CHAPTER - IV

BAYESIAN INFERENCE AND FORECASTING WITH APPLICATIONS
4.1 INTRODUCTION

In this chapter the Bayesian estimation methods are applied to the data on crude death rate, infant mortality rate, neonatal mortality rate, post-neonatal mortality rate, perinatal mortality rate, collected from Sample Registration System, India. Linear trend model was used for estimating the future mortality rates and for constructing the confidence limits for the prediction values.

Data related to the above mortality rates of India is given in Annexure - 1

4.2 FORECASTING - BAYESIAN METHODS

The Bayesian analysis of forecasting of Crude Death Rate (CDR) in rural areas of India is carried out as follows:

The linear growth equation \( U_t = a + b \, t \)

where \( U_t \) = dependent variable

\( t \) = time

\( a \) = intercept

\( b \) = regression coefficient

\[ b = \frac{-\hat{b} \pm 1.96 \times \text{S.E}(\hat{b})}{\sum (t - \bar{t})^2} \]

Linear Growth Rate: \( \frac{\hat{b} \times 100}{u} = -2.0075 \)

Confidence limits for \( b \): \( \hat{b} = \hat{b} \pm 1.96 \times \text{S.E}(\hat{b}) \)

Standard error of \( b \): \( \frac{\sum (u - \bar{u})^2}{(n - 2) \sum (t - \bar{t})^2} \)

Confidence limits for \( \hat{b} \): \([-0.3336, -0.0834]\)

Mean: 10.3875

Variance: 1.0745

Standard deviation: 1.0366; Standard error of mean: 0.2592

Confidence limits for mean: \( \bar{u} \pm 2.58 \times \text{S.E}(\bar{u}) \):
= [9.7189, 11.0561]

Considering the linear trend model \( X_t = b_1 + b_2 t + \epsilon_t \), we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \epsilon_t \sim N(0, \sigma^2_\epsilon) \), we use a normal prior for \( b = [b_1 - b_2] \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is, \( b_1^{-1} = 10.3875 \)
\( b_2^{-1} = -0.2085 \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are
\[
\text{Var}(b_1) = v_{b1} = [0.0497]
\]
\[
\text{Var}(b_2) = v_{b2} = [0.0017]
\]
The prior variance covariance matrix is : \( v' = \begin{bmatrix} 0.0497 & 0 \\ 0 & 0.0017 \end{bmatrix} \)

At time '0', the mortality rate in period \( t \), is estimated to be normally distributed with mean : \( m(0+k) = b_1 + b_2 k = 10.3875 - 0.2085k \)

and variance: \( s^2(0+k) = z'(0+k)v'z(0+k) + \sigma^2_\epsilon = [1 k] \begin{bmatrix} 0.0497 & 0 \\ 0 & 0.0017 \end{bmatrix} [k] + 1.0745 \)

For example, the mortality in the 5\textsuperscript{th} year is assumed to be normal with mean and variance:
\[ M(0+5) = 9.3448; \quad S^2(0+5) = 1.1676 \]

A 95% Bayesian prediction interval for \( X_5 \) at time 0, is given by
\[
= [7.2269, 11.4628]
\]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by
\[
= [6.0691, 10.5353]
\]

Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[
z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \quad x'x = \begin{bmatrix} 12.2 & 12 & 12 & \cdots & 9.1 \end{bmatrix}
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} ; \quad g = z'x = \begin{bmatrix} 166.2 \\ 1341.8 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} -0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} ; \quad b = G^{-1}g = \begin{bmatrix} 12.16 \\ -0.2085 \end{bmatrix}
\]

The sample variance and covariance matrix is:
\[
V = G^{-1} \sigma_y^2 = \begin{bmatrix} 0.2955 & -0.0268 \\ -0.0268 & 0.0032 \end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[
v^* - 1 = v^{-1} + v^{-}
\]

\[
b^* = v^*(v^{-1}b^{-1} + v^{-1}b), \text{ where } v \text{ and } b \text{ are vectors.}
\]

\[
v^{-1} = v^{* -1} + v^{-1}
\]

\[
\begin{bmatrix} 20.1334 & 0 \\ 0 & 575.0093 \end{bmatrix} + \begin{bmatrix} 14.8906 & 126.57 \\ 126.57 & 1392.28 \end{bmatrix}
\]

\[
v^{* -1} = \begin{bmatrix} 35.0240 & 126.5705 \\ 126.5705 & 1967.285 \end{bmatrix}
\]

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The expected value of the posterior distribution of crude death rate in period \((16+k)\) is to be used as the point forecast for that period. That is,

\[
\dot{x} = M(16+k) = 8.8628 - 0.1231k
\]

The probability distribution of \(X_{16+k}\) has variance given by the equation:

\[
\sigma^2(16+k) = 0.0372 - 0.0024(16+k) + 0.0007(16+k) + 1.0745
\]

With the help of the equations (i) and (ii), we predict mean and variance at the end of 16\(^{th}\) year. The predicted values and 95% confidence limits are given in Table 4.1.

**Table 4.1: Predictions of Crude Death Rate in rural at the end of 16\(^{th}\) year:**

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>8.7397</td>
<td>23.1964</td>
<td>-0.7001</td>
<td>18.1796</td>
</tr>
<tr>
<td>18</td>
<td>8.6166</td>
<td>25.8761</td>
<td>-1.3536</td>
<td>18.5868</td>
</tr>
<tr>
<td>19</td>
<td>8.4935</td>
<td>28.7092</td>
<td>-2.0083</td>
<td>18.9954</td>
</tr>
<tr>
<td>20</td>
<td>8.3704</td>
<td>31.6957</td>
<td>-2.6642</td>
<td>19.4049</td>
</tr>
<tr>
<td>21</td>
<td>8.2473</td>
<td>34.8356</td>
<td>-3.321</td>
<td>19.8156</td>
</tr>
<tr>
<td>22</td>
<td>8.1242</td>
<td>38.1241</td>
<td>-3.9777</td>
<td>20.2262</td>
</tr>
<tr>
<td>23</td>
<td>8.0011</td>
<td>41.5766</td>
<td>-4.6368</td>
<td>20.6390</td>
</tr>
</tbody>
</table>
At the end of the 23rd year, we may revise the probability distribution of \( b \) using either equation (i) all 16 data points in the least-squares computation and revise the original prior, or (ii) use only the data points for years 17 through 23 in the least-squares analysis and revise the probability distribution computed after the first 16 observations.

The Bayesian analysis of forecasting of Infant Mortality Rate (IMR) in rural areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b t \),

* where \( U_t \) = dependent variable \\
* \( t \) = time \\
* \( a \) = intercept \\

\[ b = \text{regression coefficient} = \frac{\sum (t u_t - \bar{u} \bar{t})}{\sum (t - \bar{t})^2} = -2.2087 \]

Linear Growth Rate: \( \frac{\hat{b}}{u_t} \times 100 = -2.5963 \)

Confidence limits for \( \hat{b} \): \( \hat{b} = \hat{b} \pm t_{0.025} \times \text{S.E.} (\hat{b}) \)

Standard error of \( b \): \[ \frac{(u - \bar{u})^2}{(n - 2) \sum (t - \bar{t})^2} \]

0.3921

Confidence limits for \( \hat{b} \): [-3.5550, -0.8623]

Mean: 85.0687; Variance: 124.4383

Standard deviation: 11.1552; Standard error of mean: 2.7887

Confidence limits for mean: \( \bar{u} \pm 2.58 \times \text{S.E.} (\bar{x}) = [77.8736, 92.2638] \)

Considering the linear trend model \( X_t = b_1 + b_2 t + s_t \): we use the results in the following paragraphs:
We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \epsilon_i \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1 - b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is,

\[
\begin{align*}
    b_1^{-1} &= 85.0687 \\
    b_2^{-1} &= -2.2087
\end{align*}
\]

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

\[
\begin{align*}
    \text{Var}(b_1) &= \nu_{11} = 5.7522 \\
    \text{Var}(b_2) &= \nu_{22} = 0.2014
\end{align*}
\]

The prior variance covariance matrix is:

\[
\begin{bmatrix}
    5.7522 & 0 \\
    0 & 0.2014
\end{bmatrix}
\]

At time '0', the mortality rate in period \( t \), is estimated to be normally distributed with mean: \( m(0+k) = b_1^{-1} + b_2^{-1}k = 85.0687 - 2.2087k \)

And variance is given by \( s^2(0+k) = z'(0+k)\nu z(0+k) + \sigma^2 \)

\[
\begin{bmatrix}
    5.7522 & 0 \\
    0 & 0.2014
\end{bmatrix} + 124.4383
\]

For example, the mortality in the 5\(^{th}\) year is assumed to be normal with mean and variance:

\[
M(0+5) = 74.025 \quad S^2(0+5) = 135.2256
\]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by

\[
[96.8175, 51.2332]
\]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by

\[
[87.0134, 38.9505]
\]
Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[ z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \]

\[ xx' = \begin{bmatrix} 105 & 104 & 102 & \cdots & 71.7 \end{bmatrix} \]

\[ G = zz' = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} ; \quad g = z'x = \begin{bmatrix} 1362.1 \\ 10818.4 \end{bmatrix} \]

\[ G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} ; \quad \hat{b} = G^{-1}g = \begin{bmatrix} 103.8425 \\ -2.2086 \end{bmatrix} \]

The sample variance and covariance matrix is:

\[ V = G^{-1} \sigma^2 = \begin{bmatrix} 34.2205 & -3.1109 \\ -3.1109 & 0.3659 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[ v^{*^{-1}} = v^{-1} + v^{-1} \]

\[ b^* = v^*(v^{-1}b^{-1} + v^{-1}\hat{b}), \] where v and b are vectors.

\[ v^{*^{-1}} = v^{-1} + v^{-1} \]

\[ \begin{bmatrix} 0.1738 & 0 \\ 4.9650 \end{bmatrix} + \begin{bmatrix} 0.1285 & 1.0929 \\ 1.0929 & 12.022 \end{bmatrix} \]

\[ v^{*^{-1}} = \begin{bmatrix} 0.3024 & 1.0929 \\ 1.0929 & 16.9870 \end{bmatrix} \]

\[ v^* = \begin{bmatrix} 4.8082 & -0.2772 \\ -0.2772 & 0.0767 \end{bmatrix} \]

\[ b^* = v^*v^{*^{-1}}b^{-1} + v^{-1}\hat{b} \]

\[ = \begin{bmatrix} 89.7812 \\ -1.3040 \end{bmatrix} \]
The expected value of the posterior distribution of infant mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 68.9172 - 1.3040K \) \( \ldots \) (i)

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[
z'z(t+k) + \sigma^2 \text{ and }
\]

\[
s^2(16+k) = \begin{bmatrix} 1 & 16+k \end{bmatrix} \begin{bmatrix} 4.3083 & -0.2772 \\ -0.2772 & 0.0767 \end{bmatrix} \begin{bmatrix} 1 \\ 16+k \end{bmatrix} + 124.438 \ldots (\text{ii})
\]

With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\(^{th}\) year. The predicted values are given in Table 4.2.

Table 4.2: Predictions of Infant Mortality Rate in rural at the end of 16\(^{th}\) year

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>67.6132</td>
<td>141.4878</td>
<td>44.2993</td>
<td>90.9271</td>
</tr>
<tr>
<td>18</td>
<td>66.3092</td>
<td>143.6179</td>
<td>42.8204</td>
<td>89.7979</td>
</tr>
<tr>
<td>19</td>
<td>65.0052</td>
<td>145.9014</td>
<td>41.3304</td>
<td>88.6799</td>
</tr>
<tr>
<td>20</td>
<td>63.7012</td>
<td>148.3383</td>
<td>39.8295</td>
<td>87.5729</td>
</tr>
<tr>
<td>21</td>
<td>62.3972</td>
<td>150.9286</td>
<td>37.718</td>
<td>86.4764</td>
</tr>
<tr>
<td>22</td>
<td>61.0932</td>
<td>153.6723</td>
<td>36.7961</td>
<td>85.3903</td>
</tr>
<tr>
<td>23</td>
<td>59.7892</td>
<td>156.5694</td>
<td>35.2645</td>
<td>84.3142</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Neò Ntal Mortality Rate (NNMR) in rural areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b \cdot t \),

where \( U_t = \text{dependent variable} \).

