Chapter - III

THEORY AND METHODOLOGY
3.1. Radial DEA Models:

In Indian commercial Banking non-performance assets are commonly observed with different degrees of seriousness. This is a bad output, needs to be controlled. In out approach of DEA, the possibility of further augmentation of good outputs and reduction of bad output are enquired for a bank. If radial DEA is chosen as a basic tool, the output problem of commercial banks may be postulated as,

Max $\theta$

subject to $\sum_{j=1}^{n} \lambda_j x_j \leq x_{i0}, \quad i = 1, 2, \ldots, k$

$\sum_{j=1}^{n} \lambda_j \mu_j \geq \theta \nu_{r0}, \quad r = 1, 2, \ldots, m \quad (3.1.1)$

$\sum_{j=1}^{n} \lambda_j \mu_j \leq \nu_{r0}, \quad r = m + 1$

$\lambda_j \geq 0$

The commercial bank, DMU$_0$ combines $k$ inputs to produce $m$ good outputs and one bad output ($r = m + 1$). The linear programming problem (3.1.1) gives input treatment to the bad output. Any attempt to solve (3.1.1) enquires further augmentation of good outputs giving input treatment to the bad output, viz., the non-performing assets thus, holding the inputs and bad output constant, DMU$_0$ optimizes its radial parameter $\theta$ for output expansion. If
further output expansion is possible ($\theta' > 1$) the DMU<sub>0</sub> is output technical inefficient, otherwise ($\theta' = 1$) output technical efficient.

We have $k+m+1$ dual variables. These variables are called the input and output multipliers. The dual of (3.1.1) may be expressed as follows:

$$\text{Min } Z = \sum_{i=1}^{k} v_i x_i + \mu_{m+1,0}$$

subject to

$$\sum_{j=1}^{m} \mu_j u_{r_j} = 1 \quad \cdots \cdots \cdots \cdots \cdots (3.1.2)$$

$$\sum_{j=1}^{m} \mu_j u_{r_j} - \sum_{i=1}^{k} v_i x_i - \mu_{m+1,1,0} \leq 0 \quad , j=1,2, \ldots \ldots n$$

$$v_i \geq 0, \quad \mu_i \geq 0$$

This model needs to be discussed. Bad output is viewed as input in output DEA problem; good outputs are increased radially, while inputs and bad outputs are held constant. $v_i$ and $\mu_i$ are respectively the input and output multipliers, which satisfy the non-negative restrictions. The equality constraint in the dual problem,

$$\sum_{j=1}^{m} \mu_j u_{r_j} = 1 \ , \text{ implies that the radial parameter } \theta \text{ in the primal problem is unrestricted for sign. Problem (3.1.2) is the CCR linear programming problem, originated from the CCR fractional programming problem. The constraint space varies from one commercial bank to another; therefore, the efficiencies computed are not global efficiencies. The DMU}_0 \text{ for}$$
which problem (3.1.2) is solved picks multiplier weights of its inputs and outputs, favor it the most.

Let $v^*_i$ and $\mu^*_i$ be input and output multipliers respectively obtained as an optimal solution of the dual problem (3.1.2).

$$Z^* = \text{Min } Z \text{ measures the output technical efficiency of DMU}_0.$$  

$$\frac{\partial Z^*}{\partial x_{i0}^*} = v^*_i, \quad i=1,2,\ldots,k$$  

$$\frac{\partial Z^*}{\partial \mu^*_{m+1,0}} = \mu^*_{m+1}$$  

$v^*_i$ measures the contribution of $i^{th}$ input that DMU$_0$ employs to the output technical efficiency, as measured by $(\theta^*)^{-1}$ in the primal problem. From the fundamental theorem of duality, the extreme values of primal and dual problems are equal, consequently we have the following

$$\theta^* = \sum_{i=1}^{k} v^*_i x_{i0} + \mu^*_{m+1,0}$$  

$$\theta^* \geq 1$$

DMU$_0$ is viewed as output technical efficient, if $\theta^* = 1$. If $v_i$ increases, $\theta$ also increases leading to output technical inefficiency. The bad output is viewed as input and its multiplier weight is given by $\mu^*_{m+1}$. As increase in $\mu^*_{m+1}$, inflates $\theta$ leading to more output technical inefficiency.

The linear programming model (3.1.2) holds bad output constant, seeks only good output expansion radially.
Implicit in economic data are returns to scale, which may be increasing, constant or decreasing. If the bank data on inputs and outputs are influenced by variable returns to scale, the commercial banks are found operating under non-homogeneous environment. A bank is said to be scale efficient if it enjoys constant returns to scale. If a bank’s returns to scale are increasing, it is scale inefficient because it failed to utilize the scale advantages fully by expanding its scale of operation. If returns to scale of a bank are decreasing the bank experiences output losses due to diseconomies of scale. If possible, such DMUs have to reduce their scale of operation.

If the constraint \( \sum_{j=1}^{m} \lambda_j = 1 \) is augmented to the primal problem (3.1.1), the DEA problem identifies variable returns to scale.

