salaries are defined to include all remuneration in monetary terms and also payable more or less regularly in each pay period to workers as compensation for the work done during the accounting year. It includes (a) direct wages and salary, (b) remuneration for the period not worked, (c) bonus and ex gratia payment paid both at regular and less frequent intervals. It also excludes inputs on the benefits in kind, employer's contribution to old age benefits and other social security benefits changes, direct expenditure on maternity benefits creches and other group benefits. The wages are expressed in terms of group value.
And, the two derived variables are given by (i) wage rate and (ii) rate of return on capital, which are given by

(i) \[ \text{Wage Rate} = \frac{\text{Wages and salaries including Employer's contribution}}{\text{Number of persons employed}} \]

(ii) \[ \text{Rate of return of capital} = r = \frac{u - w}{f} \]

Here \( u \) = value added
\( w \) = wages and salaries including Employer's contribution
\( f \) = fixed capital

1.7 ORGANIZATION OF THE PRESENT STUDY:-

The plan of the study, documents on how the main objectives of the present study are achieved within the frame work.

Chapter-I provides an introduction and which also deals with the understanding of the term "efficiency" and its measurement, the aims of the present study, source of statistical data and the variable chosen for the study, apart from the plan of the study.

Chapter-II gives a brief review of literature on various frontier production and cost functions and these are reviewed mainly under two heads, namely, (i) Deterministic frontier production function and (ii) Stochastic frontier production function respectively.
Chapter-III deals with constructing and solving linear programming problems to estimate the technical, scale, allocative and overall productive efficiencies of manufacturing sectors.

Chapter-IV is an empirical investigation in which an attempt has been made to measure various technical efficiencies such as (i) Input Pure Technical Efficiency, (ii) Input Overall Technical Efficiency, (iii) Input Scale Efficiency, (iv) Overall Productive Efficiency and (v) Input Allocative Efficiency for the data collected on selected variables of decision making units of total manufacturing sectors of Indian states.

Chapter-V presents the summary and conclusions based on the present work.

1.8 CHAPTER SCHEME:-

The contents of the present research work are documented in Five Chapters as listed below.

Chapter - I : Introduction
Chapter - II : Review of Literature
Chapter - III : Theory and Methodology
Chapter - IV : Empirical Investigations
Chapter - V : Summary and Conclusions.