\( t = \text{time} \)
\[ a = \text{intercept} \]

\[ b = \text{regression coefficient} \]

\[ \hat{b} = \frac{\sum_{i=1}^{n} (u_i - \bar{u})}{\sum (t - \bar{t})^2} = -1.254 \]

Linear Growth Rate: \[ \frac{\hat{b} \cdot 100}{100} = -2.3123 \]

Confidence limits for \( b \):

\[ \hat{b} = \hat{b} \pm t_{a-2\alpha} \text{S.E}(\hat{b}) \]

Standard error of \( \hat{b} \):

\[ \text{S.E}(\hat{b}) = \sqrt{\frac{(u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2}} = 0.3490 \]

Confidence limits for \( \hat{b} \):

\[ [-2.0044, -0.5036] \]

Mean: 54.2312

Variance: 38.6582

Standard deviation: 6.2175

Standard error of mean: 1.5544

Confidence limits for mean: \( \bar{u} \pm 2.58 \text{S.E}(\bar{u}) = [50.2209, 58.2416] \)

Considering the linear trend model \( X_t = b_1 + b_2 t + \epsilon_t \): we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \epsilon_t \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1, b_2] \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is, \( b_1^{-1} = 54.2313 \)

\( b_2^{-1} = -1.254, \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.
The variances are:

\[
\begin{align*}
\text{Var}(b_1) &= v_{11} = [1.7868] \\
\text{Var}(b_2) &= v_{22} = [0.0626]
\end{align*}
\]

The prior variance covariance matrix is:

\[
\begin{bmatrix}
1.7868 & 0 \\
0 & 0.0626
\end{bmatrix}
\]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: \( m(0+k) = b_1^{-1} + b_2^{-1}k = 10.3875 + -0.2085 \) (k)

And variance is given by
\[
s^2(0 + k) = z'(0 + k)\nu'(0 + k) + \sigma^2_e
\]

\[
= [1 \ k] \begin{bmatrix} 1.7868 & 0 \\ 0 & 0.0626 \end{bmatrix} + 38.6582
\]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:

\[
M(0+5) = 47.9614 \\
S^2(0+5) = 42.0095
\]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by

\[
= [35.2577, 60.6651]
\]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by

\[
= [28.2971, 55.0859]
\]

Using this information, posterior distribution at \( t=16 \) is computed using least squares method as follows:

\[
z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix}
\]

\[
x'x = [65.5 \ 63.6 \ 62 \ \cdots \ 44]
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \quad g = z'x = [867.7] \begin{bmatrix} 6949.1 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} \quad \hat{b} = G^{-1}g = [64.89] \begin{bmatrix} -1.2539 \end{bmatrix}
\]
The sample variance and covariance matrix is:

\[ V = \begin{pmatrix} 10.6310 & -0.9664 \\ -0.9664 & 0.1137 \end{pmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[ \nu^{* -1} = \nu^{\prime -1} + \nu^{-1} \]

\[ b^* = \nu^*(\nu^{-1}\nu^{-1} + \nu^{\prime -1}b) \]

where \( \nu \) and \( b \) are vectors.

\[
\begin{align*}
\nu^{* -1} &= \nu^{\prime -1} + \nu^{-1} \\
&= \begin{bmatrix} 0.5596 & 0 \\ 0 & 15.9823 \end{bmatrix} + \begin{bmatrix} 0.4138 & 3.518 \\ 3.518 & 38.698 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\nu^* &= \begin{bmatrix} 0.9734 & 13.5180 \\ 3.5180 & 54.6803 \end{bmatrix}
\end{align*}
\]

\[ b^* = \nu^*\nu^{-1}\nu^{-1} + b^* \]

\[
\begin{align*}
\nu^* &= \begin{bmatrix} 56.9067 \\ -0.7403 \end{bmatrix}
\end{align*}
\]

The expected value of the posterior distribution of neonatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 56.9067 + (-0.7403)(16+k) \) \ ...(i)

\[
\begin{align*}
\hat{x} &= 45.0619 - 0.7403k
\end{align*}
\]

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[ z'(t+k)\nu^*(t+k)+\sigma^2_k \]

\[ s^2(16+k) = \begin{bmatrix} 1.3384 & -0.0861 \\ -0.0861 & 0.0238 \end{bmatrix} 16+k \]

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With the help of the equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year. The predicted values are given in Table 4.3

Table 4.3 Predictions of neonatal mortality rate in rural at the end of 16\textsuperscript{th} year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>44.3216</td>
<td>43.9475</td>
<td>31.3282</td>
<td>57.3150</td>
</tr>
<tr>
<td>18</td>
<td>43.5813</td>
<td>44.6083</td>
<td>30.4906</td>
<td>56.6720</td>
</tr>
<tr>
<td>19</td>
<td>42.841</td>
<td>45.3167</td>
<td>29.6467</td>
<td>56.0353</td>
</tr>
<tr>
<td>20</td>
<td>42.1007</td>
<td>46.0727</td>
<td>29.0566</td>
<td>55.4046</td>
</tr>
<tr>
<td>21</td>
<td>41.3604</td>
<td>46.4717</td>
<td>27.999</td>
<td>54.7217</td>
</tr>
<tr>
<td>22</td>
<td>40.6201</td>
<td>47.7275</td>
<td>27.0794</td>
<td>54.1608</td>
</tr>
<tr>
<td>23</td>
<td>39.8798</td>
<td>48.6263</td>
<td>26.2113</td>
<td>53.5474</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Post Neonatal Mortality Rate (PNMR) in rural areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b t \),

where \( U_t \) = dependent variable

\( t \) = time

\( a \) = intercept

\( b \) = regression coefficient \( \hat{b} = \frac{\sum (u_t - \bar{u}_n) t}{\sum (t - \bar{t})^2} = -0.9563 \)

Linear Growth Rate: \( \frac{\hat{b}}{u} \times 100 = -3.1132 \)

Confidence limits for \( b \) : \( \hat{b} = \hat{b} \pm t_{\alpha/2 \times n} \) S.E (\( \hat{b} \) )
Standard error of \( \hat{b} \) is \( \frac{\sqrt{(u_k - \bar{y})^2}}{(n-2)\sum (t_i - \bar{t})^2} \) = 0.2965

Confidence limits for \( \hat{b} \): [-1.5938, -0.3188]

Mean: 30.7187

Variance: 27.8976

Standard deviation: 5.2818

Standard error of mean: 1.3204

Confidence limits for mean: \( \bar{y}_t \pm 2.58 \text{ S.E.} \times \bar{y}_t = [27.3119, 34.1255] \)

Considering the linear trend model \( X_t = b_1 + b_2 \cdot t + \varepsilon_t \): we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \varepsilon_t \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1 - b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is, \( b_1^{-1} = 30.7187 \)

\( b_2^{-1} = -0.9563 \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are

\[ \text{Var}(b_1) = \nu_{11} = [1.2895] \]

\[ \text{Var}(b_2) = \nu_{22} = [0.0451] \]

The prior variance covariance matrix is: \( \nu' = \begin{bmatrix} 1.2896 & 0 \\ 0 & 0.0451 \end{bmatrix} \)
At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: \( m(0+k) = b_1 + b_2 k = 30.7187 + -0.9563(k) \)
And variance is given by \( s^2(0+k) = z'(0+k)vz(0+k) + \sigma_v^2 \)
\[
\begin{bmatrix}
1.2896 & 0 \\
0 & 0.0451
\end{bmatrix}
\begin{bmatrix}
k
\end{bmatrix}
+ 27.8976
\]
For example, the mortality in the 5th year is assumed to be normal with mean and variance:
\[
\begin{align*}
M(0+5) &= 25.9371 \\
S^2(0+5) &= 30.3160
\end{align*}
\]
A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by:
\[
[15.1454, 36.7288]
\]
Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by
\[
[9.7769, 32.5340]
\]
Using this information, posterior distribution at \( t = 16 \) is computed using least squares method as follows:
\[
z' = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & 2 & 3 & \cdots & 16
\end{bmatrix}
\]
\[
x'x = \begin{bmatrix}
39.1 & 40.5 & 40.1 & \cdots & 27.7
\end{bmatrix}
\]
\[
G = z'z = \begin{bmatrix}
16 & 136 \\
136 & 1496
\end{bmatrix} \quad ; \quad g = z'x = \begin{bmatrix}
491.5 \\
3852.6
\end{bmatrix}
\]
\[
G^{-1} = \begin{bmatrix}
0.275 & -0.025 \\
-0.025 & 0.0029
\end{bmatrix} \quad ; \quad \hat{b} = G^{-1}g = \begin{bmatrix}
38.8475 \\
-0.9563
\end{bmatrix}
\]
The sample variance and covariance matrix is:
\[
V = G^{-1}\sigma^2 = \begin{bmatrix}
7.6718 & -0.6974 \\
-0.6974 & 0.0820
\end{bmatrix}
\]
The parameters of the posterior distribution are computed using the following equations:

\[ v^{e-1} = v^{e-1} + v^{-1} \]

\[ b^e = v^e (v^{-1} b^{-1} + v^{-1} \hat{b}) \], where \( v \) and \( b \) are vectors.

\[ v^{e-1} = v^{e-1} + v^{-1} \]

\[
\begin{bmatrix}
0.7754 & 0 \\
0 & 22.1469
\end{bmatrix} +
\begin{bmatrix}
0.5735 & 4.8749 \\
4.8749 & 53.6246
\end{bmatrix} =
\begin{bmatrix}
1.3489 & 4.8749 \\
4.8749 & 75.7715
\end{bmatrix}
\]

\[ v^e = \begin{bmatrix} 1.3489 & -0.0621 \\ -0.0621 & 0.0172 \end{bmatrix} \]

\[ b^e = v^e b^{-1} + v^{-1} \hat{b} \]

\[
\begin{bmatrix}
32.7592 \\
-0.5646
\end{bmatrix}
\]

The expected value of the posterior distribution of post neonatal mortality rate in period (16+k) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 32.7592 + (-0.5646)(16+k) \)

\[ = 23.7256 - 0.5646k \quad ... \text{(i)} \]

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[ z'(t + k) v^e z(t + k) + \sigma^2_z \text{ and} \]

\[ s^2(16+k) = [I \ 16+k] \begin{bmatrix}
0.9658 & -0.0621 \\
-0.0621 & 0.0171
\end{bmatrix} \begin{bmatrix}
1 \\
16+k
\end{bmatrix} + 27.8916 \quad ... \text{(ii)} \]

With the help of this equations (i) and (ii), we predict mean and variance at the end of 16th year.
Table 4.4 Predictions of post neonatal mortality rate in rural at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>23.161</td>
<td>31.6939</td>
<td>12.1267</td>
<td>34.1953</td>
</tr>
<tr>
<td>18</td>
<td>22.5964</td>
<td>32.1682</td>
<td>11.4799</td>
<td>33.7129</td>
</tr>
<tr>
<td>19</td>
<td>22.0318</td>
<td>32.6767</td>
<td>10.8277</td>
<td>33.2358</td>
</tr>
<tr>
<td>20</td>
<td>21.4672</td>
<td>33.2194</td>
<td>10.1705</td>
<td>32.7639</td>
</tr>
<tr>
<td>21</td>
<td>20.9026</td>
<td>33.7963</td>
<td>9.5082</td>
<td>32.2969</td>
</tr>
<tr>
<td>22</td>
<td>20.338</td>
<td>34.4074</td>
<td>8.8411</td>
<td>31.8349</td>
</tr>
<tr>
<td>23</td>
<td>19.7734</td>
<td>35.0527</td>
<td>8.1692</td>
<td>31.3776</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Peri Natal Mortality Rate (PENMR) in rural areas of India is carried out as follows:

The linear growth equation: $U_t = a + b t$

where $U_t = \text{dependent variable}$

$t = \text{time}$

$a = \text{intercept}$

$b = \text{regression coefficient}$

$$b = \frac{\sum t u_t - n \bar{u}}{\sum (t - \bar{t})^2} = -0.75$$

Linear Growth Rate: $\ast 100 = -1.5629$

Confidence limits for $b$: $\hat{b} = b \pm t_{n-1,\alpha} \text{ S.E.}(\hat{b})$.