Max \( \theta \)

subject to \( \sum_{j=1}^{m} \lambda_j x_{ij} \leq x_{i0}, \quad i = 1, 2, \ldots, k \)

\( \sum_{j=1}^{m} \lambda_j u_{rj} \geq \theta u_{r0}, \quad r = 1, 2, \ldots, m \) \quad \ldots \ldots \quad (3.1.3) \)

\( \sum_{j=1}^{m} \lambda_j u_{rj} \leq u_{r0}, \quad r = m+1 \)

\( \sum_{j=1}^{m} \lambda_j = 1 \)

\( \lambda_j \geq 0, \quad \forall j \)

\( \theta \) is unrestricted for sign.
The dual of the problem (3.1.3) can be obtained as follows:

\[
\text{Min } Z = \frac{1}{\lambda} \sum \mu_i x_i + \mu_{m+1} u_{h0} - w_0 \quad \ldots \ldots \ldots \ldots (3.1.4)
\]

subject to

\[
\sum_{i=1}^{\lambda} \mu_i u_i - \sum_{i=1}^{\lambda} \mu_i x_i - \mu_{m+1} u_{h0} - w_0 \leq 0, \quad j=1,2,\ldots,n
\]

\[
\sum_{i=1}^{\lambda} \mu_i u_i = 1
\]

An optimal solution of problem (3.1.4) reveals apart from output technical efficiency, returns to scale increasing, constant or decreasing. It is the sign of \( w_0 \) that reveals returns to scale.

\( 0 \Rightarrow \) returns to scale are constant

\( w_0^* = \lambda \quad < 0 \Rightarrow \) decreasing returns to scale

\( > 0 \Rightarrow \) increasing returns to scale

The DEA problem (3.1.1) admits constant returns to scale alone.

Some times merger of commercial banks is contemplated. One of the factors which favour merger is the presence of increasing and constant returns to scale. If one bank is enjoying constant returns to scale, a sensitivity analysis is required for increases in what levels of inputs and outputs the bank remains to be scale efficient, so that the merger with some other bank is economical.

**Additive DEA Models:** To measure efficiency an alternative is the additive model. There are several additive models of which we choose the following:
Maximize \( Z = \sum_{i=1}^{n} s^i_i + \sum_{r=1}^{n} s^r_i \)

subject to \( \sum_{j=1}^{n} \lambda_j x_{ij} + s^i_i = x_{i0}, \quad i=1,2,\ldots,k \) \hspace{1cm} (3.1.5)

\( \sum_{j=1}^{n} \lambda_j u_{rj} - s^r_i = u_{r0}, \quad r=1,2,\ldots,m \)

\( \sum_{j=1}^{n} \lambda_j = 1 \)

\( \lambda_j \geq 0. \)

A commercial bank is additive efficient if and only if \( s^- = 0 \) and \( s^{++} = 0 \). \( s^- \) and \( s^{++} \) are the input and output slack vectors respectively obtained by optimizing the additive model.

![Figure (3.1.1)](image)

The DMU that operates at \( D \) is inefficient. The line segments \( AB \) and \( BC \) constitute a production frontier that admits variable returns to scale.
\[ s_i^- = x_{i0} - \sum_{j=1}^{k} \lambda_j x_{ij}, \quad i = 1, 2, \ldots, k \]

\[ s_r^+ = \sum_{j=1}^{m} \lambda_j u_{rj} - u_{r0}, \quad r = 1, 2, \ldots, m \]

\( s_i^- \) and \( s_r^+ \) measure respectively the input excess and output short fall respectively.

\[ x_0 = x_0 - s \]

and \[ u_0 = u_0 + s^+ \] (3.1.6)

are additive efficient input and output vectors of DMU_0.

Optimizing the L_1 norm the input and output of DMU_0 are projected from D to B whose coordinates are, \((\hat{x}_0, \hat{u}_0)\). In the presence of bad output such as non-performing assets a commercial bank's additive optimization problem may be expressed as,

Maximize \( \phi = \sum_{i=1}^{k} s_i^- + \sum_{r=1}^{m} s_r^+ + s_{m+1}^- \)

subject to

\[ \sum_{j=1}^{k} \lambda_j x_{ij} + s_i^- = x_{i0}, \quad i = 1, 2, \ldots, k \]

\[ \sum_{j=1}^{m} \lambda_j u_{rj} - s_r^+ = u_{r0}, \quad r = 1, 2, \ldots, m \] (3.1.7)
\[ \sum_{j=1}^{n} \lambda_j u_j + s_j^- = u_{r0}, \quad r = m + 1 \]

\[ \sum_{j=1}^{n} \lambda_j = 1 \]

\[ s_j^- \geq 0, \quad s_j^+ \geq 0, \quad s_{m+1} \geq 0. \]

The DEA model (3.1.7) differs from the DEA problem (3.1.1) in several respects. The DEA model (3.1.1) holds inputs and non-performing assets constant and seeks output augmentation. The DEA model (3.1.7) seeks reduction in inputs and bad output and augmentation of good outputs, simultaneously.
The additive DEA model is translation invariant. A DEA model is said to be translation invariant if original input and output values are subjected to a translation, the optimal solution of the original DEA model is the same as the translated DEA model. The radial DEA models are not translation invariant in general.

