CHAPTER V

HYDEROMAGNETIC UNSTEADY HELE-SHAW FLOW OF A RIVLIN -ERICKSEN FLUID THROUGH POROUS MEDIA

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1. INTRODUCTION

In most of the investigations of elastico-viscous fluids, the flow has been considered slow and parameters characterising the elastic properties of the fluid have been assumed small Infact the increase emergence of non-Newtonian fluids such as the molten plastics, pulps, emulsions, aqueous solutions of polyacrylamid and polyisobutylene, etc., as important raw materials and chemical products in a large variety of industrial processes has stimulated a considerable attention in recent years to the study of non-Newtonian fluids and their related transport processes. General stress-strain relations are expressed by highly complicated non-linear differential equations, to work out solutions for such a class of fluids even for slow flows is not an easy task. The stress-strain velocity relations of classical hydrodynamics and the Rheological behaviour of the non-Newtonian liquids has been studied by Rivlin [8]

In the field of classical theory of fluid mechanics the problem of Hele-Shaw flow has been considered by a good number of investigators. Lee and Fung [7], Buckmaster [2],

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Lamb [6] and Thompson [12] have discussed the steady Hele-Shaw flows of viscous incompressible fluids by assuming the pressure gradient to be constant. Swaminathan [11] and Johri [4] have discussed the unsteady Hele-Shaw flow of viscous and visco-elastic Rivlin-Ericksen incompressible fluid respectively under the time dependent pressure gradient. Gupta et al. [3] have studied the unsteady Hele-Shaw flow of a Newtonian fluid and a visco-elastic fluid through porous media. Recently Singh and Sharma [9] have investigated the unsteady Hele-Shaw flow of a viscous fluid between two parallel porous walls in the presence of magnetic field applied perpendicular to the fluid flow. Kumar and Singh [5] have studied the MHD Hele-Shaw flow of a Maxwell fluid between two parallel walls past a fixed circular cylinder under time depending pressure gradient through porous media. Bikash Chandra Ghosh and P.R.Sengupta [1] have studied the Hele-Shaw flow of visco-elastic Walter's B-type fluid through porous media between two parallel walls past a fixed circular cylinder under time dependent pressure gradient.

In this chapter we consider the unsteady Hele-Shaw flow of an elastico viscous Rivlin-Ericksen conducting fluid between two parallel walls, $z=\pm h$, past a fixed circular cylinder, $x^2+y^2=a^2$, $-h\le z\le h$, under the time dependent pressure gradient through porous medium in the presence of a uniform transverse magnetic field. We have discussed the cases when the pressure gradient is (i) proportional to e^{int}, (ii) 0 for $t<0$ and equal to a constant for $t\geq 0$ and (iii) proportional to e^{-nt}. The expressions for the velocity component u and v of the fluid in x and y-directions are derived in Section 3. The effects of magnetic parameter, visco-elastic parameter, porosity parameter, time, Reynolds number and suction parameter on the above said physical quantities are discussed in Section 4.

2. Rheological equations of state

The constitutive equation for a Rivlin-Ericksen fluid is

$$T = -PI + \Phi_1 d + \Phi_2 b + \Phi_3 d^2$$

where $T = \|T_{ij}\|$, T_{ij} is the stress tensor

$I = \|\delta_{ij}\|$, δ_{ij} is the Kroneckar delta

$d = \|d_{ij}\|$

$d_{ij} = (1/2)\ (w_{i,j} + w_{j,i})$ = deformation rate tensor

$b = \|b_{ij}\|$, $b_{ij} = a_{i,j} + a_{j,i} + 2\ W_{m,i}\ W_{m,j}$ = Viscoelastic parameter

with $\bar{a}_i = \dfrac{\partial w_i}{\partial t} + w_{ij} w_j$ = acceleration vector

The Φ_1, Φ_2, Φ_3 are coefficients of viscosity, visco-elasticity and cross-viscosity respectively and these are in general functions of temperature, material properties and invariants of d, b, d^2. For many liquids acqueous solutions of polyacrylamid and polybutylene ϕ_1, ϕ_2, ϕ_3 may be taken as constant.

