Chapter – II

REVIEW OF LITERATURE
2.0. Introduction:

A commercial bank is an organization controlled by government regulations which change from time to time. Its dual objectives are profit maximization and risk minimization. The commercial banks compete with several other financial institutions to handle the savings and credit needs of customers.

The banking industry is influenced by the fiscal policies of the government and the monetary policies of the central bank. Never the less its transactions are affected by the business cycles. Apart from these broad factors, local factors, and the philosophical attitudes of the management also influence the bank's business.

Fiscal policy intercepts income, savings and expenditure of the economy. Monetary policy intervenes into lending and investment activities of the commercial banks.

Commercial banking has traditionally remained for many years a protected industry in many emerging economies and India was not an exception. Regulated deposit and lending rates and restrictions on competition enabled the industry to sail smoothly and securely. Lending and deposit taking have continued to remain two major activities.
If economy is on the path of rapid expansion and inflation is mounting at an alarming rate, reduction of government expenditure and increase in taxes combined with increase in rate of interest appear to bring down the rate of inflation and the expansion can be slowed on the other hand, if recession is observed, so that the economy is contracting, increase in government expenditure, reduction of taxes and cutting down the interest rates lead to the expansion of the economy. Thus, business cycles and interest cycles affect the business of the commercial banks.

The commercial banks and other similar financial institutions are to reserve a part of their deposits and the remains enable the banks to create more money which contributes to money supply. These funds are used as advances and investments.

To help economic recovery in periods of inflation and recession fiscal and monetary policy must work in union. Other factors which influence commercial bank business are local such as the structure of the bank, bank’s community and the management.

Commercial banks are viewed as financial intermediaries that channel funds between savers and borrowers. Banking business is to satisfy both the users and the suppliers of bank funds. The balance sheet and income statement of banks reveal bank’s success in managing its assets and liabilities.
Banking business has been undergoing drastic changes, consequently financial stability has come to occupy center stage as one of the prime policy concerns facing central banks worldwide. In recent years there was a significant improvement in the performance of the commercial banking system, measured in terms of net profits and marketability.

2.1. Asset/Liability Management:

The assets and liabilities variables of schedule commercial banks* are as follows:

**Liabilities:**

1. Capital.
2. Reserves and Surplus.
3. Deposits = Demand Deposits • Saving Bank Deposits • Term Deposits.
4. Borrowings.
5. Other liabilities and provisions.

**Assets:**

1. Cash and balances with RBI.
2. Balances with banks and money at call and short notice.
3. Investments: Government securities • Approved securities • Unapproved securities.

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4. Loans and Advances: Bills purchased and Discounted + Cash Credit + Over drafts + Term loans.

5. Fixed assets.

6. Other assets.

Because banks are highly regulated, the general categories of assets are fairly consistent from bank to bank. The balance sheet lists assets from the most liquid to the least liquid.

Some researchers view banks as producers of loans and deposits accounts or by the number of accounts serviced, which is called "Production Approach".

**Intermediation approach:**

\[ \text{Value of loans} = f(\text{wage bill, capital, operations, interest}) \]

**Production approach:**

\[ \text{Number of transactions} = f(\text{operating cost, wage bill, capital deposits}) \]

Asset/liability management is coordinating the assets and liabilities in order to maximize bank profitability and stock holders' earnings. The objectives of asset/liability management are to meet liquidity needs. Planning the maturities of assets and liabilities limit their exposure to interest rate risk.
and controlling the rates earned and paid on assets and liabilities in order to maintain or maximize the spread between interest costs and interest income.

Spread and Gap management are two important strategic approaches to asset/liability management. 'Spread' is the difference between bank's interest income on assets and interest expense on liabilities. For a successful bank this should be as large as possible.

Gap matches rate sensitive assets and liabilities to achieve maximum profits over an interest rate cycle. These strategies coordinate bank's assets and liabilities given alternative movements in the interest rates. The objectives of spread management are to reduce the bank's exposure to cycle rates and to stabilize earnings over the long time.