The additive DEA constraints are,

\[ \sum_{j=1}^{n} \lambda_j x_{y} + s_{i}^{-} = x_{i0} \quad (1) \]

\[ \sum_{j=1}^{n} \lambda_j u_{\eta} - s_{r}^{+} = u_{r0} \]

Define

\[ x_{yi} = x_{yi} + \alpha, \quad (2) \]

\[ u_{\eta i} = u_{\eta i} + \beta, \]

Substitute (2) in (1) to obtain,

\[ \sum_{j=1}^{n} (x_{yi} - \alpha_i) \lambda_j + s_{i}^{-} = \sum_{j=1}^{n} x_{yi} \lambda_j + s_{i}^{-} - \alpha, \quad \text{since} \quad \sum_{j=1}^{n} \lambda_j = 1 \]

\[ = x_{i0} - \alpha_i \]

\[ \sum_{j=1}^{n} x_{yi} \lambda_j + s_{i}^{-} = x_{i0}, \quad i=1,2,\ldots\ldots.m \] (3)

Similarly, we have,

\[ \sum_{j=1}^{n} u_{\eta i} \lambda_j - s_{r}^{+} = u_{r0}, \quad r=1,2,\ldots\ldots.s \]

The optimal solution of (1) is same as the optimal solution of (3).
The dual of the DEA problem (3.1.7) may be expressed as,

\[
\text{Min } Z = \sum_{i=1}^{n} v_i x_{i0} - \sum_{j=1}^{m} \mu_j u_{j0} + \mu_{m+1} u_{m+1,0} + l_0
\]

subject to

\[
\sum_{j=1}^{m} v_j x_{ij} - \sum_{j=1}^{m} \mu_j u_{ij} + \mu_{m+1} u_{m+1,j} + l_0 \geq 0 \quad \ldots \ldots \ldots (3.1.8)
\]

\[
v_i \geq 1, \quad \mu_j \geq 1
\]

\[
i = 1, 2, \ldots \ldots k
\]

\[
j = 1, 2, \ldots \ldots m, m+1
\]

3.2. Inputs and Outputs-Parsimony Axioms:

Parsimony is required in the choice of inputs and outputs of a DEA problem. If two inputs are such that one is approximately a multiple of another, omission of one input does not alter efficiency significantly.

If an additional input is augmented to the list of already chosen inputs, the constraints space of the BCC problem shrinks.

* Let \( x_j \) and \( x_{i,j} \) be two inputs of \( j^{th} \) DMU which are such that

\[
x_j = a x_{i,j}, \quad j = 1, 2, \ldots n
\]

\[
\sum_{j=1}^{n} \lambda_j x_{i,j} \leq x_{i,0}
\]

\[
\Rightarrow \lambda_j (x_j / a) \leq x_{i,0} / a
\]

\[
\Rightarrow \lambda_j x_j \leq x_{i,0}. \text{ As such we have only one constraint, rather than two.}
\]

\[
\{(x,u) : \sum_{j=1}^{n} \lambda_j x_{i,j} \leq x_{i,0}, \ i = 1, 2, \ldots k+1, \ \sum_{j=1}^{n} \lambda_j u_{i,j} \geq u_{i,0}, \ r = 1, 2, \ldots m, \text{L}^{k+1,m}(u) \subseteq \text{L}^{k,m}(u) \}
\]

\[
\text{Min } \{ \lambda : \lambda x_{i,0} \in \text{L}^{k+1,m}(u_0) \} \geq \text{Min } \{ \lambda : \lambda x_{i} \in \text{L}^{k,m}(u_0) \}
\]

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Thus, input technical efficiencies will increase. DMUs which were not efficient earlier emerge to be efficient and the DEA looses its discriminatory power.

In terms of output sets, we have

\[ p(x^{t+1}_0) \subseteq p(x^*_0) \]

\[
\text{Max} \{ \theta : \theta u_0 \in P(x^{t+1}_0) \} \leq \text{Max} \{ \theta : \theta u_0 \in P(x^*_0) \}
\]

Thus, output technical efficiency increases, leading to more number of output efficient DMUs and the DEA suffers from lack of discriminatory power.

The consequences are the same in the introduction of an additional output to the already existing outputs.

If two inputs are found to be highly correlated, it is tempting to delete one input from DEA. Such deletion is meaningful if one input is a multiple of another. Otherwise, inclusion efficiencies may be significantly different from exclusion efficiencies.

The BCC input technical efficiencies are output translation invariant but not input translation invariant. Similarly, the BCC output
technical efficiencies are input translation invariant but not output translation invariant.

![Diagram](image)

Figure (3.2.1)

The commercial bank operating at P is technically inefficient input wise. The DMU employs input $x_0$ and produces output $u_0$. By means of radial reduction of inputs, to the level $\lambda x_0$ the producer achieves input technical efficiency, still able to produce the same level of output viz., $u_0$.