Standard error of $\hat{b} = \frac{\sum (u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2} = 0.2256$

Confidence limits for $\hat{b}$: $[-1.2351, -0.2649]$

Mean: 47.9875

Variance: 16.1518
Standard deviation: 4.0189
Standard error of mean: 1.0047
Confidence limits for mean: $\bar{x} \pm 2.58 \text{S.E}(\bar{x}) = [45.3953, 50.5797]

Considering the linear trend model $X_t = b_1 + b_2 t + \epsilon$, we use the results in the following paragraphs:

We know that $Z_1(t) = 1$ and $Z_2(t) = t$

Since we assume that $\epsilon_i \sim N(0, \sigma^2)$, we use a normal prior for $b = [b_1, b_2]'$

The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is,

$b_1^{-1} = 47.9875$

$b_2^{-1} = -0.75$

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

$\text{Var}(b_1) = \nu_1 = [0.7466]$  
$\text{Var}(b_2) = \nu_2 = [0.0261]$

The prior variance covariance matrix is: $\nu = \begin{bmatrix} 0.7466 & 0 \\ 0 & 0.0261 \end{bmatrix}$

At time ‘0’, the mortality rate in period $t$, is estimated to be normally distributed with mean:

$m(0+k) = b_1^{-1} + b_2^{-1}k = 47.9875 + -0.75 (k)$

and variance is given by $s^2(0+k) = \nu'(0+k)\nu(0+k) + \sigma^2$

$= [1 k] \begin{bmatrix} 0.7466 & 0 \\ 0 & 0.0261 \end{bmatrix} [1 k] + 16.1518$
For example, the mortality in the 5th year is assumed to be normal with mean and variance:

\[ M(0+5) = 44.2375 \]
\[ S^2(0+5) = 17.552 \]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by

\[ = [36.0260, 52.4489] \]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time ‘0’ is given by

\[ = [31.8295, 49.1454] \]

Using this information, posterior distribution at \( t = 16 \) is computed using least squares method as follows:

\[
\begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & 2 & 3 & \cdots & 16
\end{bmatrix}
\]

\[
x'x = \begin{bmatrix} 51.8 & 54.4 & 53.1 & \cdots & 39.1 \end{bmatrix}
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \quad g = z'x = \begin{bmatrix} 767.8 \\ 6271.3 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} \quad \hat{b} = G^{-1}g = \begin{bmatrix} 54.3625 \\ -0.75 \end{bmatrix}
\]

The sample variance and covariance matrix is:

\[
V = G^{-1}\sigma^2_i = \begin{bmatrix} 4.4417 & -0.4037 \\ -0.4037 & 0.0475 \end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[ v^{-1} = v'^{-1} + v^{-1} \]

\[ b^* = v'(v'^{-1}b^{-1} + v^{-1}\hat{b}) \], where \( v \) and \( b \) are vectors.

\[ v'^{-1} = v^{-1} + v^{-1} \]

\[
\begin{bmatrix}
1.3393 & 0 \\
0 & 38.2524
\end{bmatrix} + \begin{bmatrix}
0.9906 & 8.4201 \\
8.4201 & 92.6211
\end{bmatrix}
\]
The expected value of the posterior distribution of perinatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \(\hat{x} = M(16+k) = 49.5877 + (-0.4428)(16+k)\)

\[
\hat{x} = 49.5877 - 0.4428k
\]

The probability distribution of \(X_{16+k}\) has variance given by the equation:

\[
s^2(16+k) = \left[ 0.5592 \quad -0.0359 \right] \begin{bmatrix} 1 \\ 16+k \end{bmatrix} + 16.1518 \quad \ldots \quad (i)
\]

With the help of these equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year.

Table: 4.5 Predictions of perinatal mortality rate in rural at the end of 16\textsuperscript{th} year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted Mean</th>
<th>Predicted Variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>18.3515</td>
<td>42.0601</td>
<td>5.6402</td>
<td>31.0628</td>
</tr>
<tr>
<td>18</td>
<td>18.6262</td>
<td>41.6173</td>
<td>5.982</td>
<td>31.2704</td>
</tr>
<tr>
<td>19</td>
<td>18.9207</td>
<td>41.1745</td>
<td>6.3439</td>
<td>31.4975</td>
</tr>
<tr>
<td>20</td>
<td>19.235</td>
<td>40.7317</td>
<td>6.7261</td>
<td>31.7439</td>
</tr>
<tr>
<td>21</td>
<td>19.5691</td>
<td>40.2889</td>
<td>7.1283</td>
<td>32.0099</td>
</tr>
<tr>
<td>22</td>
<td>19.923</td>
<td>39.8461</td>
<td>7.5508</td>
<td>32.2953</td>
</tr>
<tr>
<td>23</td>
<td>20.2967</td>
<td>39.4033</td>
<td>7.9934</td>
<td>32.3033</td>
</tr>
</tbody>
</table>
The Bayesian analysis of forecasting of Crude Death Rate (CDR) in urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b_t \)

where \( U_t \) = dependent variable
\( t \) = time
\( a \) = intercept
\( b \) = regression coefficient
\[
\hat{b} = \frac{\sum (u_t - \bar{u})}{\sum (t - \bar{t})^2} = -0.0931
\]

Linear Growth Rate: \( \frac{\hat{b}}{u_t} \times 100 = -1.3778 \)

Confidence limits for \( b \): \( \hat{b} \pm t_{n-2a%} \text{ S.E} (\hat{b}) \)

Standard error of \( \hat{b} = \frac{\sum (u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2} = 0.0009 \)

Confidence limits for \( \hat{b} \): \([-0.1590,-0.0271]\)

Mean: 6.7562

Variance: 0.2986

Standard deviation: 0.5465

Standard error of mean: 0.1366

Confidence limits for mean: \( \bar{u} \pm 2.58 \text{ S.E} (\bar{u}) = [6.4038, 7.1087] \)

Considering the linear trend model \( X_t = b_1 + b_2 t + \varepsilon_t \): we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \varepsilon_t \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1 - b_2] \)
The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is, 

$$b_1^{-1} = 6.7562$$
$$b_2^{-1} = -0.0931$$

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

$$\text{Var}(b_1) = \nu_{11} = [0.0138]$$
$$\text{Var}(b_2) = \nu_{22} = [0.0004]$$

The prior variance covariance matrix is:

$$\nu' = \begin{bmatrix} 0.0138 & 0 \\ 0 & 5E-04 \end{bmatrix}$$

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean $m(0+k) = b_1^{-1} + b_2^{-1}k = 50.65 - 1.3356k$ and variance is given by $s^2(0+k) = z'(0+k)v'z(0+k)+\sigma^2$.

$$= [1 \ k] \begin{bmatrix} 0.0138 & 0 \\ 0 & 5E-04 \end{bmatrix} [1 \ k] + 0.2986$$

For example, the mortality in the 5th year is assumed to be normal with mean and variance:

$$M(0+5) = 6.2908$$
$$S^2(0+5) = 0.3245$$

A 95% Bayesian prediction interval for $X_5$, at time 0, is given by

$$= [5.1743, 7.4073]$$

Similarly a 95% Bayesian prediction for $X_{10}$, made at time '0' is given by

$$= [4.6481, 7.0026]$$
Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[
z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix}
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \quad g = z'x = \begin{bmatrix} 108.1 \\ 887.2 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} \quad \hat{b} = G^{-1}g = \begin{bmatrix} -7.5475 \\ -0.0931 \end{bmatrix}
\]

The sample variance and covariance matrix is:

\[
V = G^{-1}\sigma^2 = \begin{bmatrix} 0.0821 & -0.0075 \\ -0.0075 & 0.0008 \end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[
v^*-1 = v'^{-1} + v^{-1}
\]

\[
b^* = v^*(v'^{-1}b^{-1} + v^{-1}\hat{b}) \quad \text{where v and b are vectors.}
\]

\[
v'^{-1} = v'^{-1} + v^{-1}
\]

\[
v^* = \begin{bmatrix} 72.4430 & 0 \\ 0 & 2068.97 \end{bmatrix} + \begin{bmatrix} 53.5789 & 455.4 \\ 455.4 & 5010 \end{bmatrix}
\]

\[
v^* = \begin{bmatrix} 126.022 & 455.421 \\ 455.421 & 7078.6 \end{bmatrix}
\]

\[
v^* = \begin{bmatrix} 0.0103 & -0.0007 \\ -0.0007 & 0.0002 \end{bmatrix}
\]

\[
b^* = v^*[v'^{-1}b^{-1} + v^{-1}\hat{b}]
\]

\[
b^* = \begin{bmatrix} 6.9548 \\ -0.0549 \end{bmatrix}
\]

The expected value of the posterior distribution of crude death rate in period (16+k) is to be used as the point forecast for that period.
That is, \( \dot{x} = M(16+k) = 6.0764 - 0.0549k \) \( ... (i) \)

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[
\sigma^2(16+k) = \left[I_{16+k} \begin{bmatrix} 0.0103 & -0.0007 \\ -0.0007 & 0.0002 \end{bmatrix}_{16+k} + 0.2986 \right] \quad \text{... (ii)}
\]

With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\(^{th}\) year.

**Table: 4.6 Predictions of crude death rate in urban at the end of 16\(^{th}\) year.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>6.0215</td>
<td>0.3429</td>
<td>4.8738</td>
<td>7.1692</td>
</tr>
<tr>
<td>18</td>
<td>5.9666</td>
<td>0.3485</td>
<td>4.8095</td>
<td>7.1237</td>
</tr>
<tr>
<td>19</td>
<td>5.9117</td>
<td>0.3545</td>
<td>4.7448</td>
<td>7.0787</td>
</tr>
<tr>
<td>20</td>
<td>5.8568</td>
<td>0.3609</td>
<td>4.6793</td>
<td>7.0343</td>
</tr>
<tr>
<td>21</td>
<td>5.8019</td>
<td>0.3677</td>
<td>4.6134</td>
<td>6.9904</td>
</tr>
<tr>
<td>22</td>
<td>5.747</td>
<td>0.3749</td>
<td>4.5469</td>
<td>6.9471</td>
</tr>
<tr>
<td>23</td>
<td>5.6921</td>
<td>0.3825</td>
<td>4.492</td>
<td>6.9043</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Infant Mortality Rate (IMR) in urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b \times t \),

where \( U_t \) = dependent variable,

\( t \) = time

\( a \) = intercept
b = regression coefficient $\hat{b} = \frac{\sum (u_i - \bar{u})}{\sum (t - \bar{t})^2} = -1.3356$

Linear Growth Rate: $\frac{\hat{b} \cdot 100}{u_i} = -2.6369$

Confidence limits for $b$: $\hat{b} = b \pm t_{n-2\alpha/2} \text{S.E.}(\hat{b})$

Standard error of $\hat{b} = \frac{\sqrt{\sum (u - \bar{u})^2}}{[(n-2)\sum (t - \bar{t})^2]} = 0.1497$

Confidence limits for $\hat{b}$: [-2.1677, -0.5035]