Most of the banks' assets and liabilities are rate sensitive. Gap management is an asset/liability strategy that analyzes the rate sensitivity of assets and liabilities. Stability in net interest spread and net operating income occur when rate sensitive assets equal rate sensitive liabilities. If interest sensitive assets exceed interest sensitive liabilities, then the gap is asset sensitive, otherwise the gap is liability sensitive. Every debt instrument of a bank carries some degree of risk. Credit risk and interest risk are the two basic types of risks in bank investments and loans.
Credit risk arises if the borrowers of loan or the issuer of a security fails to pay interest and the principal amount. Such defaults tend to increase the non-performing assets of a bank. Banks can reduce the credit risk by a careful review of the borrower’s present financial position and his past credit history.

Investment issues are also subjected to some degree of credit risk when the issuer of the investment securities fails to meet the promises in the debt contract due to the interruption or reduction of income. The organization’s credit rate drops down and its borrowing costs will increase.

The other prominent risk is interest rate risk. A commercial bank’s liabilities and assets are greatly influenced by the rate of interest, which subjected to major fluctuations can produce a devastating impact on bank profitability if a bank fails to protect itself.

A bank requires strong profits to pay dividends to stock holders and build stock holders equity, to offset losses due to non-performing assets, to pay operating expenses and to expand products and services. ‘Return on Assets’ and ‘Return on Equity’ are the performance or profitability measures of a commercial bank. Carefully balancing the credit risk, banks should improve the qualities of their assets and liabilities.
The concept of ‘risk concentration’ has become a focal point for all the commercial banks. Large credit exposure to an individual customer or a group of related borrowers is to be avoided; otherwise, the bank will liquidate if the borrowers fail to repay the advances given to them. The recommendation of the World Bank is 25 percent of bank’s capital is the limit for an individual large exposure, in India this limit is only 15 percent.

For commercial banks which gone for public issue a two stage optimization is desirable*. ‘Profitability’ and ‘Marketability’ are the two issues concerned to the banks which have gone for public issue. ‘Profit’ and ‘Revenue’ are the outputs of stage (1) optimization and theses outputs are treated as inputs in stage (2) optimization, concerned with marketability.

2.2. Indian Banking:

Indian Banking was dominated by private ownership; profit and return to investments were the performance indicators. This scenario was observed prior to 1969. Commercial banks were nationalized, subsequently, with additional objectives of optimizing social benefit and geographical expansion to meet the needs of the people. The commercial banks strive hard towards cost effectiveness, maximizing profit, consumer services, mobilization of demands

and credit advancement in rural and backward regions. In the recent past the commercial banks have diversified their activities consisting credit cards business, Insurance business, investment in mutual funds and so on.

Globalization allowed many foreign commercial banks to operate on Indian soil. The changes that are taking world wide continued to give shocks to the banking system resulted in an expansion of banking services both in range, volume and non-performing assets. Entry of foreign banks and new private sector banks resulted in competition and every commercial bank has to rise to international standards and evolve suitable competitive strategies.

Banking sector reforms from time to time are to be initiated to make the Indian commercial banks internationally competitive.

Measurement of efficiency and ranking* of Indian Commercial Banks are of serious concern not only to the management of an individual bank but also to the policy maker. Non-performing assets (NPA) reflect the risk factor of the concerned commercial bank. The performance measures treat good and bad outputs differently by valuing good outputs and ignoring bad outputs. Undesirable output such as NPA is not freely (costlessly) disposed. Therefore,

weak disposability of bad outputs is a suitable hypothesis in the presence of NPA. Weak disposability of bad outputs imply bad output reduction is possible through the reduction of good outputs, incurring a loss of good outputs.