Input technical efficiency: $\text{ITE} = \frac{\lambda x_0}{x_0} = \lambda$

Let us shift the origin from O to Z, this can not bring any change in the input technical efficiency. Thus, input technical efficiency is translation invariant in DEA outputs.
The origin is shifted from $O$ to $Z$ and the new frontier production function is constituted by the line segments $ZE$, $EC$ and $CD$.

Let $Z = \lambda_2 x_0$

The input technical efficiency implied by the new frontier is $\lambda_2 \neq \lambda_1$. The BCC input technical efficiency is not input translation invariant.

In a production process of a DMU it is difficult to set output targets, if one solves the input technical efficiency problems. For an identified list of outputs the simple correlation coefficient is found between each pair of outputs. If high correlation is found between a pair of outputs, one of the outputs is included into DEA output list.
Similarly, it can be shown that output technical efficiency is input translation invariant. In short run the plant can not be expanded and inputs can not be increased. For a given set of inputs, it is enquired if further output augmentation is possible. In output DEA simple correlations are found between each pair of inputs. If high correlation is noticed between any pair of inputs one of the inputs is dropped in DEA input list.

If it is desirable to reduce input excesses and output shortfalls simultaneously, an additive DEA is proposed and solved. In additive DEA inputs are reduced and outputs are increased simultaneously. Since additive DEA is translation invariant in both inputs and outputs the choice of DEA inputs and outputs may be based on simple correlation coefficients.

As DEA allows flexibility in the choice of weights on inputs and outputs, the greater the number of factors included, the lower is the level of discrimination. To achieve a reasonable level of discrimination the DMUs shall be 2mk, where m and k respectively represent the number of outputs and inputs.

In the context of Indian banking Saha and Ravisankar* consider the following Inputs and Outputs:

Inputs:

(i) Number of branches,
(ii) Number of employees,
(iii) Establishment Expenditure,
(iv) Non-establishment expenditure (excluding interest expenditure).

Outputs:

(i) Deposits,
(ii) Advances,
(iii) Investments,
(iv) Spread,
(v) Total income,
(vi) Non-interest income,
(vii) Working Funds.

There are two approaches to measure efficiency of commercial banks, (a) the production approach and (b) the intermédiaion approach.

The inputs and outputs of the production approach are*.

Inputs:

(i) Number of branch and account managers,
(ii) Number of administrative and commercial staff,
(iii) Number of tellers,
(iv) Operational costs (excluding staff costs).

Outputs:

(i) Total value of deposits,
(ii) Total value of loans,
(iii) Total value of balance sheet business,
(iv) Number of general service transactions.

Input Prices:

(i) Salary of branch/account managers,
(ii) Salary of administrative/commercial staff,
(iii) Salary of tellers.

The inputs and outputs of intermedation approach are,

Inputs:

(i) Non-interest costs,
(ii) Interest costs from deposits,
(iii) Interest costs from loans.
Outputs:

(i) Total value of loans,
(ii) Total value of off-balance sheet business,
(iii) Total value of deposits.

Output Prices (Revenue):

(i) Fund transfer price of deposits,
(ii) Interest earned from loans,
(iii) Income from off-balance sheet business.

Camanho and Dyson* measured cost efficiency with price uncertainty DEA, employing four inputs and one output.

Inputs:

(i) Number of branch and accounting managers,
(ii) Number of administrative and commercial staff,
(iii) Number of tellers,
(iv) Operational costs excluding staff cost.

Output:

Number of general service transactions.

Input prices:

(i) Average salary and fringe benefits of branch and account managers,

(ii) Average salary of fringe benefits of administrative/commercial staff,

(iii) Average salary and fringe benefits of tellers.

The intermediation approach was originally developed by Sealey and Lindleeg.

Inputs:

(i) Deposits,

(ii) Labour,

(iii) Capital.

Outputs:

(i) Total loans,

(ii) Securities.

Elena Beccalli and Barbara casu*, in their study on efficiency and stock performance used a variation of intermediary approach in their choice of inputs and outputs.

Inputs:

(i) Average cost of labour,

(ii) Deposits and

(iii) Capital.

* Elena Beccali, Barbara Casu, Clandia Girardone, “Efficiency and Stock Performance in European Banking”.

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Outputs:

(i) Total loans (Reflects lending activities).

(ii) Non-lending activities.

3.3. Radial Measure of Eco-Efficiency:

We give the value of non-performing assets the input status and enquire for its proportional reduction assuming that the inputs and good outputs remain to be the same.

The BCC formulation takes the form,

\[
\text{Min } \lambda - \varepsilon \left[ \sum_{i=1}^{k} s_i^- + \sum_{r=1}^{m} s_r^+ + s_{m+1}^- \right]
\]

subject to \( \sum_{j=1}^{n} \lambda_j y_j + s_i^- = x_{i0}, \ i = 1,2, \ldots, k \)

\[
\sum_{j=1}^{n} \lambda_j u_{rj} - s_r^+ = u_{r0}, \ r = 1,2, \ldots, m \quad (3.3.1)
\]

\[
\sum_{j=1}^{n} \lambda_j u_{m+j} - \lambda u_{m+1,0} + s_{m+1}^- = 0
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\lambda_j \geq 0.
\]

A commercial bank for which the above problem is solved is said to be eco-efficient if and only if,
\[ s_{i} = s_{r} = s_{m+1} = 0, \quad i = 1,2, \ldots, k; \quad r = 1,2, \ldots, m \]

and \( \lambda' = 1 \), where the asterisk marks indicate that the solution is optimal.

\[ \lambda' u_{m+1,0} - s_{m+1} = \sum_{j=1}^{k} \lambda'_{j} u_{m+1,j}, \]

estimates possible bad output reduction given the inputs and good outputs.

