Chapter 1

INTRODUCTION
1.0. General Introduction:

Many production processes produce not only desirable, but undesirable outputs. For example, pollutants produced by a cement factory are undesirable outputs which pollute earth’s atmosphere. In Indian Banking ‘non-performing assets’ is a menace to the progress of any Bank. Due to the domination of NPA several banks liquidated, merged with anchor banks.

The proposed study analyses the performance of Indian banking sector, on the premise that all banks are not equally efficient, entertain non-performing assets.

There are approaches in literature aimed at measuring ecological efficiency with an implicit assumption that the production units are scale efficient.

1.1. Efficiency Measurement:

To measure efficiency of production, under constant returns to scale CCR (1978) formulated a multiplier problem.

\[
\text{Max } Z = \frac{\sum_{i=1}^{m} \mu_i u_{x_i}}{\sum_{i=1}^{n} \nu_i x_{r_i}} = \frac{U_o}{X_o}
\]

\[
\sum_{j=1}^{n} \frac{\mu_j \mu_i}{v_i x_j} \leq 1, \quad j = 1, 2, \ldots, n. \quad \text{(1.1.1)}
\]

\[\mu_i, v_i \geq 0\]

Problem (1.1.1) is a fractional programming problem. Using Charnes and Cooper transformation, it can be expressed as a linear programming problem.

Max \[\sum_{i=1}^{n} \mu_i \mu_{i0}\]

subject to \[\sum_{i=1}^{n} \mu_i \mu_{i0} - \sum_{i=1}^{n} v_i x_i \leq 0\] \quad \text{------------------- (1.1.2)}

\[\sum_{i=1}^{n} v_i x_{i0} = 1\]

\[\mu_i, v_i \geq 0\]

Problem (1.1.2) has a dual, called the envelopment problem*:

Min \[\lambda\]

subject to \[\sum_{i=1}^{n} \lambda_i x_i \geq \lambda x_{i0}, \quad i = 1, 2, \ldots, m\]

\[\sum_{i=1}^{n} \lambda_i \mu_i \geq \mu_{i0}, \quad r = 1, 2, \ldots, s\]

\[\lambda_i \geq 0\] \quad \text{------------------- (1.1.3)}

If one of these problems is feasible, then the other also is feasible. Problem (1.1.3) is always feasible.

\[ \lambda_i = 0, \quad \forall j \neq 0, \lambda_j = 1, \lambda = 1 \] is a feasible solution to the envelopment problem, which implies that problem (1.1.3) is also feasible.

From the fundamental theorem of duality the two extremities of the objective functions are equal.

\[ \text{Max} \sum \mu_i u_{i,0} = \text{Min} \lambda \]

1.2. Models for Measuring Eco-Efficiency:

The following models** were explored to measure eco-efficiency of decision making units (DMUs).

MODEL –A:

\[ \delta_A = \text{Max} \frac{\sum \mu_i u_{i,0} - \sum \mu_i u_{i,0}}{\sum v_i x_{i,0}} \]

* \( \lambda_i = 0, \quad \forall j \neq 0, \lambda_j = 1, \lambda = 1 \) is a feasible solution of (1.1.3). Therefore, the solution space for either of problem is non-empty.

subject to \[
\sum_{i=1}^{t} \mu_{i} u_{i} - \sum_{i=t+1}^{s} \mu_{i} u_{i} \leq 1 \]
\[
\sum_{i=1}^{t} v_{i} x_{i} \]
\[
v_{i}, \mu_{i} \geq 0
\]

Model-A presumes that there are s outputs altogether produced by the commercial banks, of which k outputs are desirable and (s-k) outputs are undesirable. That each of these outputs is produced employing m inputs and n commercial banks are competing.

The contributions to the efficiency score by good outputs are measured by the multipliers, \( \mu_{r} (\geq 0) \) and bad outputs by \( -\mu_{r} (\geq 0) \). The constraints of (1.2.1), although appear to be in fractional form, they can be easily transformed into linear form.

By Charnes, Cooper transformation the objective function can also be expressed in linear form, leading to the following linear programming problem:

\[
\delta = \max \sum_{i=1}^{t} \mu_{i} u_{i} - \sum_{i=t+1}^{s} \mu_{i} u_{i}
\]

subject to \[
\sum_{i=1}^{t} \mu_{i} u_{i} - \sum_{i=t+1}^{s} \mu_{i} u_{i} + \sum_{i=1}^{t} v_{i} x_{i} \leq 0 \]
\[
\sum_{i=1}^{t} v_{i} x_{i} = 1
\]
\[
\mu_{i}, v_{i} \geq 0
\]