Mean: 50.65

Variance: 47.5346

Standard deviation: 6.8945

Standard error of mean: 1.7236

Confidence limits for mean: $\bar{u} \pm 2.58 \text{S.E.}(\bar{x}) = [46.203, 55.097]$

Considering the linear trend model $X_i = b_1 + b_2 t + \varepsilon_i$, we use the results in the following paragraphs:

We know that $Z_1(t) = 1$ and $Z_2(t) = t$

Since we assume that $\varepsilon_i \sim N(0, \sigma_i^2)$, we use a normal prior for $b = [b_1 - b_2]'$

The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is, $b_1' = 50.65$

$\hat{b}_2' = -1.3356$
The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:
\[
\text{Var}(b_1) = \nu'_1 = 2.1973 \\
\text{Var}(b_2) = \nu'_2 = 0.0769
\]

The prior variance covariance matrix is:
\[
\nu' = \begin{bmatrix} 2.1973 & 0 \\ 0 & 0.0769 \end{bmatrix}
\]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: \( m(0+k) = b_1^{-1} + b_2^{-1} k = 50.65 - 1.3356 k \) and variance is given by
\[
s^2(0+k) = z'(0+k)\nu'z(0+k) + \sigma^2 = [1 \ k] \begin{bmatrix} 2.1973 & 0 \\ 0 & 0.0769 \end{bmatrix} [1 \ k] + 47.5346
\]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:
\[
M(0+5) = 43.9721 \\
S^2(0+5) = 51.6554
\]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by
\[
= [29.8852, 58.0589]
\]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by
\[
= [22.4413, 52.1469]
\]

Using this information, posterior distribution at \( t=16 \) is computed using least squares method as follows:
\[
z' = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & 2 & 3 & \ldots & 16 \end{bmatrix} \\
P = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} ; \ g = z'x = \begin{bmatrix} 810.4 \\ 6434.3 \end{bmatrix} \\
P^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} ; \ \hat{b} = P^{-1}g = \begin{bmatrix} 62.0025 \\ -1.3356 \end{bmatrix}
\]
The sample variance and covariance matrix is:

\[
V = G^{-1} \sigma_i^2 = \begin{bmatrix}
13.0720 & -1.1884 \\
-1.1884 & 0.1398
\end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[
\nu_s = \nu^{-1} + v^{-1}
\]

\[
b^s = v^s (v^{-1} b^{-1} + v^{-1}) , \text{ where } v \text{ and } b \text{ are vectors.}
\]

\[
\nu_s = \nu^{-1} + v^{-1}
\]

\[
\begin{bmatrix}
0.4551 & 0 & 0.3365 & 2.8610 \\
0 & 12.9978 & 2.8610 & 31.4718
\end{bmatrix}
\]

\[
\nu_s = \begin{bmatrix}
0.7917 & 2.8610 \\
2.8610 & 44.4696
\end{bmatrix}
\]

\[
\nu_s = \begin{bmatrix}
1.6457 & -0.1059 \\
-0.1059 & 0.0293
\end{bmatrix}
\]

\[
b^s = v^s [v^{-1} b^{-1} + v^{-1}]
\]

\[
= \begin{bmatrix}
53.4996 \\
-0.7885
\end{bmatrix}
\]

The expected value of the posterior distribution of infant mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \(\hat{x} = M(16+k) = 40.8836 - 0.7885k\) \... (i)

The probability distribution of \(X_{16+k}\) has variance given by the equation:

\[
z' z(t + k) v^s z(t + k) + \sigma_i^2 \text{ and}
\]

\[
s'(16+k) = [1 \ 16 + k \begin{bmatrix}
1.6457 & -1.1059 \\
-1.1059 & 0.0293
\end{bmatrix} 1 + 47.5346 \ ... (ii)
\]
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year.

**Table: 4.7 Predictions of infant mortality rate in urban at the end of 16\textsuperscript{th} year.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>40.0951</td>
<td>54.0474</td>
<td>25.6858</td>
<td>54.5044</td>
</tr>
<tr>
<td>18</td>
<td>39.3066</td>
<td>54.8611</td>
<td>24.7892</td>
<td>53.8239</td>
</tr>
<tr>
<td>19</td>
<td>38.5181</td>
<td>55.7334</td>
<td>23.8858</td>
<td>53.1504</td>
</tr>
<tr>
<td>20</td>
<td>37.7296</td>
<td>56.6643</td>
<td>22.9756</td>
<td>52.4836</td>
</tr>
<tr>
<td>21</td>
<td>36.9411</td>
<td>57.6538</td>
<td>22.0588</td>
<td>51.8234</td>
</tr>
<tr>
<td>22</td>
<td>36.1526</td>
<td>58.7019</td>
<td>21.1357</td>
<td>51.1696</td>
</tr>
<tr>
<td>23</td>
<td>35.3641</td>
<td>59.8086</td>
<td>20.2063</td>
<td>50.5219</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Neo-natal Morality Rate in urban areas of India is carried out as follows:

The linear growth equation: $U_t = a + b \times t$

where $U_t$ = dependent variable

$\quad t = \text{time}$

$\quad a = \text{intercept}$

$\quad b = \text{regression coefficient} = \frac{\sum_{i=1}^{n}(u_i - \overline{u})(t - \overline{t})}{\sum(t - \overline{t})^2} = -0.6246$

Linear Growth Rate: \[
\frac{\hat{b}}{u_t \times 100} = -2.0694
\]

Confidence limits for $b$: $\hat{b} = \hat{b} \pm t_{n-2,\alpha} \times \text{S.E.}(\hat{b})$

Standard error of $\hat{b}$: $\frac{(u - \overline{u})^2}{(n-2)\sum(t - \overline{t})^2} = 0.0342$
Confidence limits for $\hat{b}$: $[-1.0224, -0.2267]$

Mean: $30.1812$

Variance: $10.8656$

Standard deviation: $3.2963$

Standard error of mean: $0.8241$

Confidence limits for mean: $\bar{x} \pm 2.58 \text{S.E.}(\bar{x}) = [28.0551, 32.3074]$

Considering the linear trend model $X_t = b_1 t + b_2 t + \varepsilon$: we use the results in the following paragraphs:

We know that $Z_1(t) = 1$ and $Z_2(t) = t$

Since we assume that $\varepsilon \sim N(0, \sigma^2)$, we use a normal prior for $b = [b_1 - b_2]$

The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is, $b_1^{-1} = 30.1813$

$b_2^{-1} = -0.6246$

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

$$\text{Var}(b_1) = \nu_1 = 0.5023$$

$$\text{Var}(b_2) = \nu_2 = 0.0176$$

The prior variance covariance matrix is:

$$\nu = \begin{bmatrix} 0.5023 & 0 \\ 0 & 0.0176 \end{bmatrix}$$

At time '0', the mortality rate in period $t$, is estimated to be normally distributed with mean: $m(0+k) = b_1^{-1} + b_2^{-1}k = 30.1813 - 0.6246k$

and variance is given by $s^2(0+k) = z'(0+k)\nu z(0+k) + \sigma^2$

$$= [1 \ k] \begin{bmatrix} 0.5023 & 0 \\ 0 & 0.0176 \end{bmatrix} [1 \ k] + 10.8656$$

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For example, the mortality in the 5th year is assumed to be normal with mean and variance:

\[
\begin{align*}
M(0+5) &= 27.0584 \\
S^2(0+5) &= 11.8075
\end{align*}
\]

A 95% Bayesian prediction interval for \(X_5\), at time 0, is given by

\[
[20.3234, 33.7934]
\]

Similarly a 95% Bayesian prediction for \(X_{10}\), made at time ‘0’ is given by

\[
[16.8344, 31.0368]
\]

Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[
z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix}
\]

\[
xz' = [36.2 \quad 33.3 \quad 34.6 \quad \cdots \quad 24.9]
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \quad ; \quad g = z'x = \begin{bmatrix} 482.9 \\ 3892.3 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} \quad ; \quad \hat{b} = G^{-1}g = \begin{bmatrix} 35.49 \\ -0.6245 \end{bmatrix}
\]

The sample variance and covariance matrix is:

\[
V = G^{-1}\sigma^2_e = \begin{bmatrix} 2.9880 & -0.2716 \\ -0.2716 & 0.03195 \end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[
\nu' = \nu^{-1} + v^{-1}
\]

\[
\hat{b}^* = \nu'(\nu'^{-1}\hat{b} - v^{-1}\hat{b}), \text{ where } v \text{ and } b \text{ are vectors.}
\]
The expected value of the posterior distribution of neonatal mortality rate
in period $(16+k)$ is to be used as the point forecast for that period.

That is, $\hat{x} = M(16+k) = 25.6146 - 0.3687k$ \hspace{1cm} (i)

The probability distribution of $X_{16+k}$ has variance given by the equation:

$z'(t+k) \sigma^2 z(t+k) + \sigma_i^2$ and \hspace{1cm} (ii)

$s^2(16+k) = \begin{bmatrix} 1 & 16 + k \\ 0.3762 & -0.0242 \\ -0.0242 & 0.0067 \end{bmatrix} \begin{bmatrix} 1 \\ 16 + k \end{bmatrix} + 10.8656$

With the help of these equations (i) and (ii), we predict mean and variance
at the end of 16th year.
Table 4.8: Predictions of neonatal mortality rate in urban at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>25.2459</td>
<td>12.3553</td>
<td>18.3565</td>
<td>32.1353</td>
</tr>
<tr>
<td>18</td>
<td>24.8772</td>
<td>12.5414</td>
<td>17.9361</td>
<td>31.8183</td>
</tr>
<tr>
<td>19</td>
<td>24.5085</td>
<td>12.7409</td>
<td>17.5124</td>
<td>31.5046</td>
</tr>
<tr>
<td>20</td>
<td>24.1398</td>
<td>12.9538</td>
<td>17.0855</td>
<td>31.1941</td>
</tr>
<tr>
<td>21</td>
<td>23.7711</td>
<td>13.1801</td>
<td>16.6554</td>
<td>30.8868</td>
</tr>
<tr>
<td>22</td>
<td>23.4024</td>
<td>13.4198</td>
<td>16.2223</td>
<td>30.5825</td>
</tr>
<tr>
<td>23</td>
<td>23.0337</td>
<td>13.6729</td>
<td>15.7862</td>
<td>30.2812</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Post Neonatal Mortality Rate in urban areas of India is carried out as follows:

The linear growth equation: $U_t = a + b \cdot t$

where $U_t$ = dependent variable

t = time

a = intercept

$b = \text{regression coefficient} \; \hat{b} = \frac{\sum tU_t - n\bar{U}}{\sum (t - \bar{t})^2} = -0.7289$

Linear Growth Rate: $\frac{a}{u_t} \cdot 100 = -3.5920$

Confidence limits for $b: \hat{b} = \hat{b} \pm t_{n-2e} \cdot \text{S.E}(\hat{b})$
Standard error of \( \hat{b} = \frac{\sum(u - \bar{u})^2}{(n-2)\sum(t - \bar{t})^2} = 0.2310 \)

Confidence limits for \( \hat{b} : \) \([-1.2257, -0.2322]\)

Mean: 20.2937

Variance: 16.9379

Standard deviation: 4.1155

Standard error of mean: 1.0289

Confidence limits for mean: \( \bar{u} \pm 2.58 \text{S.E} (\bar{x}) = [17.6392, 22.9483] \)

Considering the linear trend model \( X_t = b_1 + b_2 t + \epsilon_t : \) we use the results in the foregoing paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \epsilon_t \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1 - b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is, \( b_1^{-1} = 20.2938 \)

\( b_2^{-1} = -0.7289 \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.
The variances are:

\[ \text{Var}(b_1) = \nu_1 = 0.7829 \]
\[ \text{Var}(b_2) = \nu_2 = 0.0274 \]