3. Formulation and Solution of the problem:

We consider the unsteady Hele-Shaw flow of an elastico-viscous Rivlin-Ericksen conducting fluid between two parallel walls, $z=\pm h$, past a fixed circular cylinder, $x^2 + y^2 = a^2$, $-h \le z \le h$, under the time dependent pressure gradient through porous medium in the presence of a uniform transverse magnetic field. We assume that the fluid is of small electrical conductivity with the magnetic Reynolds number much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field (Sparrow and Cess [10]). In the absence of any input electric field, the equations governing the motion of the conducting Rivlin-Ericksen fluid in the Hele-Shaw cell under the influence of a uniform transverse magnetic field are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\nu + \beta\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2} - \frac{\nu}{K}u - \frac{\sigma\mu_e^2 H_0^2}{\rho}u \qquad (3.1)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \left(\nu + \beta\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\nu}{K}v - \frac{\sigma\mu_e^2 H_0^2}{\rho}v \qquad (3.2)$$

where u and v are the components of the fluid velocity in the x and y directions respectively, t the time, ρ the constant fluid density, p the fluid pressure, ν the coefficient of kinematic viscosity, K permeability of the porous medium, β the kinematic viscoelasticity, σ the electrical conductivity of the fluid, μ_e the magnetic permeability and H_0 is the intensity of the magnetic field introduced in the z-direction.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.3}$$

If $\beta \to 0$ the above equations reduce to the case of incompressible viscous conduting fluid

The boundary conditions are

$$u = 0, \ v = 0 \ \text{at} \ z = \pm h \tag{3.4}$$

We introduce the following non-dimensional quantities

$$u' = \frac{u}{U}, \quad v' = \frac{v}{U}, \quad z' = \frac{z}{h}, \quad p' = \frac{p}{\rho U^2},$$
$$x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad t' = \frac{t\nu}{h^2} \tag{3.5}$$

where u is the characteristic velocity. In view of the equation (3.5), the equations (3.1) to (3.3) reduce to (after dropping superscripts *)

$$\frac{\partial u}{\partial t} = - R_e \frac{\partial p}{\partial x} + \left(1 + s \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial z^2} - a_1 u - Mu \tag{3.6}$$

$$\frac{\partial v}{\partial t} = - R_e \frac{\partial p}{\partial y} + \left(1 + \frac{\partial}{\partial t}\right) \frac{\partial^2 v}{\partial z^2} - a_1 v - Mv \tag{3.7}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.8}$$

where $R_e = \frac{Uh}{\nu}$ Reynold number

$s = \frac{\beta}{h^2}$ Viscoelastic parameter

$a_1 = \frac{h^2}{K}$ Porosity parameter

$M = \frac{\sigma\mu_e^2 H_0^2 h^2}{}$ Magnetic parameter

The non-dimensional boundary conditions are

$$u = 0, \quad v = 0 \quad \text{at} \quad z = \pm 1 \tag{3.9}$$

Using the equation (3.8), the equations (3 6) and (3 7) give

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \quad 0 \tag{3.10}$$

Hence p is a function of x, y, z and t.

$$\text{Let} \quad u = f\,(t,z)\,\frac{\partial \phi}{\partial x} \quad \text{and} \quad v = f\,(t,z)\,\frac{\partial \phi}{\partial y} \tag{3.11}$$

where ϕ is any function of x and y

Substituting (3.11) in (3.8) we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{3.12}$$

From (3.6), (3.7) and (3.11) we obtain

$$P_x = (\phi_x/R_e)\;[f_{zz} + s f_{tzz} - f_t - (a_1 + M)\; f] \tag{3 13}$$

$$P_y = (\phi_y/R_e)\ [f_{zz} + sf_{tzz} - f_t - (a_1 + M)\ f] \qquad (3.14)$$

where $f_{zz} = \dfrac{\partial^2 f}{\partial z^2}$, $f_{tzz} = \dfrac{\partial^3 f}{\partial t \partial z^2}$, $f_t = \dfrac{\partial f}{\partial t}$

$P_x = \dfrac{\partial p}{\partial x}$, $P_y = \dfrac{\partial P}{\partial y}$ and $\phi_x = \dfrac{\partial \phi}{\partial x}$

Integrating (3.13) and (3.14) we obtain

$$P = (\phi/R_e)(f_{zz} + sf_{tzz} - f_t - (a_1 + M)\ f) + \psi(t) \qquad (3.15)$$

where $\psi(t)$ is an arbitrary function of time

Case (i) : Periodic motion

Let the pressure gradient be proportional to e^{int}.