Inflationary pressures and higher nominal interest rates increased the opportunity value of depositor's funds. Regulations prevented the traditional institutions from offering their services. The external debt crisis which surfaced in 1991 made India vulnerable in meeting its international payments obligations and the mounting rate of inflation were two major causes of deregulation. The Narsimham Committee pointed that, ".... with increased deregulation of industry and the emerge of new competitive conditions the responsibilities developing on the financial system in mobilizing resources and allocating them effectively and responding flexibility to emerging situations would be much greater. An efficient and market-oriented financial system could thus be regarded as complement to market-based decision making in the real sector".

**Characteristics of Indian Banking:**

1. Collective bargaining, recruitment are at Industry level, had little to do with wages or service conditions.

2. Decisions regarding technology were taken under the guidelines provided by the RBI.
3. Diversification of activities had to be approved by RBI.

Balance sheets are presented since 1992 on the basis of new guidelines

(i) NPAs had gone up 24% of loan portfolio.
(ii) Only 15 banks declared profits and 13 banks declared losses.
(iii) Half of the public sector banks had negative net worth.

Non-Performing assets:

The burden of NPAs was a millstone round the necks of the banks. The government decided to inject capital the RBI entered into an understanding with the banks and the government put Rs.125, 000 million. At the end of 1995, 13 banks have achieved a capital adequacy ratio of at least 8% while another between 4 to 8 percent. Three banks were below 4%. Amending the banking companies’ acquisition and transfer act, banks were allowed to go to capital market for capitalization. The second most significant step is the gradual reduction of Cash Revenue Ratio (CRR) and Statutory Liquidity Ratio (SLR) to meet government deficits. This step was recommended by the Narasimham Committee.

Non-performing assets are due to political interference, willful defaults, target lending and fraudulent behavior of banks themselves.
The commercial banks should concentrate on somehow reducing the amount and number of accounts of NPA category.

In the banking post nationalized era banks were unfamiliar with the outcome of their new activities that could result in losses and even carried risks of total failure. Attempts were not made to provide cushion to abort the shocks. The commercial banks shall be of serious concern to the evaluation proposals, monitoring of accounts and so on. Lending shall be based on risk management techniques. Banks such as Barclays Bank in England has successfully implemented such polices with suitably tailored technology (lending advisor software's). It is also mandatory to critically review the organizational structure.

Technology shall be lead to simplifications without compromises of safety and security. New technology in terms of softwares and computers led to excess staff and down sizing has become necessary. However, if expansion of a commercial bank is to take place, the experienced excess staff can take charge of the new wings or branches.

For the last 25 years, performance planning and budgeting has been the major management tool in use. Every area of work-lending, investing and deposit mobilization and the associated work systems will undergo radical changes quite quickly. Tough decisions are desirable. The staff whose

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intransigence and negativism threaten the whole process will have to be dealt with immediately and decisively.

Lack of coordination with man power plans and capital asset plans lead to the growth of NPA.

- Asset-led growth is required
- Empowerment of the employees is the need of the hour.

A demoralized managerial staff cannot be expected to deliver a competitive environment.

Planning mechanism needs to be revitalized. Successful implementation of new policies requires continuous, open and honest communication at all levels ... employees shall be told the implications of the various changes.

Employees of commercial banks do not accept changes easily. If organizations fail to adapt the proposed structural changes it only leads to collapse of the whole structure.

In the minds of employees the management shall promote a state of mind that accepts changes towards gaining strategic opportunities setting aside the past.
RBI extended help since 1992 to reduce non-performing assets. The commercial banks are instructed to classify the NPA accounts as per the guidelines issued by it. RBI instructed banks to classify the accounts into four categories.

1. Standard Assets,
2. Sub-standard Assets,
3. Doubtful Assets,
4. Loss Assets.

The remedial actions differ from one asset type to another.