The prior variance covariance matrix is:

\[ \nu' = \begin{bmatrix} 0.7829 & 0 \\ 0 & 0.0274 \end{bmatrix} \]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: \( m(0+k) = b_1^{-1} + b_2^{-1}k = 30.1813 - 0.6246k \)
and variance is given by

\[ s^2(0+k) = z'(0+k)\nu z(0+k) + \sigma^2 \]

\[ = \begin{bmatrix} 0.7829 & 0 \\ 0 & 0.0274 \end{bmatrix} + 16.9379 \]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:

\[ M(0+5) = 16.6489 \]
\[ S^2(0+5) = 18.4063 \]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by

\[ = [8.24, 25.0578] \]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by

\[ = [4.1379, 21.8701] \]

Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[ z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \]
\[ xz' = [25.8 \ 27.3 \ 27.5 \ \ldots \ 17.4] \]
\[ G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \]
\[ g = z'x = \begin{bmatrix} 324.7 \\ 2512.1 \]
The sample variance and covariance matrix is:

\[ G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} ; \quad \hat{b} = G^{-1} g = \begin{bmatrix} 26.49 \\ -0.7289 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[ v^{*\text{-1}} = v^{-1} + v^{-1} b \]
\[ b^* = v^*(v^{-1}b^{-1} + v^{-1}b) \text{, where } v \text{ and } b \text{ are vectors.} \]

\[ v^{*\text{-1}} = v^{-1} + v^{-1} \]
\[ = \begin{bmatrix} 1.2772 & 0 \\ 0 & 36.4771 \end{bmatrix} + \begin{bmatrix} 0.9446 & 8.0293 \\ 8.0293 & 88.3223 \end{bmatrix} \]

\[ v^{*\text{-1}} = \begin{bmatrix} 2.2218 & 8.0293 \\ 8.0293 & 124.7994 \end{bmatrix} \]

\[ v^* = \begin{bmatrix} 0.5864 & -0.0377 \\ -0.0377 & 0.0104 \end{bmatrix} \]

\[ b^* = v^* v^{-1} b + v^{-1} \hat{b} \]
\[ = \begin{bmatrix} 21.8491 \\ -0.4304 \end{bmatrix} \]

The expected value of the posterior distribution of post neonatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 14.9627 - 0.4304k \)

The probability distribution of \(X_{16+k}\) has variance given by the equation:

\[ z'(t+k)v^* z(t+k) + \sigma^2_i \text{ and} \]

\[ s^2(16+k) = [1 \quad 16+k] \begin{bmatrix} 0.5864 & -0.0377 \\ -0.0377 & 0.0104 \end{bmatrix} [1 \quad 16+k] + 16.9379 \]
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year.

Table 4.9: Predictions of post neonatal mortality rate in urban at the end of 16\textsuperscript{th} year

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>14.5323</td>
<td>19.2481</td>
<td>5.9333</td>
<td>23.1313</td>
</tr>
<tr>
<td>18</td>
<td>14.1019</td>
<td>19.5367</td>
<td>5.4386</td>
<td>22.7652</td>
</tr>
<tr>
<td>19</td>
<td>13.6721</td>
<td>19.8461</td>
<td>4.9405</td>
<td>22.4037</td>
</tr>
<tr>
<td>20</td>
<td>13.2411</td>
<td>20.1763</td>
<td>4.4372</td>
<td>22.0450</td>
</tr>
<tr>
<td>21</td>
<td>12.8107</td>
<td>20.5273</td>
<td>3.9305</td>
<td>21.6909</td>
</tr>
<tr>
<td>22</td>
<td>12.3803</td>
<td>20.8991</td>
<td>3.4201</td>
<td>21.3405</td>
</tr>
<tr>
<td>23</td>
<td>11.9499</td>
<td>21.2917</td>
<td>2.9897</td>
<td>20.9939</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Peri Neonatal Morality Rate in urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b t \),

where \( U_t \) = dependent variable
\( t \) = time
\( a \) = intercept
\( b \) = regression coefficient
\( \hat{b} = \frac{\sum (t \cdot u_t - \bar{t} \cdot \bar{u}_t)}{\sum (t - \bar{t})^2} = -0.4507 \)

Linear Growth Rate: \( \frac{\hat{b}}{u} \cdot 100 = -1.4329 \)
Confidence limits for \( b : \hat{b} = \hat{b} \pm t_{n-2\alpha} \text{S.E.}(\hat{b}) \)

Standard error of \( \hat{b} = \left[ \frac{(u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2} \right] = 0.0351 \)

Confidence limits for \( \hat{b} : \) \([-0.8536, -0.0478]\]

Mean: \(31.4562\)

Variance: \(11.1453\)

Standard deviation: \(3.3384\)

Standard error of mean: \(0.8346\)

Confidence limits for mean: \(\bar{u} \pm 2.58 \text{S.E.}(\bar{u}) = [29.3029, 33.6096]\)

Considering the linear trend model \( X_t = b_1 + b_2 t + \epsilon \), we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \epsilon \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1, b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.

That is, \( b_1^{-1} = 31.4563 \)
\( b_2^{-1} = -0.4507 \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.
The variances are:

\[ \text{Var}(b_1) = \nu_{11} = 0.5152 \]
\[ \text{Var}(b_2) = \nu_{22} = 0.0180 \]

The prior variance covariance matrix is:

\[ \nu = \begin{bmatrix} 0.5152 & 0 \\ 0 & 0.0180 \end{bmatrix} \]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean:
\[ m(0+k) = b_1^{-1} + b_2^{-1}k = 9.5813 - 0.1739(k) \]

and variance is given by:

\[ s^2(0+k) = z'(0+k)\nu z(0+k) + \sigma^2 = [1 \ k] \begin{bmatrix} 0.5152 & 0 \\ 0 & 0.0180 \end{bmatrix} [1 \ k] + 11.1453 \]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:

\[ M(0+5) = 29.2025 \]
\[ S^2(0+5) = 12.1114 \]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by
\[ = [22.3815, 36.0237] \]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by
\[ = [19.7569, 34.1408] \]

Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[ z' = [1 \ 1 \ 1 \ --- \ 1] \]
\[ 1 \ 2 \ 3 \ --- \ 16 \] ; \[ x'x = [32.7 \ 32.4 \ 34.5 \ ... \ 24.7] \]

\[ G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix} \] ; \[ g = z'x = [503.3] \]
\[ [4124.8] \]
\[ G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix} \quad b = G^{-1}g = \begin{bmatrix} 35.2875 \\ -0.4507 \end{bmatrix} \]

The sample variance and covariance matrix is:
\[ V = G^{-1}\sigma^2_e = \begin{bmatrix} 3.0649 & -0.2786 \\ -0.2786 & 0.0327 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:
\[ \nu^{*-1} = \nu^{-1} + \nu^{-1} \]
\[ b^* = v^T(\nu^{-1}b^{-1} + \nu^{-1}\hat{b}), \quad \text{where } \nu \text{ and } b \text{ are vectors.} \]
\[ \nu^{*-1} = \nu^{-1} + \nu^{-1} \]
\[ \begin{bmatrix} 1.9410 & 0 \\ 0 & 55.4357 \end{bmatrix} + \begin{bmatrix} 1.4355 & 12.2024 \\ 12.2024 & 134.2271 \end{bmatrix} = \begin{bmatrix} 3.3766 & 12.2024 \\ 12.2024 & 189.6628 \end{bmatrix} \]
\[ \nu^* = \begin{bmatrix} 0.3858 & -0.0248 \\ -0.0248 & 0.0068 \end{bmatrix} \]
\[ b^* = v^T[\nu^{-1}b^{-1} + \nu^{-1}\hat{b}] \]
\[ = \begin{bmatrix} 32.4179 \\ -0.2661 \end{bmatrix} \]

The expected value of the posterior distribution of perinatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 28.1603 - 0.2661k \) \( \ldots (i) \)

The probability distribution of \( X_{16+k} \) has variance given by the equation:
\[ z^Tz(t+k)b^*z(t+k) + \sigma^2_e \]
\[ s^2(16+k) = \begin{bmatrix} 0.3859 & -0.0248 \\ -0.0248 & 0.0068 \end{bmatrix} \begin{bmatrix} 1 \\ 16 + k \end{bmatrix} + 11.1453 \] \( \ldots (ii) \)
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16th year.

Table: 4.10 Predictions of perinatal mortality rate in urban at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>27.8942</td>
<td>12.6532</td>
<td>20.9223</td>
<td>34.8662</td>
</tr>
<tr>
<td>18</td>
<td>27.6281</td>
<td>12.8416</td>
<td>20.6045</td>
<td>34.6518</td>
</tr>
<tr>
<td>19</td>
<td>27.362</td>
<td>13.0436</td>
<td>20.2833</td>
<td>34.4407</td>
</tr>
<tr>
<td>20</td>
<td>27.0959</td>
<td>13.2592</td>
<td>19.959</td>
<td>34.2328</td>
</tr>
<tr>
<td>21</td>
<td>26.8298</td>
<td>13.4884</td>
<td>19.6314</td>
<td>34.0282</td>
</tr>
<tr>
<td>22</td>
<td>26.5637</td>
<td>13.7312</td>
<td>19.3008</td>
<td>33.8266</td>
</tr>
<tr>
<td>23</td>
<td>26.2976</td>
<td>13.9876</td>
<td>18.9672</td>
<td>33.628</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Crude Death Rate in rural and urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b \cdot t \),
where \( U_t \) = dependent variable
\( t \) = time
\( a \) = intercept
\( b \) = regression coefficient

\[
b = \frac{\sum t u_t - n \bar{u}_t}{\sum (t - \bar{t})^2} = -1.1739
\]

Linear Growth Rate: \( \frac{\hat{b} \cdot 100}{u_t} = -1.8157 \)

Confidence limits for \( b \):

\[
\hat{b} = \hat{b} \pm t_{n-2} \cdot S.E(\hat{b})
\]
Standard error of \( \hat{b} = \frac{\sum(u - \bar{u})^2}{(n-2)\sum(t - \bar{t})^2} = 0.0024 \)

Confidence limits for \( \hat{b} \): \([-0.2794, -0.0685]\]

Mean: 9.5812

Variance: 0.7629

Standard deviation: 0.8735

Standard error of mean: 0.2184

Confidence limits for mean: \( \bar{u} \pm 2.58 \text{ S.E (}\bar{x}\text{)} = [9.0178, 10.1446] \)

Considering the linear trend model \( X_t = b_1 + b_2 t + \varepsilon_t \) we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \varepsilon_t \sim N(0, \sigma^2) \), we use a normal prior for \( b = [b_1 - b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean; calculated from the data.