$$\text{Let} \quad f_{zz} + sf_{tzz} - f_t - (a_1 + M)\ f = -AR_e\ e^{int} \qquad (3.16)$$

where A is a given constant

$$\text{Let} \quad f(t, z) = e^{int}\ F(z) \qquad (3.17)$$

Then F satisfies the equation

$$\frac{d^2F}{dz^2} \quad \frac{(in + a_1 + M)}{(1 + isn)}\ F = \frac{-A\ R_e}{(1 + isn)} \qquad (3.18)$$

Now the boundary conditions are

$$F = 0 \quad \text{at} \quad Z = \pm 1 \qquad (3.19)$$

Solving equation (3.18) using (3.19) we obtain

$$F = \frac{AR_e}{(in + a_1 + M)} \quad X$$

$$\frac{[\text{Cos h } (c \cos B) \text{ Cos } (c \sin B) + i \text{ Sin h } (c \cos B) \text{ Sin } (c \sin B) - [\text{Cos h } (cz \cos B) \text{ Cos } (cz \sin B) + i \text{ Sin h } (cz \cos B) \text{ Sin } (cz \sin B)]]}{\text{Cos h } (c \text{ Cos } B) \text{ Cos } (c \text{ Sin } B) + i \text{ Sin h } (c \text{ Cos } B) \text{ Sin } (c \text{ Sin } B)} \qquad (3.20)$$

Where

$$C = \frac{\sqrt{(a_1 + M)^2 + n}}{[((a_1 + M)^2 + sn^2)^2 + n^2 (1 - (a_1 + M) s)^2]^{1/4}}$$

$$\text{Tan } 2B = \frac{n [1 - (a_1 + M) s]}{(a_1 + M + s^2 n^2)} \qquad (3.21)$$

The function $\phi(x,y)$ can be calculated by solving (3.12) subject to the condition

$$u \cos\theta + v \sin\theta = 0 \text{ on } r = a$$

$$\text{or} \qquad \frac{\partial\phi}{\partial r} = 0 \text{ on } \qquad r = a \qquad (3.22)$$

where $x = r\cos\theta$, $y = r\sin\theta$ and $\frac{\partial\phi}{\partial x} \to 1$, $\frac{\partial\phi}{\partial y} \to 0$

as $|x|$, $|y| \to \infty$

$$\text{Hence } \phi(x,y) = [r + (a^2/r)] \cos\theta \qquad (3.23)$$

From (3.11), (3.17), (3.20) and (3.23) we obtain

$$u_{real} = \frac{AR_e}{\{(a_1+M)^2 + n^2\} \quad (a_3^2 + b_3^2)} \times$$

$$[\{(a_1+M) \cos nt + n \sin nt\} (a_3^2 + b_3^2 - a_3a_2 - b_2b_3) - \{(a_1+M) \sin nt - n \cos nt\} (a_2b_3 - a_3b_2)] \times$$

$$\left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2}\right] \qquad (3.24)$$

$$v_{real} = \frac{AR_e}{\{(a_1+M)^2 + n^2\} \quad (a_3^2 + b_3^2)} \times$$

$$[\{(a_1+M) \cos nt + n \sin nt\} \, (a_3^2 + b_3^2 - a_2a_3 -$$

$$b_2b_3) - \{(a_1+M) \sin nt - n \cos nt\} \, (a_2b_3 - a_3b_2)] \times$$

$$\left[- \frac{2a^2xy}{(x^2 + y^2)^2} \right] \tag{3.25}$$

where $a_2 = \cos h \, (cz \cos B) \cos (cz \sin B)$

$b_2 = \sin h \, (cz \cos B) \sin (cz \sin B)$

$a_3 = \cos h \, (c \cos B) \cos (c \sin B)$

$b_3 = \sin h \, (c \cos B) \sin (c \sin B)$

and u_{real}, v_{real} denote the real parts of u, v respectively. When n=0, we obtain

$$u = \frac{AR_e}{(a_1+M)^2 \quad (a_3^2 + b_3^2)} [(a_1+M) \, (a_3^2 + b_3^2 - a_2a_3 - b_2b_3)] \times$$

$$\left[1 - \frac{a^2 \, (x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.26}$$

$$v = \frac{AR_e}{(a_1+M)^2 \quad (a_3^2 + b_3^2)} [(a_1+M) \, (a_3^2 + b_3^2 - a_2a_3 - b_2b_3)] \times$$

$$\left[- \frac{2a^2xy}{(x^2 + y^2)^2} \right] \tag{3.27}$$

which is the solution for steady flow in the Hele-Shaw cell If $M=0$, $a_1=0$, $s=0$ we obtain $c=n^{1/2}$ and $B = \pi/4$