Apart from advances given to customers by the banks, banks themselves may invest and create assets which may also suffer in value in due course of time. A major part of the banks' investment comprises government securities. Approved securities can be divided into two categories, permanent and current. Such investments which the bank holds till the maturity are permanent. Current investments are those which a bank buy and sell on daily basis. Capital adequacy measures are essential to safeguard the banks in a helpless situation.

Efficiency measurement examines the balance sheets of banks and assets the state of health of Indian commercial banks. The parameters required for efficiency measurements are

1. Return on average assets,
2. Investment income/average assets,
3. Investment on loans/average assets,

4. Total interest income/average assets, and so on.

Peer Groups:

Banking is relatively homogeneous and banks with similar asset size tend to be similar with respect to their structure and operation. Comparison of one bank’s performance with others is useful.

Why the performance of a bank differs from its peer group? The variations could be attributed etc, variation in lending to various sectors.

Liquid Asset Schedule Analysis:

Low levels of liquid assets, compared to those of peer groups may indicate excessive risk taking. Liquidity ratios measure the risk of not being able to accommodate customer needs.

Deregulation steps:

- Reduction of Cash-Reserve Ratio
- Reduction of Statutory Liquidity Ratio
- Reduction of Maximum lending rate
- Institutional debts
- Prudential restrictions
- Income regulation provisions.
CAMEL Rating-Performance Indicators:

'Capital, Assets, Management, Earnings and Liquidity' ratios.

(i) \[
\frac{\text{Capital}}{\text{Total assets}}
\]
\[G_c - G_r \geq 0, \quad G_c \geq G_r\]

This is a capital adequacy ratio. Capital provides a cushion to absorb losses. A low ratio value in relation to the peer group average indicates limited ability to withstand losses and/or future economic turns.

(ii) \[
\frac{\text{Delinquent loans}}{\text{Total assets}}
\] : This is an asset quality ratio.

This should be as small as possible. It indicates not only control but also potential losses.

(iii) \[
\frac{\text{Operating expenses} - \text{Provisions for loan losses} - \text{Invest expenses}}{\text{Average assets}}
\]

Larger values in relation to the peer group values indicate that operating expenses are not adequately controlled.

(iv) \[
\frac{\text{Net Income after transfers}}{\text{Average assets}}
\]

This ratio is indicator of profitability.
2.3. **DEA Variables in Different Approaches:**

Financial ratios such as return on equity, return on assets were two important ratios traditionally used as efficiency indicators of a commercial bank in a competitive environment. With the emerge of data envelopment analysis that provides a benchmark frontier relative to which an inefficient bank is compared with DEA can handle multiple inputs and multiple outputs with comfortable ease.

Silva Portela et.al* have chosen a range directional model to study commercial banks efficiency. Their DEA variables were,

**Inputs:**

(1) Number of clients (t)

(2) Rent (t)

**Outputs:**

(1) $\Delta$ Number of clients (t)

(2) $\Delta$ Value current account (t)

(3) $\Delta$ Value other resources (t)

(4) $\Delta$ Value Titles deposited (t)

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(5) Δ Value Credit over bank (t)
(6) Δ Value Credit Associates (t)
(7) Δ Number of Transactions (t).

Δ - Change in value between the start and the end of period t.
t - Time period.

To study Canadian Commercial Banks performance, Paradi et.al* have chosen the following inputs and outputs.

Inputs:

(1) Managers
(2) Account Managers
(3) Assistants
(4) Secretaries
(5) Cash Managers
(6) IT Expense
(7) Rent
(8) Other Non-interest expense.

Outputs:

(1) Deposits
(2) Loans

(3) Operating services (Fee income)

(4) Account maintenance.

Based on Value added models, Camanho et al.* made a study of cost efficiency of commercial banks. In their endeavor they made use of DEA, whose inputs and outputs are as follows:

Production approach

Inputs:

(1) Number of branch and Account Managers

(2) Number of administrative and commercial staff

(3) Number of tellers

(4) Operational cost (excluding staff costs).