That is,
\[
\begin{align*}
\hat{b}_1 &= 9.5812 \\
\hat{b}_2 &= -0.1739
\end{align*}
\]

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:
\[
\begin{align*}
\text{Var}(b_1) &= \nu_{11} = 0.0352 \\
\text{Var}(b_2) &= \nu_{22} = 0.0012
\end{align*}
\]
The prior variance covariance matrix is: \[ \Sigma = \begin{bmatrix} 0.0352 & 0 \\ 0 & 0.0012 \end{bmatrix} \]

At time ‘0’, the mortality rate in period \( t \), is estimated to be normally distributed with mean: \( m(0+k) = b_1 + b_2 k = 9.5812 - 0.1739 \)
And variance is given by \( s^2(0+k) = z'(0+k)\Sigma z(0+k) + \sigma^2 \)

For example, the mortality in the 5th year is assumed to be normal with mean and variance:
\[ M(0+5) = 8.7114 \]
\[ S^2(0+5) = 0.8291 \]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by
\[ = [6.9267, 10.4961] \]
Similarly a 95% Bayesian prediction for \( X_{10} \), made at time ‘0’ is given by
\[ = [36.7400, 80.2239] \]

Using this information, posterior distribution at \( t=16 \) is computed using least squares method as follows:
\[ z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \]
\[ x'x = \begin{bmatrix} 1 \end{bmatrix} \]
\[ G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix}; \quad g = z'x = \begin{bmatrix} 153.3 \\ 1243.9 \end{bmatrix} \]
\[ G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix}; \quad \hat{b} = G^{-1}g = \begin{bmatrix} 11.06 \\ -0.1739 \end{bmatrix} \]
The sample variance and covariance matrix is:

\[ V = G^{-1} \sigma^2_z = \begin{bmatrix} 0.2098 & -0.0191 \\ -0.0191 & 0.2995 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[
\begin{align*}
\psi^{-1} &= \psi^{-1} + \chi^{-1} \\
\beta^* &= \psi^* (\psi^{-1} \beta^* + \chi^{-1} \tilde{b}) , \text{ where } \psi \text{ and } \beta \text{ are vectors.} \\
\psi^* &= \psi^* + \chi^{-1} \\
\beta^* &= \psi^* \left( \psi^{-1} \beta^* + \chi^{-1} \tilde{b} \right) \\
&= 9.9524 \\
&= -0.1027
\end{align*}
\]

The expected value of the posterior distribution of infant mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 8.3092-0.1027k \) \( \ldots \) (i)

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[
\begin{align*}
&z' z(t+k) \psi^* z(t+k) + \sigma^2_z \text{ and} \\
&s^2(16+k) = [1 \quad 16+k \quad \begin{bmatrix} 0.0264 & -0.0017 \\ -0.0017 & 0.0005 \end{bmatrix} \quad 1 ] + 0.7629 \quad \ldots \text{(ii)}
\end{align*}
\]
With the help of the equations (i) and (ii), we predict mean and variance at the end of 16th year.

Table: 4.11 Predictions of infant mortality rate in rural and urban at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>8.2065</td>
<td>0.876</td>
<td>6.372</td>
<td>10.0409</td>
</tr>
<tr>
<td>18</td>
<td>8.1038</td>
<td>0.8901</td>
<td>6.2546</td>
<td>9.953</td>
</tr>
<tr>
<td>19</td>
<td>8.0011</td>
<td>0.9052</td>
<td>6.1363</td>
<td>9.8658</td>
</tr>
<tr>
<td>20</td>
<td>7.8984</td>
<td>0.9213</td>
<td>6.0171</td>
<td>9.7797</td>
</tr>
<tr>
<td>21</td>
<td>7.7957</td>
<td>0.9384</td>
<td>5.897</td>
<td>9.6944</td>
</tr>
<tr>
<td>22</td>
<td>7.763</td>
<td>0.9565</td>
<td>5.7761</td>
<td>9.6099</td>
</tr>
<tr>
<td>23</td>
<td>7.5903</td>
<td>0.9756</td>
<td>5.6544</td>
<td>9.5262</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Infant Mortality Rate in rural and urban areas of India is carried out as follows:

The linear growth equation: $U_t = a + bt$,

where $U_t = \text{dependent variable}$

$t = \text{time}$

$a = \text{intercept}$

$b = \text{regression coefficient} \hat{b} = \frac{\sum t u_t - n \bar{u}}{\sum (t-i)^2} = -1.9987$

Linear Growth Rate: $\frac{\hat{b} \times 100}{u_t} = -2.5471$

Confidence limits for $b$: $\hat{b} = \hat{b} \pm t_{n-2} \times \text{S.E}(\hat{b})$
Standard error of  
\[ \hat{b} = \frac{\sum(u - \bar{u})^2}{(n-2)\sum(x - \bar{x})^2} \]  
= 0.3209

Confidence limits for \( \hat{b} \):  
[-0.7806,-3.2167]

Mean: 78.4687
Variance: 101.857
Standard deviation: 10.0924
Standard error of mean: 2.5231
Confidence limits for mean: \( \bar{u} \pm 2.58\text{S.E} \bar{x} = [71.9591,84.9783] \)

Considering the linear trend model \( X_i = b_1 + b_2 t + \varepsilon_i \): we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)

Since we assume that \( \varepsilon_i \sim N(0,\sigma^2) \), we use a normal prior for \( b = [b_1 - b_2]' \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.
That is,  
\[ b_1^{-1} = 78.4687 \]
\[ b_2^{-1} = -1.9987 \]

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:
\[ \text{Var}(b_1) = \nu_1 = 4.7083 \]
\[ \text{Var}(b_2) = \nu_2 = 0.1648 \]
The prior variance covariance matrix is: 

\[ V = \begin{bmatrix} 4.7083 & 0 \\ 0 & 0.1648 \end{bmatrix} \]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: 

\[ m(0+k) = b_1 + b_k = 78.4687 - 1.9986 \]

And variance is given by 

\[ s^2(0+k) = z'(0+k)v'(0+k) + \sigma_z^2 \]

For example, the mortality in the 5\textsuperscript{th} year is assumed to be normal with mean and variance: 

\[ M(0+5) = 68.4753 \]
\[ S^2(0+5) = 110.6867 \]

A 95\% Bayesian prediction interval for \( X_5 \), at time 0, is given by 

\[ [47.8546, 89.0961] \]

Similarly a 95\% Bayesian prediction for \( X_{10} \), made at time '0' is given by 

\[ [36.7400, 80.2239] \]

Using this information, posterior distribution at \( t=16 \) is computed using least squares method as follows:

\[ z' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \]
\[ x'x = \begin{bmatrix} 96 & 95 & 94 & \cdots & 65.9 \end{bmatrix} \]
\[ G = z'z = \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix}; \quad g = z'x = \begin{bmatrix} 1255.5 \\ 9992.2 \end{bmatrix} \]
\[ y^{-1} = \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix}; \quad b = G^{-1}g = \begin{bmatrix} 95.4575 \\ -1.9986 \end{bmatrix} \]

The sample variance and covariance matrix is:
The parameters of the posterior distribution are computed using the following equations:

\[ v_{\text{post}} = v_{\text{prior}} + v \]

\[ b_{\text{post}} = v_{\text{prior}} b_{\text{prior}} + v b \]

where \( v \) and \( b \) are vectors.

\[
\begin{bmatrix}
0.2124 & 0 & 0.1571 & 1.3352 \\
0 & 6.0658 & 1.3352 & 14.687
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.3694 & 1.3352 \\
1.3352 & 20.7531
\end{bmatrix}
\]

\[ v_{\text{post}} = \begin{bmatrix} 3.5264 & -0.2269 \\ -0.2269 & 0.0627 \end{bmatrix} 
\]

\[ b_{\text{post}} = v_{\text{prior}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} 
\]

The expected value of the posterior distribution of infant mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 63.8531-1.18k \) \hspace{1cm} (i)

The probability distribution of \( X_{16+k} \) has variance given by the equation

\[ z'(t+k)v_{\text{post}}z(t+k) + \sigma^2 \]

and

\[ s^2(16+k) = [1 \hspace{1cm} 16+k] \begin{bmatrix} 3.5265 & -0.2269 \\ -0.2269 & 0.0627 \end{bmatrix} [1 \hspace{1cm} 16+k] + 101.857 \] \hspace{1cm} (ii)
With the help of the equations (i) and (ii), we predict mean and variance at the end of 16th year.

Table: 4.12 Predictions of infant mortality rate in rural and urban at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>62.6731</td>
<td>115.7892</td>
<td>41.5825</td>
<td>83.7637</td>
</tr>
<tr>
<td>18</td>
<td>61.4931</td>
<td>117.5299</td>
<td>40.2445</td>
<td>82.7417</td>
</tr>
<tr>
<td>19</td>
<td>60.3131</td>
<td>119.396</td>
<td>38.8965</td>
<td>81.7297</td>
</tr>
<tr>
<td>20</td>
<td>59.1331</td>
<td>121.3875</td>
<td>37.5387</td>
<td>80.7275</td>
</tr>
<tr>
<td>21</td>
<td>57.9531</td>
<td>123.5044</td>
<td>36.1712</td>
<td>79.7351</td>
</tr>
<tr>
<td>22</td>
<td>56.7731</td>
<td>125.7467</td>
<td>34.7943</td>
<td>78.7519</td>
</tr>
<tr>
<td>23</td>
<td>55.5931</td>
<td>128.1144</td>
<td>33.4083</td>
<td>77.7778</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Neonatal Mortality Rate (NNMR) in rural and urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b t \),

Where \( U_t \) = dependent variable

\( t = \) time

\( a = \) intercept

\[ b = \text{regression coefficient} = \frac{\sum t u_t - n t \bar{u}_t}{\sum (t - \bar{t})^2} = -1.1382 \]

Linear Growth Rate: \( \frac{\Delta}{u_t} \times 100 = -2.2919 \)

Confidence limits for \( b \): \( \hat{b} = \hat{b} \pm t_{n-2} \times \text{S.E}(\hat{b}) \)
Standard error of \[ \hat{b} = \frac{\sum (u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2} = 0.0989 \]

Confidence limits for \( \hat{b} \): [-1.8143,-0.4621]

Mean: 49.6625
Variance: 31.3825
Standard deviation: 5.6020
Standard error of mean: 1.4005
Confidence limits for mean: \( \bar{u} \pm 2.58 \text{S.E (} \bar{x}) \)
\[ = [46.0492, 53.2758] \]

Considering the linear trend model \( X_i = b_1 + b_2 t + \epsilon_i \) : we use the results in the following paragraphs:

We know that \( Z_1(t) = 1 \) and \( Z_2(t) = t \)
Since we assume that \( \epsilon_i \sim N(0, \sigma_i^2) \), we use a normal prior for \( b = [b_1 - b_2]^T \)

The prior means of \( b_1 \) and \( b_2 \) are taken as the mid-points of the limits for \( \hat{b} \) and mean, calculated from the data.
That is, \( b_1^{-1} = 49.6625 \)
\( b_2^{-1} = -1.1382 \)

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:
\[ \text{Var}(b_1) = \nu_{11} = 1.4507 \]

96
The prior variance covariance matrix is: 

\[
\begin{bmatrix}
1.4506 & 0 \\
0 & 0.0508
\end{bmatrix}
\]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: 

\[
m(0+k) = b_1^{-1} + b_2^{-1}k = 49.6625 - 1.1382k
\]

And variance is given by: 

\[
\sigma^2(0+k) = \Sigma(0+k)\nu(0+k) + \sigma^2
\]

\[
= [1 \ k] \begin{bmatrix}
1.4506 & 0 \\
0 & 0.0508
\end{bmatrix} \begin{bmatrix}
1 \\
k
\end{bmatrix}
\]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:

- M(0+5) = 43.9713
- S^2(0+5) = 34.1029

A 95% Bayesian prediction interval for X_5, at time 0, is given by

\[
= [32.5253, 55.4173]
\]

Similarly a 95% Bayesian prediction for X_10, made at time '0' is given by

\[
= [26.2118, 50.3484]
\]

Using this information, posterior distribution at t=16 is computed using least squares method as follows:

\[
z' = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \end{bmatrix}
\]

\[
x'x = \begin{bmatrix} 1 & 2 & 3 & \ldots & 16 \end{bmatrix}
\]

\[
G = z'z = \begin{bmatrix} 16 & 136 \\
136 & 1496 \end{bmatrix} ; \quad g = z'x = \begin{bmatrix} 794.6 \\
6367.1 \end{bmatrix}
\]

\[
G^{-1} = \begin{bmatrix} 0.275 & -0.025 \\
-0.025 & 0.0029 \end{bmatrix} ; \quad \hat{b} = G^{-1}g = \begin{bmatrix} 59.3375 \\
-1.1382 \end{bmatrix}
\]
The sample variance and covariance matrix is:

\[
V = G^{-1}\sigma_x^2 = \begin{bmatrix}
8.6302 & -0.7845 \\
-0.7845 & 0.0923
\end{bmatrix}
\]

The parameters of the posterior distribution are computed using the following equations:

\[
\begin{align*}
&= \nu^{-1} + v^{-1} \\
&= \nu^{-1} = \nu(v^{-1}b^{-1} + v^{-1}b), \text{ where } v \text{ and } b \text{ are vectors.} \\
\nu' &= \nu^{-1} + \nu^{-1} \\
&= \frac{0.6893}{1.1991} \begin{bmatrix} 0.5098 & 4.3336 \\ 19.6876 & 47.6699 \end{bmatrix} \\
&= \begin{bmatrix} 0.6893 & 0 \\ 0 & 19.6876 \end{bmatrix} \begin{bmatrix} 0.5098 & 4.3336 \\ 4.3336 & 47.6699 \end{bmatrix} \\
&= \begin{bmatrix} 0.6893 & 4.3336 \\ 4.3336 & 67.3575 \end{bmatrix} \\
&= \begin{bmatrix} 1.0865 & -0.0699 \\ -0.0699 & 0.0193 \end{bmatrix} \\
&= \nu v^{-1}b^{-1} + v^{-1}b \\
&= \begin{bmatrix} 52.0910 \\ -0.6720 \end{bmatrix}
\]

The expected value of the posterior distribution of neonatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \(\hat{x} = M(16+k) = 41.339 - 0.6720k\) ... (i)

The probability distribution of \(X_{16+k}\) has variance given by the equation:

\[
\sigma_x^2 = \nu v^{-1}b^{-1} + v^{-1}b + \sigma_x^2
\]

\[
s^2(16+k) = \left[1 \quad 16+k \right] \begin{bmatrix} 1.0865 & -0.0699 \\ -0.0699 & 0.0193 \end{bmatrix} \begin{bmatrix} 1 \\ 16+k \end{bmatrix} + 31.3825 \quad \text{n} \quad \text{(ii)}
\]
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year.