In this case we obtain the velocity components u and v for unsteady flow of viscous incompressible fluid

$$u = \frac{AR_e}{n \quad (a_3^2 + b_3^2)} X$$

$$[(a_3^2 + b_3^2 - a_2a_3 - b_2b_3) \sin nt + (a_2b_3 - a_3b_2)$$

$$\cos nt] \; X \left[1 - \frac{a^2 \; (x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.28}$$

$$v = \frac{AR_e}{n \quad (a_3^2 + b_3^2)} X$$

$$[(a_3^2 + b_3^2 - a_2a_3 - b_2b_3) \sin nt + (a_2b_3 - a_3b_2)$$

$$\cos nt] \; X \left[- \frac{2a^2xy}{(x^2 + y^2)^2}\right] \tag{3.29}$$

These results agree with results obtained by Swaminathan [11]

Case (ii): Impulsive motion

The motion starts from a state of rest. In the equation (3.15) we assume that

$$f_{zz} + sf_{tzz} - f_t - (a_1+M) f = -AR_e H(t) \tag{3.30}$$

where $H(t) = 0$ when $t < 0$ (3.31a)

$= 1$ for $t \geq 0$ (3.31b)

We define the Laplace transform

$$\overline{f} = \int_0^\infty f\, e^{-pt}\, dt \tag{3.32}$$

with the inversion

$$f = \frac{1}{2\pi i} \int_{r-i\infty}^{r+i\infty} \overline{f}\, e^{pt}\, dp \tag{3.33}$$

using the equation (3.32), the equation (3.30) is transformed to

$$\frac{d^2\overline{f}}{dz^2} - \frac{(P + a_1 + M)}{(1 + sp)}\, \overline{f} = - \frac{AR_e}{p(1 + sp)} \tag{3.34}$$

The transformed boundary conditions are

$$\overline{f}\ (P,\ \pm 1) = 0 \tag{3.35}$$

Solving the equation (3.34) by using (3.35) we obtain

$$\overline{f}\ (p, z) = \frac{AR_e}{P\ (P + a_1 + M)} \left[1 - \frac{Cosh\sqrt{\dfrac{P + a_1 + M}{1 + sp}}\, z}{Cosh\sqrt{\dfrac{P + a_1 + M}{1 + sp}}} \right] \tag{3.36}$$

Inverting (3.36) using the inversion formula (3.33) we obtain

$$f\ (t, z) = \frac{AR_e}{(a_1 + M)} \left[1 - \frac{Cosh\sqrt{a_1 + M}\, z}{Cosh\sqrt{a_1 + M}} \right.$$

$$\left. - \sum_{n=1}^{\infty} \frac{16(-1)^n\ (a_1 + M)\ e^{-a_n t}\ CosK_n z}{\Pi\ (2n+1)\ [\Pi^2\ (2n+1)^2 + 4\ (a_1 + M)]} \right] \tag{3.37}$$

where $K_n = (\pi/2)\ (2n+1)$

$$a_n = \frac{\Pi^2\ (2n+1)^2 + 4\ (a_1 + M)}{4 + \Pi^2\ (2n+1)^2\ s}\ ,\ n = 0 \text{ or any integer}$$

using (3.11), (3.23) and (3.37) we obtain

$$u = \frac{AR_e}{(a_1 + M)}\left[1 - \frac{Cosh\sqrt{a_1 + M}\ z}{Cosh\sqrt{a_1 + M}} - \sum_{n=1}^{\infty} \frac{16(-1)^n\ (a_1+M)\ e^{-a_n t}\ CosK_n z}{\Pi\ (2n+1)\ [\Pi^2\ (2n+1)^2 + 4\ (a_1 + M)]}\right] \times \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2}\right] \quad (3.38)$$

$$V = \frac{AR_e}{(a_1 + M)}\left[1 - \frac{Cosh\sqrt{a_1 + M}\ z}{Cosh\sqrt{a_1 + M}} - \sum_{n=1}^{\infty} \frac{16(-1)^n\ (a_1+M)\ e^{-a_n t}\ CosK_n z}{\Pi\ (2n+1)\ [\Pi^2\ (2n+1)^2 + 4\ (a_1 + M)]}\right] \times \left[- \frac{2a^2xy}{(x^2 + y^2)^2}\right] \quad (3.39)$$

The time after which the motion starting from rest becomes steady may be obtained by requiring that the exponential interms in (3.38) and (3 39) be sufficiently small. Thus setting $(\pi^2 t/4s^2) \sim 10$, the exponent of e is $e^{10} = 0.000045$.