Outputs:

(1) Total value deposits

(2) Total value loans

(3) Total value of off balance sheet business

(4) Number of general service transactions.

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Value added approach:

Inputs:

(1) Non-interest costs

(2) Interest costs from deposits

(3) Interest costs from loans.

Outputs:

(1) Total value deposits

(2) Total value loans

(3) Total value of off balance sheet business

(4) Number of general service transactions.

To compare technical efficiencies of Malaysian commercial banks, Nasser Katib and Mathews* proposed suitable DEA models. Their inputs and outputs were as follows:

Inputs:

(1) Number of employees

(2) Net fixed assets after accounting for depreciation.

Outputs:

(1) Total loans and advances

(2) Total deposits

(3) Investments.

In measuring efficiency of commercial banks, there is no general consensus in the choice of input and output variables.

An observation of asset and liability statement of commercial bank induces us to choose a large number of inputs and outputs. But such a choice leads to the dimensionality problem in data envelopment analysis. If the production possibility set is convex, allows for inefficiency and satisfies minimum extrapolation, we have,

\[ \text{GR}(u, x) = \left\{ (u, x) : \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j u_j \geq u, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\} \]

where \( x_j \in R^1_+ \), \( u_i \in R^1_+ \), \( j=1,2,\ldots \ldots \ n \).

are the input and output vectors of jth commercial bank. There are n banks in competition and GR stands for Graph of the production possibilities.

2.4. Sensitivity of DEA results:

If an additional input is augmented the production possibility set is more constrained. We can write the production possibility set in a different way as follows*:

\[
\text{GR}(u,x) = \left\{ (u,x): \sum_{j=1}^{s} \lambda_j x_{jj} \leq x_j, \quad i = 1,2,\ldots,k, \quad \sum_{j=1}^{s} \lambda_j u_{r} \geq u_r, \quad r = 1,2,\ldots,s, \quad \sum_{j=1}^{s} \lambda_j = 1 \right\}
\]

\[
\text{GR}(u,x^t) = \left\{ (u,x^t): \sum_{j=1}^{s} \lambda_j x_{jj} \leq x_j, \quad i = 1,2,\ldots,k+1, \quad \sum_{j=1}^{s} \lambda_j u_{r} \geq u_r, \quad r = 1,2,\ldots,s, \quad \sum_{j=1}^{s} \lambda_j = 1 \right\}
\]

\[
\text{GR}(u,x^t) \supseteq \text{GR}(u,x)
\]

If input efficiency is to be calculated we solve,

\[
F_1(x_o,u_o) = \text{Min } \lambda
\]

subject to

\[
\sum_{j=1}^{s} \lambda_j x_{jj} \leq \lambda x_{oo}, \quad i = 1,2,\ldots,k.
\]

\[
\sum_{j=1}^{s} \lambda_j u_{r} \geq u_{r0}, \quad r = 1,2,\ldots,s, \quad \sum_{j=1}^{s} \lambda_j = 1, \quad \lambda_j \geq 0.
\]

Thus, introduction of an extra input inflates the input technical efficiencies of the decision making units. Consequently, more number of commercial banks emerge with 100% efficiency score.

If output efficiency evaluation is desirable, we solve,
\[ F_0(u_o, x_o) = \text{Max } \theta \]

subject to
\[ \sum_{j=1}^{\infty} \lambda_j x_y \leq \lambda x_{r0}, i = 1,2,\ldots,k. \]
\[ \sum_{j=1}^{s} \lambda_j u_o \geq u_{r0}, r = 1,2,\ldots,s. \]  
\hspace{1cm} \text{(2.4.2)}
\[ \sum_{j=1}^{s} \lambda_j = 1. \]
\[ F_0(u_o, x_o) \geq F_0(u_o, x_o^1) \]  
\hspace{1cm} \text{(2.4.3)}

Even if the added input is irrelevant, the output technical efficiencies are inflated and more DMUs emerge with 100\% efficiency score.