Table 4.13 Predictions of neonatal mortality rate in rural and urban at the end of the 16\textsuperscript{th} year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>40.667</td>
<td>35.6701</td>
<td>28.9611</td>
<td>52.3729</td>
</tr>
<tr>
<td>18</td>
<td>39.995</td>
<td>36.2058</td>
<td>28.2014</td>
<td>51.7886</td>
</tr>
<tr>
<td>19</td>
<td>39.323</td>
<td>36.7801</td>
<td>27.4363</td>
<td>51.2097</td>
</tr>
<tr>
<td>20</td>
<td>38.651</td>
<td>37.393</td>
<td>26.6656</td>
<td>50.6364</td>
</tr>
<tr>
<td>21</td>
<td>37.979</td>
<td>38.0445</td>
<td>25.8897</td>
<td>50.0683</td>
</tr>
<tr>
<td>22</td>
<td>37.307</td>
<td>38.7346</td>
<td>25.1085</td>
<td>49.5055</td>
</tr>
<tr>
<td>23</td>
<td>36.635</td>
<td>39.4633</td>
<td>24.3223</td>
<td>48.9476</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Post Neonatal Mortality Rate in rural and urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b \cdot t \),

where \( U_t \) = dependent variable
\( t \) = time
\( a \) = intercept
\( b \) = regression coefficient

\[ b = \frac{\sum t \cdot u_t - n \bar{u}_t}{\sum (t - \bar{t})^2} = -0.9169 \]

Linear Growth Rate: \( \frac{\Delta u}{\bar{u}_t} \times 100 = -3.1955 \)
Confidence limits for $b$: $\hat{b} = \hat{b} \pm t_{n-2,\alpha/2} \cdot \text{S.E}(\hat{b})$

Standard error of $\hat{b} = \frac{\sum (u - \bar{u})^2}{(n-2)\sum (t - \bar{t})^2} = 0.0802$

Confidence limits for $\hat{b}$: $[-1.5256, -0.3082]$

Mean: 28.6937

Variance: 25.4393

Standard deviation: 5.0437

Standard error of mean: 1.2609

Confidence limits for mean: $\bar{u} \pm 2.58 \cdot \text{S.E}(\bar{u}) = [25.4405, 31.947]$

Considering the linear trend model $X_t = b_1 + b_2 t + \epsilon$, we use the results in the following paragraphs:

We know that $Z_1(t) = 1$ and $Z_2(t) = t$

Since we assume that $\epsilon_i \sim N(0, \sigma^2)$, we use a normal prior for $b = [b_1 - b_2]'$

The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is, $b_1' = 28.6938$

$\quad b_2' = -0.9169$

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

$\text{Var}(b_1) = \nu_{11}' = 1.1759$
The prior variance covariance matrix is: 
\[ \mathbf{v'} = \begin{bmatrix} 
1.1759 & 0 \\
0 & 0.0412 
\end{bmatrix} \]

At time '0', the mortality rate in period t, is estimated to be normally distributed with mean: 
\[ m(0+k) = b_1^{-1} + b_2^{-1}k = 28.6938 - 0.9169k \]
and variance is given by 
\[ s^2(0+k) = z'(0+k)\mathbf{v'}z(0+k) + \sigma^2 \]
\[ = [1 \ k] \begin{bmatrix} 
1.1759 & 0 \\
0 & 0.0412 
\end{bmatrix} k + 25.4393 \]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:
\[ M(0+5) = 24.1092 \]
\[ S^2(0+5) = 27.6446 \]

A 95% Bayesian prediction interval for X5, at time '0' is given by 
\[ = [13.8038, 34.4145] \]
Similarly a 95% Bayesian prediction for X_{10}, made at time '0' is given by 
\[ = [8.6589, 30.3902] \]
Using this information, posterior distribution at t=17 is computed using least squares method as follows:
\[ z' = [1 \ 1 \ 1 \ \ldots \ 1] \]
\[ x'x = \begin{bmatrix} 
36.6 & 37.7 & 37.7 & \ldots & 25.7 
\end{bmatrix} \]
\[ G = z'z = \begin{bmatrix} 
16 & 136 \\
136 & 1496 
\end{bmatrix}; \quad g = z'x = 459.1 \quad 3590.6 \]
\[ G^{-1} = \begin{bmatrix} 
0.275 & -0.025 \\
-0.025 & 0.0029 
\end{bmatrix} \quad \hat{b} = G^{-1}g = \begin{bmatrix} 
36.4875 \\
-0.9169 
\end{bmatrix} \]
The sample variance and covariance matrix is:

\[ V = G^{-1}\sigma_e^2 = \begin{bmatrix} 6.9958 & -0.6359 \\ -0.6359 & 0.0748 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[ v^{*-1} = v^{-1} + v^{-1} \]

\[ b^* = v^*(v^{-1}b^{-1} + v^{-1}\hat{b}) \], where \( v \) and \( b \) are vectors.

\[ v^{*-1} = v^{-1} + v^{-1} \]

\[
\begin{bmatrix}
0.8504 & 0 & 0.6289 & 5.3401 \\
0 & 24.2871 & 5.3461 & 58.8067
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.4793 & 5.3461 \\
5.3461 & 83.0938
\end{bmatrix}
\]

\[ v^* = \begin{bmatrix}
0.8807 \\
-0.0567
\end{bmatrix}, \quad b^* = v^*[v^{-1}b^{-1} + v^{-1}\hat{b}] \]

\[
\begin{bmatrix}
30.6501 \\
-0.5413
\end{bmatrix}
\]

The expected value of the posterior distribution of post neonatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \[ \hat{x} = M(16+k) = 21.9893 - 0.5413(16+k) \] \( \ldots (i) \)

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[ s^2(x) = (16+k)(0.8807 - 0.0567) + 25.4393 \] \( \ldots (ii) \)
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16th year.

Table: 4.14 Predictions of post neonatal mortality rate in rural and urban at the end of 16th year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>21.448</td>
<td>28.9295</td>
<td>10.9059</td>
<td>31.9901</td>
</tr>
<tr>
<td>18</td>
<td>20.9067</td>
<td>29.3656</td>
<td>10.2855</td>
<td>31.5279</td>
</tr>
<tr>
<td>19</td>
<td>20.3684</td>
<td>29.8331</td>
<td>9.6629</td>
<td>31.0739</td>
</tr>
<tr>
<td>20</td>
<td>19.8241</td>
<td>30.332</td>
<td>9.0294</td>
<td>30.6187</td>
</tr>
<tr>
<td>21</td>
<td>19.2828</td>
<td>30.8623</td>
<td>8.3942</td>
<td>30.1713</td>
</tr>
<tr>
<td>22</td>
<td>18.7415</td>
<td>31.424</td>
<td>7.7543</td>
<td>29.7287</td>
</tr>
<tr>
<td>23</td>
<td>18.2002</td>
<td>32.0171</td>
<td>7.1098</td>
<td>29.2906</td>
</tr>
</tbody>
</table>

The Bayesian analysis of forecasting of Perinatal Mortality Rate in rural and urban areas of India is carried out as follows:

The linear growth equation: \( U_t = a + b \cdot t \),

where \( U_t \) = dependent variable

t = time

a = intercept

\[ b = \text{regression coefficient} \hat{b} = \frac{\sum t u_t - t \bar{u}_t}{\sum (t - \bar{t})^2} = -0.9138 \]

Linear Growth Rate: \( \frac{\hat{a}}{u_t} \cdot 100 = -2.0651 \)

Confidence limits for \( b \): \( \hat{b} = \hat{b} \pm t_{n-2} S.E(\hat{b}) \)
Standard error of $\hat{b} = \frac{\sum(u - \bar{u})^2}{\sum\sum(t - \bar{t})^2} = 0.0985$

Confidence limits for $\hat{b}$: [-1.5886,-0.2390]

Mean: 44.25
Variance: 31.2613
Standard deviation: 5.5912

standard error of mean: 1.3978

Confidence limits for mean: $\bar{u}, \pm 2.58 \text{S.E}(\bar{u}) = [40.6437, 47.8563]$

Considering the linear trend model $X_t = b_1 + b_2 t + \varepsilon_t$, we use the results in the following paragraphs:

We know that $Z_1(t) = 1$ and $Z_2(t) = t$

Since we assume that $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, we use a normal prior for $b = [b_1 - b_2]'$

The prior means of $b_1$ and $b_2$ are taken as the mid-points of the limits for $\hat{b}$ and mean, calculated from the data.

That is, $b_1 = 44.25$
$b_2 = -0.9138$

The prior standard deviations are assumed to be one-sixth of the range since six standard deviations for all practical purposes constitute the spread of a normal distribution.