Case (iii): Presure gradient is proportional to e^{-nt}

Let $f_{zz} + sf_{tzz} - f_t - (a_1+M)\, f = -AR_e\, e^{-nt}$ (3.40)

where n is a positive integer and A is a given constant

Suppose $f(t,z) = e^{-nt}\, F(z)$ (3.41)

Now the equation (3.40) reduces to

$$\frac{d^2F}{dz^2} + \frac{[n - (a_1+M)]}{(1 - sn)} F = - \frac{AR_e}{(1 - sn)} \tag{3.42}$$

The boundary conditions are

$F = 0$ at $z = \pm 1$ (3.43)

Solving the equation (3.42) by using (3.43), we get

$$F(z) = \frac{AR_e}{[n - (a_1 + M)]} \left[\frac{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}}\, z}{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}}} - 1 \right] \tag{3.44}$$

Using (3.11), (3.23), (3.41) and (3.44) we obtain

$$u = \frac{AR_e}{[n - (a_1 + M)]}\, e^{-nt} \left[\frac{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}}\, z}{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}}} - 1 \right] \times \left[1 - \frac{a^2\,(x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.45}$$

$$V = \frac{AR_e}{[n - (a_1 + M)]} e^{-nt} \left[1 - \frac{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}} z}{Cosh\sqrt{\frac{n - (a_1 + M)}{1 - sn}}} \right] X$$

$$\left[- \frac{2a^2xy}{(x^2 + y^2)^2} \right] \qquad (3.46)$$

Subcase (1). $M=0, a_1 \neq 0, s \neq 0, n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscoelastic fluid are

$$U = \frac{AR_e}{(n - a_1)} e^{-nt} \left[\frac{Cosh\sqrt{\frac{n - a_1}{1 - sn}} z}{Cosh\sqrt{\frac{n - a_1}{1 - sn}}} - 1 \right] X$$

$$\left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3.47)$$

$$V = \frac{AR_e}{(n - a_1)} e^{-nt} \left[1 - \frac{Cosh\sqrt{\frac{n - a_1}{1 - sn}} z}{Cosh\sqrt{\frac{n - a_1}{1 - sn}}} \right] X$$

$$\left[- \frac{2a^2xy}{(x^2 + y^2)^2} \right] \qquad (3.48)$$

Subcase (ii). $M \neq 0$, $a_1 = 0$, $s \neq 0$, $n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscoelastic fluid under the influnce of a uniform transverse magnetic field are

$$u = \frac{AR_e}{(n - M)} e^{-nt} \left[\frac{\cosh\sqrt{\frac{n - M}{1 - sn}}\, z}{\cosh\sqrt{\frac{n - M}{1 - sn}}} - 1 \right] \times$$

$$\left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3.49)$$

$$v = \frac{AR_e}{(n - M)} e^{-nt} \left[1 - \frac{\cosh\sqrt{\frac{n - M}{1 - sn}}\, z}{\cosh\sqrt{\frac{n - M}{1 - sn}}} \right] \times$$

$$\left[\frac{2a^2 xy}{(x^2 + y^2)^2} \right] \qquad (3\ 50)$$

Subcase (iii) $M \neq 0$, $a_1 \neq 0$, $s = 0$, $n \neq 0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible fluid under the influence of a uniform transverse magnetic field are

$$u = \frac{AR_e}{[n-(a_1+M)]} e^{-nt} \left[\frac{Cosh\sqrt{n-(a_1+M)}\ z}{Cosh\sqrt{n-(a_1+M)}} - 1 \right] X$$

$$\left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \quad (3.51)$$

$$v = \frac{AR_e}{[n-(a_1+M)]} e^{-nt} \left[1 - \frac{Cosh\sqrt{n-(a_1+M)}\ z}{Cosh\sqrt{n-(a_1+M)}} \right] X$$

$$\left[\frac{2a^2xy}{(x^2 + y^2)^2} \right] \quad (3.52)$$

Subcase (iv): $M \neq 0$, $a_1 \neq 0$, $s \neq 0$, $n = 0$

The velocity components in the case of steady Hele-Shaw flow of viscoelastic fluid under the influence of a uniform transverse magnetic field are

$$u = \frac{AR_e}{(a_1 + M)} \left[1 - \frac{Cos\sqrt{a_1+M}\ z}{Cos\sqrt{a_1+M}} \right] \left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \quad (3.53)$$

$$v = \frac{AR_e}{(a_1 + M)} \left[\frac{Cos\sqrt{a_1+M}\ z}{Cos\sqrt{a_1+M}} - 1 \right] \left[\frac{2a^2xy}{(x^2 + y^2)^2} \right] \quad (3.54)$$