Similarly, augmenting an additional output also inflates the input and output technical efficiencies of decision making units.

If an additional commercial bank is augmented to the current decision making units, the input efficiency problem may be expressed as follows:
\[ F_i(x_o, u_o)^{\text{ext}} = \text{Min } \lambda \]

subject to
\[ \sum_{j=1}^{\infty} \lambda_j x_y \leq \lambda x_{r0}, i = 1,2,\ldots,k. \]
\[ \sum_{j=1}^{s} \lambda_j u_o \geq u_{r0}, r = 1,2,\ldots,s. \]  
\hspace{1cm} \text{(2.4.4)}
\[ \sum_{j=1}^{s} \lambda_j = 1. \]
\[ F_i(x_o, u_o)^{\text{ext}} \leq F_i(x_o^1, u_o). \]
Augmentation of additional decision making unit under scores input technical efficiency fewer number of DMUs emerges to be efficient.

An efficient unit of a DEA model remains to be efficient with additional inputs and or outputs. For a fixed sample size adding more variables not only inflates DEA efficiency scores but it also potentially conceals the actual magnitude of inefficiency. Some times selection of suitable variables can be justified on theoretical basis.

With the sensitivity analysis three technical aspects can be considered, one that examines the overall relationship, the second deals with variations of individual scores and the third an assessment of appropriate efficient peers.

2.5. Efficiency Model Selection:

1) If two DEA models are considered, for each model we obtain efficiency scores. Instead of the magnitudes of the efficiency scores we consider their ranks. If the results are robust or insensitive to the model specification, the spearman’s rank correlation coefficient shall be statistically significant. More generally, if there are k specifications of DEA models, the null hypothesis is that the probability distributions of efficiency rankings are the same across s specifications. We can use the Friedman test statistic (non-parametric) that approximates a Chi-square distribution with s-1 degrees of freedom. If the null hypothesis is not rejected then the results are robust.
(2) We focus our attention on the magnitudes of efficiency scores. If s models are postulated for each bank we obtain s efficiency scores. If the standard deviation or range of the efficiency scores is relatively small then the DEA results are robust. Units receiving relatively large variation in efficiency scores across model specifications require further examination.

(3) Identification of appropriate peers is essential for an inefficient commercial bank. The inefficient bank imitates the management style of the peer banks which are best practice performers.

(4) Policy makers need to know whether inefficiencies exist across units in the organization, before appropriate policies can be designed and implemented to assist inefficient commercial banks. Sensitivity tests alone provide insights whether DEA results are robust across model specifications.

2.6. DEA-Assurance Region:

Data envelopment analysis (DEA) is a technique devised by Charnes, Cooper and Rhodes* to measure efficiency of decision making units, efficiency expressed in a ratio form. Numerator and denominator of efficiency ratio are

virtual output to virtual input. DEA handles multiple inputs and multiple outputs with comfortable ease. The outputs produced by decision making unit are combined, expressed as a weighted sum. The inputs combined in production are also expressed as a weighted sum. The weighted sums of inputs and outputs are called virtual output and virtual input respectively. Virtual output per unit of virtual input foregone is maximized, constraining such ratios of all DMUs, including the DMU whose efficiency evaluation is currently being under taken, not to exceed unity, the weights so derived are called input and output multipliers. The CCR optimization problem is, therefore, a fractional programming problem. However, applying the Charness Cooper transformation the fractional programming problem can be transformed into a linear programming problem.

The dual of the multiplier problem is the envelopment problem which is always feasible. Consequently the multiplier problem is also feasible and the extremities of the objective functions are equal for optimal solutions of primal and dual problems. Most of the applications of DEA involve economic variables. Often economic data are subjected to returns to scale. Returns to scale of a decision making unit may be increasing or decreasing or constant. Such DMU whose returns to scale are constant is scale efficient DMU. To achieve scale efficiency the scale of operation of increasing returns to scale DMU shall expand and that of decreasing returns to scale DMU shall contract.
Merger of banks is recommended if one of the qualifications of them is the presence of increasing returns to scale.