\( \mu_{r} \) (\( r = 1, 2, \ldots, k \)) measures the marginal contribution of \( r^{th} \) good output to the total efficiency score while this contribution is non-negative. the similar
contribution of \( r^{\#} \) \((r=k+1,\ldots,s)\) output is negative. The dual of (1.2.2) is, an
envelopment problem, expressed as,

\[
\theta_4 = \text{Min } \lambda
\]

subject to

\[
\sum_{j=1}^{n} \lambda_j x_{q} + s^+_r = \lambda x_{r_0}, \quad i = 1,2,\ldots,m.
\]

\[
\sum_{i=1}^{n} \lambda_i u_{r} - s^-_r = u_{r_0}, \quad r = 1,2,\ldots,k.
\]

\[
\sum_{i=1}^{n} \lambda_i u_{r} + s_r = u_{r_0}, \quad r = k+1,\ldots,s. \quad \text{----------- (1.2.3)}
\]

\( \lambda_j \geq 0, s^+_r, s^-_r, s_r \geq 0 \)

In the multiplier problem (1.2.2), it is conceived that the shadow prices
(multiplier weights) of undesirable outputs are non-positive.

Undesirable outputs in most of the cases are not freely disposed off. For example, in the case of non-performing assets (NPA), interests on
loans are unsteadily recovered, some times the bank waves interest fully or
partially. If a bank now and then receives interest on NP assets, or a lumpsome
amount by selling the assets of default DMU winning a legal battle, if there is
one, the NPA also generates income. Thus, the shadow prices of them may also
be viewed as non-negative. Thus, at least a fraction of NP assets are performing
assets.

\[
\mu_r u_{r_0}, \quad r = k+1,\ldots,s, \quad \text{is assumed to be the contribution of } r^{\#} \text{ non-}
\]

performing assets to the efficiency score. To control the weights we normalize
the inverses of undesirable outputs.
\[ \tilde{\sum} u_r \] is the \( r \)th undesirable output of DMU\(_j\)

\[ r = k + 1, \quad \text{s.t.} \]

Define \( \tilde{u}_{r0} \)

\[ \tilde{\sum} u_r \]

where the term inside the parenthesis represents the fraction of bad output \( r \) in the total value of the same undesirable output.

Thus, the multiplier problem of Charnes, Cooper and Rhodes may be stated as follows:

**MODEL-B:**

\[ \delta_u = \text{Max} \left| \tilde{\sum} \mu_r u_{r0} + \tilde{\sum} \mu_r \tilde{u}_{r0} \right| \]

subject to \[ \tilde{\sum} \mu_r u_{r0} + \tilde{\sum} \mu_r \tilde{u}_{r0} - \tilde{\sum} v_j x_{j0} \leq 0, \quad j = 1, \ldots, n \quad \text{----- (1.2.4)} \]

\[ \tilde{\sum} v_j x_{j0} = 1. \]

\[ \mu_r \geq 0, v_j \geq 0. \]

The dual of (1.2.4) may now be expressed as.
\[ \theta_s = \text{Min} \lambda \]

subject to
\[ \sum_{i=1}^{s} \lambda_i x_{i}^* - s_i^* = \lambda x_{i0}, i = 1,2,\ldots,m. \]
\[ \sum_{r=1}^{k} \lambda_r \mu_r + s_r^* = u_{r0}, r = 1,2,\ldots,k. \]
\[ \sum_{r=1}^{s} \lambda_r \bar{u}_r + s_r^* = \bar{u}_{r0}, r = k+1,\ldots,s. \]

\[ \lambda_i \geq 0. \]

The multiplier weights that emerge from the optimal solution of problem (1.2.4) may be interpreted as shadow prices, for example, in (1.2.4), the objective function,
\[ \sum \mu_i u_{i0}, \]  
is proposed to be maximized.

The general term may be interpreted as ‘shadow’ revenue of \( r^{th} \) output of the DMU \( 0 \). If \( u_{r0} \) is given the quantity interpretation, then \( \mu_r \) is the shadow price of \( r^{th} \) output.

**MODEL-C:**

The undesirable outputs may be viewed as inputs. since in input approach the producer seeks the possibility of further reduction in inputs. It is desirable to reduce undesirable outputs, like what one does with the inputs.

The CCR multiplier problem may now be stated as follows:

\[ \text{Max} \sum \mu_i u_{i0} \]
subject to \[ \sum_{i=1}^{r} v_{i}x_{i0} + \sum_{r+1}^{g} \mu_{p}u_{p} = 1 \] \[ \sum_{i=1}^{r} \mu_{a}u_{a} - \sum_{i=r+1}^{g} \mu_{p}u_{p} - \sum_{i=1}^{m} v_{i}x_{i} \leq 0, j = 1,2,\ldots,n \]

\[ v_{i} \mu_{a} > 0. \]

The dual problem of (1.2.6) is,

\[ \hat{\alpha} = \text{Min } \hat{\lambda} \]

subject to \[ \sum_{i=1}^{r} \hat{\lambda}_{a}x_{i0} - \hat{\lambda}_{a}x_{i}, i = 1,2,\ldots,m \]

\[ \sum_{i=1}^{r} \hat{\lambda}_{a}u_{a} - \hat{\lambda}_{a}u_{a}, r = 1,2,\ldots,k \]

\[ \sum_{i=1}^{r} \hat{\lambda}_{a}u_{a} + \hat{\epsilon}_{a} = u_{m}, r = k + 1,\ldots,s. \]

\[ \hat{\lambda} \geq 0. \]

We can estimate eco-efficiency, controlling input variation and assuming that good outputs are freely disposable.