The dual of the above model is given by,

\[
\begin{align*}
\text{Max}_{(\mu, v)} & \quad \sum_{r=1}^{m} \mu_{r} u_{r,0} - \sum_{i=1}^{k} v_{i} x_{i,0} - b_{0} \\
\text{subject to} & \quad \sum_{r=1}^{m} \mu_{r} u_{r,j} - \sum_{i=1}^{k} v_{i} x_{i,j} - b_{0} \leq 0, \quad j = 1,2, \ldots, n \quad \ldots \quad (3.3.2) \\
& \quad \mu_{r+1}, u_{m+1,0} = 1 \\
& \quad v_{i} \geq \varepsilon, \mu_{r} \geq \varepsilon.
\end{align*}
\]

where \( \varepsilon > 0 \). \( b_{0} \) is unrestricted for sign.

\( v_{i} \) and \( \mu_{r} \) are input and output multipliers respectively.

The fractional programming problem equivalent to the above is,

\[
\begin{align*}
\text{Max}_{(\mu, v)} & \quad \sum_{r=1}^{m} \mu_{r} u_{r,0} - \sum_{i=1}^{k} v_{i} x_{i,0} \\
\text{subject to} & \quad \sum_{r=1}^{m} \mu_{r} u_{r,j} - \sum_{i=1}^{k} v_{i} x_{i,j} \leq 1, \quad j = 1,2, \ldots, n.
\end{align*}
\]

\( \left( \sum_{j=1}^{k} \lambda'_{j} x_{j}, \sum_{r=1}^{m} \lambda'_{r} u_{r,j} \right) \) represent the inputs and outputs of a composite DMU that dominates DMU\(_{0}\), whose eco-efficiency is under evaluation.
Deep Eco-Efficiency:

The deep eco-efficiency of a commercial bank is measured by solving the BCC formulated linear programming problem, assuming that all the inputs take unit values. Consequently, the problem takes the form,

\[
\text{Min } \lambda - \varepsilon \left( \sum_{r=1}^{n} s^{-}_r + \sum_{r=1}^{m} s^{+}_r + s^{-}_{m+1} \right)
\]

subject to

\[
\sum_{j=1}^{n} \lambda_j u_{r,0} - s^{-}_r = u_{r,0} , \ r=1,2,...,m
\]

\[
\sum_{j=1}^{n} \lambda_j u_{m+1,0} - \lambda u_{m+1,0} + s^{-}_{m+1} = 0 \quad \text{............... (3.3.3)}
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\lambda_j \geq 0.
\]

The CCR dual of the above primal may be expressed as,

\[
\text{Max } (\mu, \nu) \sum_{r=1}^{n} \mu_r u_{r,0} - \mu_{m+1} u_{m+1,0} - b_0
\]

subject to

\[
\sum_{r=1}^{n} \mu_r u_{r,0} - \mu_{m+1} u_{m+1,0} - b_0 \leq 0 , \ j=1,2,\ldots,n
\]

\[
\mu_{r+1}, u_{m+1,0} = 1
\]

\[
\nu_i, \mu_r \geq \varepsilon , i=1,2,\ldots,k, \ r=1,2,\ldots,m.
\]
The fractional equivalent of the above problem is as follows:

\[
\begin{align*}
\operatorname*{Max}_{(\mu, v)} & \sum_{r=1}^{m} \mu_r u_{r0} - \mu_{m+1} u_{m+1,0} - b_0 \\
\text{subject to} & \sum_{r=1}^{m} \mu_r u_{rj} - \mu_{m+1} u_{m+1,j} - b_0 - \mu_{m+1} u_{m+1,0} \leq 1, \, j=1,2,\ldots,n. \quad \text{... (3.3.4)}
\end{align*}
\]

\[v_i, \mu_r \geq \varepsilon.\]

**Technical Efficiency:**

In the absence of non-performing assets DMU_0 solves the following input technical efficiency problem,

\[
\begin{align*}
\operatorname{Min} & \quad \lambda - \varepsilon \left| \sum_{L=1}^{k} s_{L}^{-} + \sum_{r=1}^{m} s_{r}^{+} + s_{m+1}^{-} \right| \\
\text{subject to} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_{i}^{-} = \lambda x_{i0}, \quad i=1,2,\ldots,k. \\
& \quad \sum_{j=1}^{n} \lambda_j u_{ij} - s_{i}^{+} = u_{i0}, \quad r=1,2,\ldots,m \\
& \quad \sum_{j=1}^{n} \lambda_j = 1
\end{align*}
\]

We need to combine the eco and input technical efficiencies.
The above figure depicts a production technology that employs a single input $x$, produces two outputs $u_s$ and $u_g$. The former is a bad output and the later is a good output. The bad output $u_s$ such as non-performing assets is assigned the status of an input. If the inputs (bad output excluded) alone are reduced proportionately, the technical efficiency is $\lambda_s$ so that the targeted inputs are $\lambda_s x_0$, where the ashtrik mark represents optimal solution.