In input oriented DEA problem we seek input reduction and in output oriented DEA problem further output augmentation.

If a DMU admits constant returns to scale in an input orientation problem, then it admits the same returns to scale (RTS) in output orientation. The output technical efficiency is reciprocal of input technical efficiency. If returns to scale are increasing in input orientation, there is no need that returns to scale are increasing in output orientation.

![Figure (2.6.1)](image)

Returns to scale are surface property of production possibility set. The ray that emanates from origin is constant returns to scale frontier. The line
segments AB and BC constitute the frontier that admits variable returns to scale. The DMU represented by P, is technically inefficient. Input technical efficiency requires horizontal projection onto the frontier at D where returns to scale are increasing. Output technical efficiency requires vertical projection onto the piecewise frontier. At E returns to scale are decreasing.

If a multiplier problem is solved it is possible that some of the multiplier weights emerge with zero values*.

where \( u_{i.o} = r^{th} output \) produced by the DMU whose efficiency is under evaluation. \( v_i \) can be interpreted as change in efficiency in response to unit change in \( r^{th} output \). In addition, if \( v_i = 0 \), nothing is contributed by the \( r^{th} output \) to the input technical efficiency. Max \( Z \).

Objective function: \( W = \sum_{i=1}^{l} \mu_i x_{i.o} \)

\[
\text{Min } W = \sum_{i=1}^{l} \mu_i x_{i.o}
\]

\( \mu_i \) measures the change in output technical efficiency in response to unit change in the \( i^{th} output \).

* Objective function: \( Z = \sum_{i=1}^{l} v_i u_{i.o} \)

\[
\text{Max } Z = \sum_{i=1}^{l} v_i u_{i.o}
\]
Zero multiplier weights imply that the respective outputs (inputs) have no role to play to determine input (output) technical efficiency which is very unlikely. To avoid zero multiplier weights the method of assurance region* is implemented. Consider the following weight restrictions.

\[ L_{1,2} \leq \frac{\mu_i}{\mu_1} \leq U_{1,2} \quad \text{------------------------ (2.6.1)} \]

First input weight is numereire

\[ \mu_1 L_{1,2} \leq \mu_i \leq \mu_1 U_{1,2} \]

We consider all the input and output multiplier weights with \( \mu_i \) and \( U_i \) as numereire.

\[ \mu_i L_{1,1} \leq \mu_i \leq \mu_i U_{1,1}, \quad i=1,2,\ldots,k \]

\[ v_j L_{1,1} \leq v_j \leq v_j U_{1,1}, \quad r=1,2,\ldots,s, \quad J \quad \text{------------------------ (2.6.2)} \]

The assurance region constraints are augmented to the multiplier constraints. The optimal solution of the AR (Assurance Region) efficiency problem yields non-zero multiplier weights. For \( j^{th} \) decision making unit, its virtual input and output may be expressed as.

\[ X_j = \mu_1 x_{1j} + \mu_2 x_{2j} + \cdots + \mu_n x_{nj} \]
\[ U_j = v_1 u_{1j} + v_2 u_{2j} + \cdots + v_n u_{nj} \]
\[ j = 1, 2, \ldots, n. \]

The multiplier \( \mu \) may be interpreted as unit cost of \( i^{th} \) input and \( \mu, x_s \) is the value of \( i^{th} \) input expended by \( j^{th} \) DMU. Thus, \( X_j \) is the shadow cost of \( j^{th} \) DMU. In a similar way we can interpret \( U_j \) as the shadow revenue of \( j^{th} \) decision making unit.

\[ \frac{\partial X_i}{\partial x_s} = \mu, \quad i = 1, 2, \ldots, k \]
\[ \frac{\partial X_i}{\partial x_{ij}} = \mu_i \]
\[ \frac{\partial X_i}{\partial x_s} \bigg|_{\partial x_{ij}} = \frac{\partial X_i}{\partial x_{ij}} = \frac{\mu_i}{\mu}, \quad i = 1, 2, \ldots, k. \]

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A producer attains equilibrium, in perfect competition if the ratio of marginal products is equal to the ratio of the corresponding input prices. If input prices are known we can use them to find assurance region weights.