Subcase (v): $M=0,\ s=0,\ a_1 \neq 0,\ n \neq 0$

The velocity components in the case of unsteady Hele-Shaw flow of viscous incompressible fluid through porous media are

$$u = \frac{AR_e}{(n - a_1)} e^{-nt} \left[\frac{Cosh\sqrt{n - a_1}\ z}{Cosh\sqrt{n - a_1}} - 1 \right] \times \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3\ 55)$$

$$v = \frac{AR_e}{(n - a_1)}\ e^{-nt} \left[1 - \frac{Cosh\sqrt{n - a_1}\ z}{Cosh\sqrt{n - a_1}} \right] \times \left[\frac{2a^2xy}{(x^2 + y^2)^2} \right] \qquad (3\ 56)$$

Subcase (vi): $M=0,\ n=0,\ a_1 \neq 0,\ s \neq 0$

The velocity components in the case of steady Hele-Shaw flow of viscoelastic fluid are

$$u = \frac{AR_e}{a_1} \left[1 - \frac{Cos\sqrt{a_1}\ z}{Cos\sqrt{a_1}} \right] \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3.57)$$

$$v = \frac{AR_e}{a_1} \left[\frac{Cos\sqrt{a_1}\ z}{Cos\sqrt{a_1}} - 1 \right] \left[\frac{2a^2xy}{(x^2 + y^2)^2} \right] \qquad (3.58)$$

Subcase (vii): $M=0$, $a_1=0$, $s\neq0$, $n\neq0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible conducting fluid are

$$u = \frac{AR_e}{n} e^{-nt} \left[\frac{Cosh\sqrt{\frac{n}{1-sn}}\, z}{Cosh\sqrt{\frac{n}{1-sn}}} - 1 \right] \left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \quad (3.59)$$

$$v = \frac{AR_e}{n} e^{-nt} \left[1 - \frac{Cosh\sqrt{\frac{n}{1-sn}}\, z}{Cosh\sqrt{\frac{n}{1-sn}}} \right] \left[\frac{2a^2 xy}{(x^2 + y^2)^2} \right] \quad (3.60)$$

Subcase (viii): $M\neq0$, $a_1=0$, $s=0$, $n\neq0$

The velocity components in the case of unsteady Hele-Shaw flow of viscous incompressible conducting fluid are

$$u = \frac{AR_e}{(n-M)} e^{-nt} \left[\frac{Cosh\sqrt{n-M}\, z}{Cosh\sqrt{n-M}} - 1 \right] \left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \quad (3.61)$$

$$v = \frac{AR_e}{(n-M)} e^{-nt} \left[1 - \frac{Cosh\sqrt{n-M}\, z}{Cosh\sqrt{n-M}} \right] \left[\frac{2a^2 xy}{(x^2 + y^2)^2} \right] \quad (3.62)$$

Subcase (ix): $M\neq0$, $a_1=0$, $n=0$, $s\neq0$

The velocity components in the case of steady Hele-Shaw flow of viscoelastic conducting fluid are

$$u = \frac{AR_e}{M}\left[1 - \frac{Cos\sqrt{M}\, z}{Cos\sqrt{M}}\right] \quad \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.63}$$

$$v = \frac{AR_e}{M}\left[\frac{Cos\sqrt{M}\, z}{Cos\sqrt{M}} - 1\right] \quad \left[\frac{2a^2xy}{(x^2 + y^2)^2}\right] \tag{3.64}$$

Subcase (x): $M \neq 0,\ s=0,\ n=0,\ a_1 \neq 0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible conducting fluid are

$$u = \frac{AR_e}{(a_1 + M)}\left[1 - \frac{Cos\sqrt{a_1+M}\, z}{Cos\sqrt{a_1+M}}\right] \quad \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.65}$$

$$v = \frac{AR_e}{(a_1 + M)}\left[\frac{Cos\sqrt{a_1+M}\, z}{Cos\sqrt{a_1+M}} - 1\right] \quad \left[\frac{2a^2xy}{(x^2 + y^2)^2}\right] \tag{3.66}$$