The variances are:

$\text{Var}(b_1) = \nu_{11} = 1.4451$
$\text{Var}(b_2) = \nu_{22} = 0.0506$
The prior variance covariance matrix is: \( \Sigma = \begin{bmatrix} 1.4451 & 0 \\ 0 & 0.0506 \end{bmatrix} \)

At time '0', the mortality rate in period \( t \), is estimated to be normally distributed with mean: 
\[ m(0+k) = h_1^{-1} + h_2^{-1} k = 44.25 - 0.9138 k \]
And variance is given by 
\[ s^2(0+k) = z'(0+k)\nu z(0+k) + \sigma_s^2 \]

\[ = [1 \ k] \begin{bmatrix} 1.4450 & 0 \\ 0 & 0.0506 \end{bmatrix} \begin{bmatrix} 1 \\ k \end{bmatrix} + 31.2613 \]

For example, the mortality in the 5th year is assumed to be normal with mean and variance:
\[
\begin{align*}
M(0+5) &= 39.6808 \\
S^2(0+5) &= 33.9713
\end{align*}
\]

A 95% Bayesian prediction interval for \( X_5 \), at time 0, is given by
\[ = [28.2570, 51.1047] \]

Similarly a 95% Bayesian prediction for \( X_{10} \), made at time '0' is given by
\[ = [23.0667, 47.1567] \]

Using this information, posterior distribution at \( t=16 \) is computed using least squares method as follows:

\[
\begin{align*}
z' &= \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & 16 \end{bmatrix} \\
x'x &= \begin{bmatrix} 48.1 & 50.1 & 49.6 & \cdots & 26.2 \end{bmatrix} \\
z'z &= \begin{bmatrix} 16 & 136 \\ 136 & 1496 \end{bmatrix}; \quad g = z'x = \begin{bmatrix} 708 \\ 5707.3 \end{bmatrix} \\
G^{-1} &= \begin{bmatrix} 0.275 & -0.025 \\ -0.025 & 0.0029 \end{bmatrix}; \quad \hat{b} = G^{-1}g = \begin{bmatrix} 52.0175 \\ -0.9138 \end{bmatrix}
\end{align*}
\]
The sample variance and covariance matrix is:

\[ V = G^{-1} \sigma_x^2 = \begin{bmatrix} 8.5968 & -0.7815 \\ -0.7815 & 0.0919 \end{bmatrix} \]

The parameters of the posterior distribution are computed using the following equations:

\[ v^{*-1} = v^{*-1} + v^{-1} \]

\[ b^* = v^{*}(v^{*-1}b^{-1} + v^{-1}b), \] where \( v \) and \( b \) are vectors.

\[ v^{*-1} = v^{*-1} + v^{-1} \]

\[
\begin{bmatrix}
0.6920 & 0 \\
0 & 19.7639
\end{bmatrix}
+ \begin{bmatrix}
0.5118 & 4.3504 \\
4.3504 & 47.8546
\end{bmatrix}
\]

\[ v^{*-1} = \begin{bmatrix} 1.2038 & 4.3504 \\ 4.3504 & 67.6186 \end{bmatrix} \]

\[ v^* = \begin{bmatrix} 1.0823 & -0.0696 \\ -0.0696 & 0.0193 \end{bmatrix} \]

\[ b^* = v^*[v^{*}b^{-1} + v^{-1}b], \]

\[ v^{*} = \begin{bmatrix} 46.1997 \\ -0.5395 \end{bmatrix} \]

The expected value of the posterior distribution of perinatal mortality rate in period \((16+k)\) is to be used as the point forecast for that period.

That is, \( \hat{x} = M(16+k) = 37.5677 - 0.5395k \) \( \ldots \) (i)

The probability distribution of \( X_{16+k} \) has variance given by the equation:

\[ z^t z(t+k) v^* z(t+k) + \sigma_i^2 \text{ and} \]

\[ s^2(16+k) = \left[ 1 \quad 16+k \right] \begin{bmatrix} 1.0823 & -0.0696 \\ -0.0696 & 0.0193 \end{bmatrix} \begin{bmatrix} 1 \\ 16+k \end{bmatrix} + 31.2613 \] \( \ldots \) (ii)
With the help of this equations (i) and (ii), we predict mean and variance at the end of 16\textsuperscript{th} year.

Table: 4.15 Predictions of peri natal morality rate in rural and urban at the end of 16\textsuperscript{th} year.

<table>
<thead>
<tr>
<th>year</th>
<th>Predicted mean</th>
<th>Predicted variance</th>
<th>Lower limit</th>
<th>Upper limit</th>
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<td>36.0912</td>
<td>24.7138</td>
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<td>24.0809</td>
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<tr>
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<td>33.7912</td>
<td>39.3517</td>
<td>21.4959</td>
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4.3 Comparison of Regression and Autoregressive Moving Average (ARMA (1,1)) Forecasting

(i) Comparative analysis of forecasting of crude death rate in rural areas of India is carried out as follows using regression and ARMA methods.

<table>
<thead>
<tr>
<th>Year</th>
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<th>LCL</th>
<th>UCL</th>
</tr>
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<tbody>
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<td>7.1921</td>
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<tr>
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<td>6.9645</td>
<td>8.5973</td>
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<tr>
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<tr>
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<td>6.5059</td>
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<tr>
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<td>6.2751</td>
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</table>

<table>
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![Graph showing comparison of regression and ARMA methods for crude death rate (CDR) in rural areas over the years 2000 to 2008.](image)
### REGRESSION

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<th>LCL</th>
<th>UCL</th>
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</thead>
<tbody>
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### ARMA

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<th>UCL</th>
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</thead>
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[Graph showing IMR (rural) trends over years]
### REGRESSION

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### ARMA

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![Graph](Image)
### Regression

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### ARMA

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![Graph showing the relationship between death rate and year. The graph includes a regression line.](image-url)
### REGRESSION

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### ARMA

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(ii) Comparative analysis of forecasting of crude death rate in urban areas of India is carried out as follows using regression and ARMA methods.

<table>
<thead>
<tr>
<th>YEAR</th>
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![Graph showing CDR(urban) with regression and ARMA methods]
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![Graph showing IMR(urban) trend with regression line](image_url)
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### Graph

- **NNMR (urban)**
  - Regression
  - ARMA


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### REGRESSION

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![Graph showing PENMR(urban) with regression and ARMA lines](image)
(iii) Comparative analysis of forecasting of crude death rate in rural and urban areas of India is carried out as follows using regression and ARMA methods.

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![Graph showing CDR (combined), regression, and ARMA trends over years]
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![Graph](https://via.placeholder.com/150)

**ARMAR(combined)**

---

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**REgression**

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**Graph**

- **Legend**
  - O --- regression
  - --- ARTIS

**Coefficients**

120
### Regression

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<tr>
<th>PNMR</th>
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### ARMA

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[Graph showing PNMIR (combined) regression]
### REGRESSION

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### ARMA

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- regression

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11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56
Comparing Slopes and Intercepts Parameters:

The procedure for testing the significance of the difference between slopes and intercepts for Rural Vs Urban, Rural Vs Combined, Urban Vs Combined for the five parameters: CDR, IMR, NNMR, PNMR, PENMR is given below.

(i) Comparison of Slopes:

Let $b_1$ and $b_2$ be the slopes of two regression equations. We must test $H_0: b_1 = b_2$.

We test the null hypothesis that the slopes for predicting support for CDR from rural and urban categories are equal.

First, we carry out two regression analysis – one using the data from rural and the other using the data from urban category. The results are given below:

<table>
<thead>
<tr>
<th>Group</th>
<th>Intercept</th>
<th>Slope</th>
<th>SE slope</th>
<th>SSE</th>
<th>SD</th>
<th>Mean</th>
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<tr>
<td>CDR(rural)</td>
<td>12.16</td>
<td>-0.209</td>
<td>0.017</td>
<td>1.333</td>
<td>1.04</td>
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<td>CDR(urban)</td>
<td>7.55</td>
<td>-0.093</td>
<td>0.018</td>
<td>1.533</td>
<td>0.55</td>
<td>6.76</td>
<td>16</td>
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</table>

Student's t-test is used for computing the difference between the two slopes. The test statistic is: $t = \frac{b_1 - b_2}{\text{SE}}$ on (N-1) df. Where $N =$ no. of observations.

The pooled residual variance and the standard error of the difference between slopes is computed by using the formulae

$$S_{b_1-b_2} = \sqrt{\frac{S_{b_1}^2 + S_{b_2}^2}{n_1 + n_2 - 1}} = \sqrt{(0.017)^2 + (0.018)^2} = 0.0247$$

$$S.E(\Delta) = \sqrt{\frac{S_{b_1}^2 + S_{b_2}^2}{SS_1} + \frac{S_{b_1}^2 + S_{b_2}^2}{SS_2}} = \sqrt{\frac{0.0925}{16(1.04)^2} + \frac{0.0925}{16(0.55)^2}} = 0.1615$$
The test statistic $t = -4.69$

This is significant on 31df ($p=0.0002$). So, we conclude that there is highly significant difference between slope parameters of rural and urban categories with regard to crude death rate.

(ii) Comparison of Intercepts:

Let the intercepts be $a_1$ and $a_2$ for rural and urban categories for CDR. The difference between the intercepts can also be tested for $H_0 : a_1 = a_2$ its significance by using the formulae given below.

$$S_{a_1-a_2}^2 = S_{a_1}^2 + S_{a_2}^2 - \frac{M_1^2}{SSx_1} - \frac{M_2^2}{SSx_2}$$

$$\frac{a_1 - a_2}{S_{a_1-a_2}}, \quad (n_1 + n_2 - 1)df.$$ 

$$S_{a_1-a_2}^2 = [0.0925| \frac{1}{16} + \frac{1}{16} + \frac{(10.4)^2}{15(1.04)^2} + \frac{(6.76)^2}{15(0.55)^2}]$$

$$t = \frac{a_1 - a_2}{S_{a_1-a_2}} = 1.1226$$

This is no significant on 31 df ($p=0.292$). So, we conclude that there is no significant difference between intercept parameters of rural and urban with regard to CDR. The test for the crude death rate in rural and combined reveal that is $1.4992(p=0.292)$, there is no significant difference between slope and intercept parameters.
And, for the CDR for urban and combined, \( t = 3.3633 \) \((p=0.0043)\), \( t=0.8233 \) \((p=0.4237)\). So, we conclude that there is highly significant difference between slope parameters and while the intercept parameter is not significant. The test for the CDR in rural and combined, the \( t \) value is -1.4992 \((p=0.1545)\) reveals that there is no significant difference between intercepts.

We test for the equality of the slope and intercept parameters for predicting support for IMR from the above three categories. The computed value of the test statistic is \( t = -3.3935 \). This is significant on 31 df \((p=0.0040)\). So, we conclude that the difference between slope parameters of rural and urban categories with regard to IMR is highly significant and the intercept parameter \( t \) value= 15.18\((p=0.1)\). So, it is not significant. The test for the IMRs in rural and combined the computed values of slope and intercept parameters are \( t=-0.7453 \) \((p=0.4676)\) and \( t=2.9575 \) \((p=0.0097)\) reveals that there is no significant difference between slopes. While the difference between intercept parameters is significant. And for the IMR for urban and combined, the test statistic \( t \) is 2.7477\((p=0.0149)\) for slope, intercept test statistic \( t \) is -12.05\((p=0.01)\). So, we conclude that there is significant difference between slope and intercept parameters.

The test for equality of slope and intercept parameter for predicting support for NNMR from the above three categories is carried out. The computed value of the test statistic for slope parameters is \( t = -5.0026 \) \((p=0.0001)\), which is highly significant. For the intercept parameters \( t= 8.592 \) \((p=0.0001)\), which is highly significant. We conclude that the difference between slope and intercept parameters are significant.
For the NNMR for rural and combined, the test statistic for slope and intercept parameters are $t=-0.9226(p=0.3708)$ and $t=1.7186(p=0.1062)$. So, we conclude that there is no significant difference between slope and intercept parameters. And, the slope and intercept parameters test statistics are $t=4.5353(p=0.0039)$ and $t=-7.2057(p=0.001)$ reveals that there is a significant difference between slope and intercept parameters in urban and combined with regard to NNMR.

The computed value of the test statistic $t=-1.664(p=0.1163)$ with regard to PNMR in rural and urban category. There is not significant difference between slope parameters. While the difference between intercept parameters $t=-6.2057(p=0.0002)$ is highly significant. For the remaining two categories rural and combined, urban and combined – the slope parameter test statistic $t=-0.1888(p=0.8527)$ is not significant. While the intercept parameter test statistic value $t=1.1079(p=0.2853)$ is not significant. In the urban and combined category the test statistic values for slope and intercepts are $t=0.9972(0.3344)$, $t=-7.2057(0.0001)$.

The test for the PENMRs rural and urban the computed values of slope and intercept parameters are $t=-1.6893(p=0.1131)$, $t=1.1079(p=0.5536)$ reveals that there is no significant difference between slopes and intercept parameters. For the remaining two categories rural and combined, urban and combined – the slope parameter test statistic $t=2.2894(p=0.6369)$ is not significant. While the intercept parameter test statistic value $t=-5.2913(p=0.001)$ is highly significant. In the last category the test statistic values for slope and intercepts are $t=-0.1888(p=0.3662)$, $t=1.1079(p=0.5536)$. 126