$L(u_g)$ is the input level set that consists of pairs of bad output and input compatible with good output $u_g$.
Holding the input constant, if the bad output is reduced proportionately, the technical efficiency achieved is $\theta^*_b$ and the targeted bad output is,

$$\theta^*_b u^0_b, \quad 0 \leq \theta^*_b \leq 1$$

For given good output levels a proportional reduction of inputs and non-performing assets require to solve the BCC linear programming problem:

$$\text{Max} \quad \lambda - \bar{\varepsilon} \left( \sum_{r=1}^{k} s^r_i + \sum_{r=1}^{m} s^r_{m+1} \right)$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_j - \lambda x_{o_0} + s^r_i = 0, \quad i=1,2,\ldots,k$$

$$\sum_{j=1}^{n} \lambda_j u^r_{o_j} - s^r_r = u_{o_0}, \quad r=1,2,\ldots,m \quad (3.3.5)$$

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\[ \sum_{j=1}^{\lambda_j} \mu_{u+1,j} + s_{u+1} = u_{u+1,0} \]

\[ \sum_{j=1}^{\lambda_j} \lambda_j = 1 \]

\[ \lambda_j \geq 0, \quad j=1,2,\ldots,n. \]

The dual of the above linear programming problem is the CCR multiplier problem.

\[ \text{Max} \quad \sum_{r=1}^{\mu_r} \mu_r u_r - b_0 \]

subject to

\[ \sum_{r=1}^{\mu_r} \mu_r u_r - \sum_{i=1}^{\mu_i} v_i x_i - \mu_{u+1,u+1,j} - b_0 \leq 0 \]

\[ \sum_{i=1}^{v_i} v_i x_i + \mu_{u+1,u+1,0} = 1 \quad \text{...............} \quad (3.3.6) \]

\[ v_i \geq \varepsilon, \quad \mu_r \geq \varepsilon, \quad \mu_{u+1} \geq \varepsilon. \]

The fractional representation of the above linear programming problem is,

\[ \text{Max} \quad Z = \frac{\sum_{r=1}^{\mu_r} \mu_r u_r - b_0}{\sum_{i=1}^{v_i} v_i x_i + \mu_{u+1,u+1,0}} \]

subject to

\[ \frac{\sum_{r=1}^{\mu_r} \mu_r u_r - b_0}{\sum_{i=1}^{v_i} v_i x_i + \mu_{u+1,u+1,j}} \leq 1 \quad \text{...............} \quad (3.3.7) \]

\[ \mu_r, \quad v_i \geq 0 \]
The target input and bad output are respectively, \( \lambda^* x_0 \) and \( \lambda^* u_b^0 \).

Various studies on the commercial banks' efficiency have chosen sets of outputs not identical. The input reduction linear programming problems are output translation invariant.

In commercial banks usually advances appear to be a fraction of deposits. Let \( z_j \) and \( k_j \) be deposits and advances respectively of \( j^{th} \) commercial bank

\[ z_j = \alpha k_j + \epsilon_j \]

where \( \epsilon_j \) is the statistical disturbance term.

\[ H_0 : \alpha = 0 \]

\[ H_1 : \alpha \neq 0 \]

By the method of least squares we obtain the estimator of \( \alpha \) and express it as follows:
\[ \hat{\alpha} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(k_j - \bar{k})}{\sum_{j=1}^{m}(k_j - \bar{k})^2} \]

If the null hypothesis is rejected, we compute the simple correlation coefficient. Sample correlation coefficient closer to unity reveals that only one of these outputs shall be chosen as a valid output.

In input approach the efficiency measures are output translation invariant. To exploit this property of BCC formulation simple correlation coefficients between pairs of output variables have to be found and carefully examined for qualification to be included in the list of output variables for the purpose of DEA.

Between advances and deposits with intercept zero slope is estimated and tested for significance, the calculated value of t-statistic is found to be 70.49, which is highly significant. The null hypothesis is rejected. The correlation coefficient (sample) is found to be 0.9932.

Ashish Saha, T.S. Ravisankar (2000)* in their attempt to rate Indian Commercial Banks identified Deposits and Advances as outputs among others.

Total income of a commercial bank is comprised of interest and non-interest income. Further, increase in loans and advances results in increased interest income the estimated correlation coefficient between ‘loans and advances’ and ‘interest income’, if closer to one signals the choice of one of these to be included in the list of DEA outputs.

The estimate of the intercept of the linear model with interest income as dependent variable and ‘loans and advances’ as independent variable is found to be not significant at 1% level of significance. Consequently, in the linear model fitted without intercept, the regression coefficient estimate is highly significant as found by its t-value equal to 166.49. The sample correlation coefficient is 0.9981 implying that one of the two variables ‘loans and advances’ and ‘interest income’ has to be included in the list of DEA variables.