The output oriented multiplier problem as proposed by Charnes, Cooper and Rhodes is as follows:
\[
\text{Min } h = \frac{\sum_{s} v_{s} x_{s0}}{\sum_{s} \mu_{s} u_{s0}}
\]

subject to \( \frac{\sum_{s} v_{s} x_{s}}{\sum_{s} \mu_{s} u_{s}} \geq 1 \) \hspace{1cm} \text{(1.2.8)}

Problem (1.2.8) is the reciprocal of problem (1.1.1). By employing Charnes, Cooper transformation the fractional programming equivalent to linear programming problem is,

\[
\text{Min } h = \sum_{s=1}^{m} v_{s} x_{s0}
\]

subject to \( \sum_{s=1}^{m} \mu_{s} u_{s0} = 1 \)

\[
\sum_{s=1}^{m} \mu_{s} u_{s} - \sum_{s=1}^{n} v_{s} x_{s} \geq 0 \hspace{1cm} \text{(1.2.9)}
\]

\[\mu_{s}, v_{s} \geq 0\]

the dual of (1.2.9) is the envelopment problem,

\[
\text{Max } \theta
\]

subject to \( \sum_{s=1}^{m} \lambda_{s} x_{s} \leq \lambda x_{s0}, \quad i = 1, 2, \ldots, m \hspace{1cm} \text{(1.2.10)} \)
\[ \sum_{r=1}^{n} \lambda_r u_{r*} \geq \theta u_{r*} \quad r = 1, 2, \ldots \]
\[ \lambda_r \geq 0 \]

Using problem (1.2.8), a number of eco-efficiency problems can be postulated as follows:

**MODEL-D:**

\[
\text{Min} \quad \frac{\sum_{r=1}^{n} v_r x_{r*}}{\sum_{r=1}^{n} \mu_r u_{r*} - \sum_{r=1}^{n} \mu_r u_{r*}}
\]
subject to \[
\frac{\sum_{r=1}^{n} v_r x_{r*}}{\sum_{r=1}^{n} \mu_r u_{r*} - \sum_{r=1}^{n} \mu_r u_{r*}} \geq 1 \quad (1.2.11)
\]

**MODEL-E:**

\[
\text{Min} \quad \frac{\sum_{r=1}^{n} v_r x_r}{\sum_{r=1}^{n} \mu_r u_{r*} + \sum_{r=1}^{n} \mu_r u_{r*}}
\]
subject to \[
\frac{\sum_{r=1}^{n} v_r x_r}{\sum_{r=1}^{n} \mu_r u_{r*} + \sum_{r=1}^{n} \mu_r u_{r*}} \geq 1 \quad (1.2.12)
\]
\[v_r, \mu_r \geq 0\]
MODEL-F:

\[
\begin{align*}
\text{Min} \quad & \frac{\sum_{i=1}^{v} x_{i0} + \sum_{i=1}^{n} \mu_{i0}}{\sum \mu_{i}} \\
\text{subject to} \quad & \frac{\sum_{i=1}^{v} x_{i} + \sum_{i=1}^{n} \mu_{i}}{\sum \mu_{i}} \geq 1 \quad \text{(1.2.13)} \\
& v, \mu \geq 0
\end{align*}
\]

MODEL-G:

\[
\begin{align*}
\text{Min} \quad & \frac{\sum_{i=1}^{v} x_{i} + \sum_{i=1}^{n} \mu_{i}}{\sum \mu_{i}} \\
\text{subject to} \quad & \sum \mu_{i} - \sum \mu_{i} - \sum_{i=1}^{v} x_{i} \geq 0 \quad \text{(12.14)}.
\end{align*}
\]

These models will be implemented to estimate ecological efficiency of Indian Commercial and Private Banks. The data are drawn from the Reserve Bank of India Bulletins, 2006.
1.3. CHAPTER SCHEME

The contents of the present research study have been presented under the following heads:

CHAPTER - I:  INTRODUCTION
CHAPTER - II:  REVIEW OF LITERATURE
CHAPTER - III: THEORY AND METHODOLOGY
CHAPTER - IV:  EMPIRICAL INVESTIGATION
CHAPTER - V:  SUMMARY AND CONCLUSIONS.