Let \( p_x \) and \( p_y \) be the unit prices of \( i^{th} \) and \( i^{th} \) inputs of \( j^{th} \) decision making unit.

\[ \text{Define } L_{i,j} = \min_{p_x / p_{y_i}} \frac{p_x}{p_y}, \quad \text{------------------------------- (2.6.5)} \]
\[ U_{i,j} = \max_{p_x / p_{y_i}} \frac{p_x}{p_y}, \quad \text{------------------------------- (2.6.6)} \]
Clearly,  \( L_{ij} \leq U_{ij} \)  \hspace{1cm} (2.6.7)

If the output prices are known, similar bounds can be developed for ratios of output prices.

To evaluate input or output technical efficiency envelopment problems can be formulated and solved. Optimal solutions are sought in intensity parameter space. If \( k \) inputs are combined to produce \( s \) outputs, in envelopment problem we find \( k+s+1 \) constraints.

\[ \lambda_j, \quad (j=1,2,\ldots,n) \]

are the intensity parameters and \( \sum_{j \neq 0} \lambda_j = 1 \), is the convexity constraint on the intensity parameters. If DMU\(_{j0}\) is extremely efficient,

\[
\lambda_j = \begin{cases} 
1 & \text{if } j = j_0 \\
0 & \text{if } j \neq j_0
\end{cases}
\]

If DMU\(_{j0}\) is inefficient,

\[
\lambda_j = \begin{cases} 
0 & \text{if } j \neq j_0 \\
\neq 0 & \text{for one or more } j = j_0
\end{cases}
\]

For the inefficient DMU\(_{j0}\) the DMUs for which \( \lambda_j \neq 0 \) are peers and each one of these peers is an extremely efficient decision making unit. The inefficient DMU is similar to the peer DMUs in respect of some characteristics.
the inefficient DMU shall try to resolve its inefficiency by making suitable adjustments in relation to the extremely efficient peer DMUs.

2.7. Discriminatory Power of DEA-Super Efficiency:

As more input and/or output variables are augmented to the DEA problem without increasing the number of DMUs, more DMUs emerge to be efficient and DEA loses its discriminatory power.

The extremely efficient decision making units consist super efficient DMUs. If a DMU is input super efficient its efficiency score exceeds 100 percent. On the other hand, if a DMU is output super efficient, its efficiency score will be less than 100 percent.

Figure (2.7.1)
AB and BC are two segments of a frontier production function. If DMU_B is removed from the reference technology, the new frontier consists AC as one of its linear segments. The producer who operates at B employs input $x_B$ and produces output $u_B$. The input super efficiency problem projects $x_B$ onto the super efficiency frontier, so that the input requirement to produce $u_B$ is $\lambda x_B$, where $\lambda > 1$. To produce output $u_B$ joined by DMU_A and DMU_C the input requirement is more than what DMU_B uses.

The input saved by DMU_B is,

$$(\lambda - 1)x_B,$$

therefore, it is super efficient*.

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The output produced by DMU_B is vertically projected onto the new frontier and the super efficiency score of DMU_B is $\theta$ ($<1$). Employing the input $x_a$, that is currently employed by DMU_B, the DMUs A and C together can produced $\alpha u_a$ ($< u_a$). Thus, the additional output produced by DMU_B is,

$$(1-\theta)u_a$$
due to super efficiency.

But, super efficiency DEA problems some times are infeasible, which occur if referent DMUs do not exit.