‘Loans and advances’ and ‘non-interest income’ are two variables qualified to be included in the list of DEA outputs.

Elena Beccali casu, in their study on efficiency and stock performance identified total loans and income from non-lending activities as outputs. The later output variable can be proxied by non-interest income.

Investment of commercial banks constitutes investments in India and abroad. The components of investment in India are Government securities, other approved securities, shares, debentures and bonds, subsidiaries and/or
joint ventures and others. Investments abroad constitute Government securities, subsidiaries and/or joint ventures and others.

M. Nasser Katib and Kent Matthews*, in their study of Malaysian Banking Sector had identified 'total loans and advances', 'total deposits' and 'investments' as outputs.

The input variables of commercial banks used in DEA are,

(i) Labour or employees,
(ii) Assets,
(iii) Deposits,
(iv) Number of branches,
(v) Stock holder's equity,
(vi) Establishment expenditure,
(vii) Non-establishment expenditure,
(viii) Interest expenditure,
(ix) Fixed assets,
(x) Work space,
(xi) Number of computers,
(xii) Number of tellers,
(xiii) Operational costs (excluding staff costs),
(xiv) Rent.

‘Deposits’ were considered to be ‘output’ in some studies and ‘input’ in some other studies. But, this study considers ‘deposits’ neither as input nor output.

From the balance sheet information of commercial banks operating on Indian soil information on the following inputs is available.

1. Number of employees,
2. Deposits,
3. Interest expended,
4. Operation expenses
5. Capital and reserves and surplus,
6. Borrowings,
7. Fixed assets
8. Other assets,
9. Total assets.

A common input consideration for most of the studies is ‘number of employees’. Total assets is the sum of ‘fixed’ and ‘other assets’, fixed assets is comprised of premises, fixed assets under construction and other fixed assets. Other assets constitute inter office adjustment, interest accrued, tax paid, stationery and stamps and others.

‘Total assets’ is choosen as an input in DEA.
3.4. Additive Models to Measure Eco-Efficiency:

If the inputs and outputs are held constant, but non-performing assets are reduced we solve the following additive models obtained tailoring the BCC model:

\[
\text{Max} \quad s^{-}_{m+1}
\]

subject to \[
\sum_{j=1}^{k} \lambda_{j} x_{j} + s^{-}_{i} = x_{i0}, \quad i=1,2,\ldots,k.
\]

\[
\sum_{j=1}^{m} \lambda_{j} u_{r0} - s^{+}_{r} = u_{r0}, \quad r=1,2,\ldots,m \quad \text{......... (3.4.1)}
\]

\[
\sum_{j=1}^{m} \lambda_{j} u_{s0,j} + s^{-}_{m+1,i} = u_{m+1,i}
\]

\[
\sum_{j=1}^{k} \lambda_{j} = 1
\]

The dual multiplier version of the above problem is as follows:

\[
\text{Min} \quad \sum_{i=1}^{k} v_{i} x_{i0} - \sum_{r=1}^{m} \mu_{r} u_{r0} + \mu_{m+1,i} u_{m+1,i} + b_{0}
\]

subject to \[
\sum_{i=1}^{k} v_{i} x_{i0} - \sum_{r=1}^{m} \mu_{r} u_{r0} + \mu_{m+1,i} u_{m+1,i} + b_{0} \geq 0, \quad j=1,2,\ldots,n.
\]

\[
v_{i} \geq 0, \quad i=1,2,\ldots,k \quad \text{.......................... (3.4.2)}
\]

\[
\mu_{r} \geq 0, \quad r=1,2,\ldots,m
\]

\[
\mu_{m+1} \geq 1
\]
Treating bad output as input, the additive model that reduces simultaneously are inputs and bad output is formulated as,

\[
\text{Max } \sum_{i=1}^{n} s_{i} - s_{m+1}
\]

subject to \[\sum_{j=1}^{n} \lambda_{j} x_{ij} = x_{i0}, \ i=1,2,\ldots,k\]

\[\sum_{j=1}^{n} \lambda_{j} y_{rj} = y_{r0}, \ r=1,2,\ldots,m \] \hspace{1cm} (3.4.3)

\[\sum_{j=1}^{n} \lambda_{j} u_{m+1j} + s_{m+1} = u_{m+1,0}\]

Dual of (3.4.3) is,

\[
\text{Min } \sum_{j=1}^{k} v_{j} x_{j0} - \sum_{r=1}^{m} \mu_{r} y_{r0} + \mu_{m+1} u_{m+1,0} + b_{0}
\]

subject to \[\sum_{j=1}^{k} v_{j} x_{ij} - \sum_{r=1}^{m} \mu_{r} y_{rj} + \mu_{m+1} u_{m+1,j} + b_{0} \geq 0\]

\[v_{i} \geq 1, \ i=1,2,\ldots,k \] \hspace{1cm} (3.4.4)

\[\mu_{m+1} \geq 1\]

\[\mu_{r} \geq 0, \ r=1,2,\ldots,m.\]

3.5. Cost Efficiency:

Pursuing the Data Envelopment Analysis (DEA), one can find not only technical efficiency, but also input cost efficiency.