Subcase (xi): $M \neq 0,\ a_1=0,\ s=0,\ n=0$

The velocity in the case of steady Hele-Shaw flow of viscous incompressible conducting fluid are

$$u = \frac{AR_e}{M}\left[1 - \frac{Cos\sqrt{M}\, z}{Cos\sqrt{M}}\right] \quad \left[1 - \frac{a^2\ (x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.67}$$

$$v = \frac{AR_e}{M}\left[\frac{Cos\sqrt{M}\, z}{Cos\sqrt{M}} - 1\right] \quad \left[\frac{2a^2xy}{(x^2 + y^2)^2}\right] \tag{3.68}$$

Subcase (xii): $a_1 \neq 0$, M=0, s=0, n=0

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible fluid through porous media are

$$u = \frac{AR_e}{a_1}\left[1 - \frac{\cos\sqrt{a_1}\, z}{\cos\sqrt{a_1}}\right]\left[1 - \frac{a^2\,(x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.69}$$

$$v = \frac{AR_e}{a_1}\left[\frac{\cos\sqrt{a_1}\, z}{\cos\sqrt{a_1}} - 1\right]\left[\frac{2a^2xy}{(x^2 + y^2)^2}\right] \tag{3.70}$$

Subcase (xiii): $s \neq 0$, M=0, a_1=0, n=0

The velocity components in the case of steady Hele-Shaw flow of viscoelastic fluid are

$$u = \frac{AR_e\,(1-z^2)}{2}\left[1 - \frac{a^2\,(x^2 - y^2)}{(x^2 + y^2)^2}\right] \tag{3.71}$$

$$v = AR_e\,(z^2 - 1)\left[\frac{a^2xy}{(x^2 + y^2)^2}\right] \tag{3.72}$$

It is interesting to note that these are the velocity components even in the case of steady Hele-Shaw flow of viscous incompressible fluid case (xv).

Subcase (xiv) : $n \neq 0,\ M=0,\ a_1=0,\ s=0$

The velocity components in the case of unsteady Hele Shaw flow of viscous incompressible fluid are

$$u = \frac{AR_e}{n} e^{-nt} \left[\frac{Cos\sqrt{n}\, z}{Cos\sqrt{n}} - 1 \right] \left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3.73)$$

$$v = \frac{AR_e}{n} e^{-nt} \left[1 - \frac{Cos\sqrt{n}\, z}{Cos\sqrt{n}} \right] \left[\frac{2a^2 xy}{(x^2 + y^2)^2} \right] \qquad (3.74)$$

Subcase (xv) : $M=0,\ a_1=0,\ s=0,\ n=0$

The velocity components in the case of steady Hele-Shaw flow of viscous incompressible fluid are

$$u = \frac{AR_e\ (1-z^2)}{2} \left[1 - \frac{a^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right] \qquad (3.75)$$

$$v = AR_e\ (z^2 - 1) \left[\frac{a^2 xy}{(x^2 + y^2)^2} \right] \qquad (3.76)$$

4. Conclusions

In figures (1) & (2) we have drawn the velocity component u against z for different values of magnetic parameter M respectively in cases (i, ii) and case (iii) discussed in Section (3). We have observed that u decreases with the increase in M in cases (i, ii), whereas u increases with the increase in M in case (iii). Further it is noticed that u decreases as z increases. In figures (3) & (4), the velocity component v is drawn against z for different values of magnetic parameter M in cases (i, ii) and case (iii) discussed in section 3 respectively. We have seen that v decreases with the increase in M in cases (i,ii) whereas it increases with the increase in M in case (iii). In figures (5)&(6) the velocity component u is drawn against z for different values of porosity parameter a_1 respectively in cases (i, ii) and case (iii) are discussed in section 3. We have noticed that u decreases with the increase in a_1 in cases (i, ii) whereas u increases with the increase in a_1 in case (iii). In Figures (7) & (8) we have drawn the velocity component v against z for different values of a_1 respectively in cases (i, ii) and case (iii). We have observed that v decreases with the increase in a_1 in cases (i, ii) whereas v increases with increase in a_1 in case (iii). Further, it is noticed that u, v decreases as z increases Figures (9) & (10) are drawn the velocity component u against z for different values of time t respectively in cases (i, ii) and in case (iii) we have observed that u decreases with the increase in t in all the three cases. In Figures (11) & (12) we have drawn the velocity component v against z for different values of time t respectively in cases (i, ii) and in case (iii) are discussed in Section 3. We have noticed that v decreases with the increase in t in cases (i, ii) whereas v increases with the increase in t in case (iii). In figures (13), (15), (14) & (16) we have drawn u, v against z for different values of viscoelastic parameters s respectively in cases (i,ii) and in case (iii). We have seen that u, v decreases with the increase in s in cases (i, ii) whereas u, v increases with the increase in s in case(iii) Figures (17), (19), (21), (23), (18), (20), (22) and (24) are drawn u, v against z for different values of suction parameter A and Reynolds number Re respectively in cases (i, ii) and in case (iii). We have noticed that u, v increases with the increase in A or R_e in all the three cases. Further it is noticed that u, v decreases with the increase in z in all the three cases.

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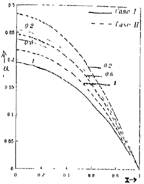

Fig 1 u against z for different M

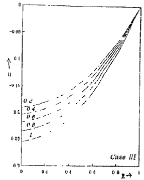

Fig 2 u against z for different M

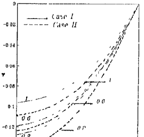

Fig 3 v against z for different M

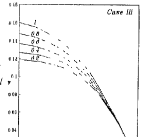

Fig 4 v against z for different M

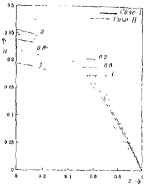

Fig. 5 u against z for different a_1

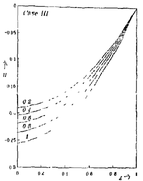

Fig 6 u against z for different a_1

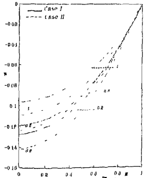

Fig 7 v against z for different a_1

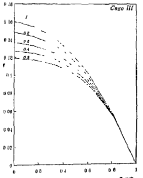

Fig 8 v against z for different a_1

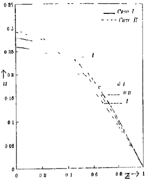

Fig 9 u against z for different t

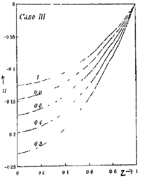

Fig 10 u against z for different t

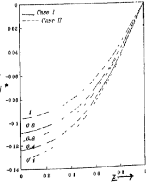

Fig.11 v against z for different t

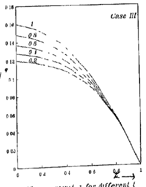

Fig 12 v against z for different t

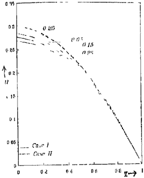

Fig. 13 u against z for different s

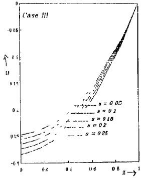

Fig 14 u against z for different s

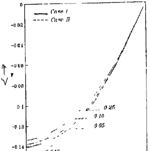

Fig 15 v against z for different s

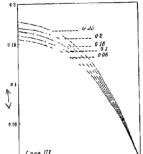

Fig 16 v against z for different s

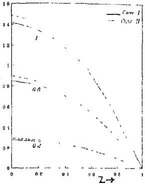

Fig 17 u against z for different A

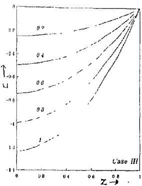

Fig 18 u against z for different A

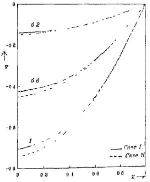

Fig 19 v against z for different A

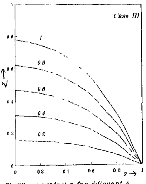

Fig 20 v against z for different A

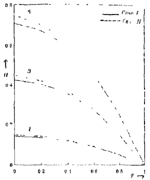

Fig 21 u against z for different Re

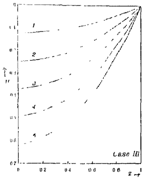

Fig.22 u against z for different Re

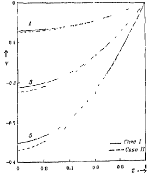

Fig 23 v against z for different Re

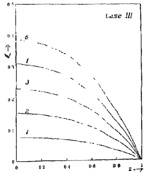

Fig 24 v against z for different Re