Estimation of cost efficiency requires data on inputs, number of employees and total assets and the input prices, wage rate and rate of return on
assets. Information on all these characteristics is available in the Reserve Bank Bulletins.

**Inputs:**

1. Number of employees
2. Total assets.

**Input Prices:**

1. Salary per employee
2. Rate of return on assets

**Outputs:**

1. Loans and advances
2. Non-interest income

**Bad output:**

Non-performing assets (NPA).

**Inputs Sets:**

Let \( L(u_o^g) \) be input level set and \( u_o^g \) be the vector of good outputs.

Define \( \hat{x}_o = [x_o, u_o^g] \)

where \( x_o \) be the vector of inputs and \( u_o^b \) be bad output, viz., non-performing assets, the bad output \( u_o^b \) is treated as input.

\[
L(u_o^b) = \left\{ \hat{x}_o : \hat{x}_o \text{ produces } u_o^b \right\}
\]
Properties of $L(u^x)$:

1. $L(0) = R_+^{exi}$, $0 \notin L(u^x)$ if $u^x \geq 0$ and positive for at least one component.

   Every input vector $\hat{x}_0$ can produce null output and null input vector can not produce positive output.

2. $\hat{x} \in L(u^x)$, $\hat{x}_i \geq \hat{x} \Rightarrow \hat{x} \in L(u^x)$

If $\hat{x}$ produces $u^x$ any input vector larger than $\hat{x}$ (larger in at least one component) can also produce $u^x$. Due to inefficiency in production, if $\hat{x}$ produces $u^x_0$ then an input vector larger than $\hat{x}$ can also produce $u^x_0$.

3. $u^x_2 \geq u^x_1 \geq 0 \Rightarrow L(u^x_2) \subseteq L(u^x_1)$

   Every input vector $\hat{x}$ that produces $u^x_2$ can also produce $u^x_1$, therefore we have: $\hat{x} \in L(u^x_2) \Rightarrow \hat{x} \in L(u^x_1)$

   $L(u^x_2) \subseteq L(u^x_1)$

* Suppose $\hat{x}$ and $\hat{x}_1$ are such that their DEA inputs are equal. But the NPA of $\hat{x}$ is smaller than the NPA of $\hat{x}_1$, when this happens $\hat{x}$ is a superior input vector than $\hat{x}_1$.
Factor Minimal Cost:

For a commercial bank DMU₀, the factor minimal cost can be estimated by solving the following linear programming problem:

\[ Q(u_0, p) = \text{Min} \left\{ p\hat{x} : \hat{x} \in L(u^*_0) \right\} \]

The input level set consistent with the axioms of convexity, inefficiency and minimum extrapolation can be expressed as,

\[ L(u^*_0) = \left\{ \hat{x} : \sum_{j=1}^{n} \lambda_j \hat{x}_j \leq \hat{x}, \sum_{j=1}^{n} \lambda_j u^*_j \geq u^*_0, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\} \]

\[ Q(u^*_0, p) = \text{Min} \left\{ p\hat{x} : \sum_{j=1}^{n} \lambda_j \hat{x}_j \leq \hat{x}, \sum_{j=1}^{n} \lambda_j u^*_j \geq u^*_0, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\} \]

Equivalently,

\[ Q(u^*_0, p) = \text{Min} \ p\hat{x} \]

subject to

\[ \sum_{j=1}^{n} \lambda_j \hat{x}_j \leq \hat{x} \]

\[ \lambda_j u^*_j \geq u^*_0 \] .............................. (3.5.1)

\[ \sum_{j=1}^{n} \lambda_j = 1 \]

\[ \lambda_j \geq 0. \]

However, in the present case the NPA price is unknown.
The factor minimal cost function inherits its properties from its input level sets.

1. \( Q(u_0^x, \lambda p) = \lambda Q(u_0^g, p) \)

   \[ Q(u_0^x, \lambda p) = \min_{\hat{x}} \left\{ \lambda p\hat{x} : \hat{x} \in L(u_0^g) \right\} \]

   \[ = \lambda \min_{\hat{x}} \left\{ \lambda p\hat{x} : \hat{x} \in L(u_0^g) \right\} \]

   \[ = \lambda Q(u_0^g, p) \]

The factor minimal cost function is linear homogeneous in input prices.

2. \( p_1 \geq p_2 \Rightarrow Q(u_0^g, p_1) \geq Q(u_0^g, p_2) \)

When prices increase, the factor minimal cost also increases.

3. \( u_1^x \geq u_2^x \Rightarrow Q(u_1^x, p) \geq Q(u_2^x, p) \)

If price vector remains stable, production of larger output vector requires larger factor minimal cost.

\[ u_1^x \geq u_2^x \]

If \( x \) produces \( u_1^x \), then it produces \( u_2^x \) also.

Thus \( x \in L(u_1^x) \Rightarrow x \in L(u_2^x) \)

\[ L(u_1^x) \subseteq L(u_2^x) \]

\[ \Rightarrow Q(u_1^x, p) \geq Q(u_2^x, p) \]

4. \( Q(u^x, p) \) is differentiable with respect to the components of \( u^x \).