INTRODUCTION
a) Fluid Mechanics

Fluid mechanics is basically the study of fluid behaviour when some external force acts over it (Fluids include liquids and gases). The subject can be divided into fluid statics, the study of fluids at rest, and fluid dynamics where the behaviour of fluid while in motion. It is a branch of continuum mechanics - a subject, which models matter without using the information that it is made out of atoms. Fluid mechanics, especially fluid dynamics, is an active field of research with many unsolved or partly solved problems. The subject is highly influencing and when modeled mathematically appears to be more complex at the very first instance. However, sometimes it can be best solved either by making realistic and suitable assumptions or else by using numerical methods, typically using computers. A modern discipline, called Computational Fluid Dynamics (CFD), is devoted to this approach for solving fluid mechanics problems. Fluid mechanics is essentially the branch of Physics which deals with the properties of fluids, namely liquids and gases, and their interaction with externally applied forces.

The study of fluid mechanics dates back at least to the days of ancient Greece, when Archimedes investigated fluid statics and buoyancy. Medieval Persian & Arab natural philosophers, including Abu Rayhan al-Biruni and Al-Khazini, combined the earlier work with dynamics to presage the later development of fluid dynamics. Rapid advancement in fluid mechanics was initiated by Leonardo da Vinci (observation and experiment), Evangelista Torricelli (barometer), Isaac Newton (viscosity) and Blaise Pascal (hydrostatics), and was later continued by Daniel Bernoulli with the introduction of mathematical fluid dynamics in Hydrodynamica (1738). The concept of inviscid flow was further analyzed mathematically by various mathematicians (Leonhard Euler, D.Alembert, Lagrange, Laplace and Poisson) while the nature of viscous flow was explored by a multitude of engineers.
including Poiseuille and Gotthilf Heinrich Ludwig Hagen. A mathematical justification was provided by Claude-Louis Navier and George Gabriel Stokes in the Navier-Stokes Equations, and development of boundary layer was later investigated by Ludwig Prandtl, while various scientists (Osborne Reynolds, Andrey Kolmogorov, Geoffrey Ingram Taylor) advanced the understanding of fluid viscosity and turbulence.

b) To Begin With

Matter is usually divided into two classes, namely - the fluids and the solids. If a portion of matter, under given thermodynamic conditions and in the absence of external forces has a definite shape, then it is termed as a solid, and if it takes the shape of the container - it is called a fluid. In other words, it is said that, in solids the different volume elements have definite relative positions and these elements change their relative positions only when external forces are applied. In fluids, different elements can change their positions even without any external force and therefore, when a fluid is put into a container; the elements re-arrange themselves and shall take the shape of the container.

Such a classification which is based on the geometry of the matter is very simple and useful but is not without shortcomings. Recent emergence of plastics which have dual behavior of solids and liquids has rendered this definition rather loose and also ambiguous. Jellies, pastes, liquid crystals etc., are other few examples which under same thermodynamic conditions sometimes have a definite shape and thus behave like solids, while at other times they do not have a definite shape thus behave like fluids.

Deformation of matter is another criterion for the classification of the matter between solids and fluids. This criterion states that, in solids, when some external force is applied, it is the deformation which is important, i.e., there exists a relationship between the external
force and the deformation caused in the matter, while in fluids, it is the rate of deformation which is important and not just the deformation, i.e., the relationship between the external force and the rate of deformation caused in the matter. Thus, it is implied to note that a portion of matter is a solid if it undergoes deformation when subjected to an external force for a long time and a fluid if it undergoes deformation continuously when subjected to an external force, however small it may be.

Such a definition for solids and fluids though looks complicated, but is quite satisfactory and rigorous. However, even this definition also has certain limitations. For example, paint - it takes the shape of the container, and so is a fluid according to first definition, but when it is applied on a door, it is observed that a thin layer of paint is pasted on the door even before it gets dried up. In all such cases, paint is being constantly subjected to a shearing force due to gravity, but no continuous deformation takes place indefinitely, because otherwise the whole paint would have flowed down the door. Therefore, it is not a fluid as per the later definition. Thus, there are certain limitations as per the definition and the way it is interpreted. Such a situation is not uncommon in physics. However, in normal circumstances any of the above two definitions will generally work.

And at times, fluids are further subdivided into three categories viz: liquids, gases and plasmas. A portion of matter is called a liquid if its expansion, when subjected to some external force or an increase in temperature, is not found to be significant, i.e., the liquids have definite volume which changes slightly when subjected to external forces or temperature differences. Thus, a liquid may not occupy whole of the space of the container.

c) Continuum Hypothesis

It is a well established fact that, matter consists of molecules or atoms which are in a constant state of random motion. Therefore, a most fundamental investigation of the motion
of a fluid will involve the study of all such molecules constituting the fluid. This is a formidable task, especially when the fluid consists of many molecules, which is usually true. For instance air - under normal conditions, has approximately as many as $4.4 \times 10^{20}$ molecules per cubic inch.

Moreover, such an analysis at microscopic level is not necessary. As most of the time, the motion of individual molecules is not of main interest, but are concerned only in investigating the observable phenomena occurring in the matter which consists of large number of molecules i.e., the statistical behavior of the molecules which is a point of deep concern. Furthermore, a fluid which consists of discrete molecules moving randomly, the mathematical tools, such as calculus, become inapplicable and even if applied, increases the complexity of the problem to a greater extent. So as to avoid this difficulty, the fluid is assumed to be continuously distributed in a given space. Thus, a fluid element can be subdivided indefinitely. Such an assumption is called continuum hypothesis. Besides other things, the continuum hypothesis makes the analysis significantly simpler. For the validity of continuum hypothesis, it is necessary to have two criteria. One is that the mean distance between two consecutive collisions of the molecules, called the mean free path, and the other is the average time taken between two consecutive collisions. If the mean free path is much less than the characteristic dimensions of the problem or some reference length of the problem (say less than 1%), then the distance between the molecules can be neglected and the fluid can be taken to be continuously distributed in the given space. But if this mean free path is comparable but less, or of the same order, or greater than the characteristic dimensions of the problem then it is essential to consider the motion of the individual particles. The regimes of fluid mechanics where the motion of the individual particles is important are the situations where large changes occur in a very small distance or the density is very low. The second criterion for the validity of continuum hypothesis is that, the average time taken by
molecules between two consecutive collisions (which in fact is the average time taken by the molecules in travelling mean free path) is much smaller than the characteristic time of observation (say the minimum time, the measuring device can detect), then the variations due to the individual molecules are statistically averaged out and thus, the phenomenon occurring in the fluid can be taken due to the continuous distribution of the matter justifying the continuum hypothesis.

Such a continuum hypothesis has been widely used in fluid dynamics as it gives quite satisfactory results even in situations where one feels that it is likely to be invalid. For instance, in situations where the mean free path is of the same order as that of the dimensions of the problem, quite satisfactory explanation has been obtained by employing the continuum hypothesis. Although, the continuum hypothesis gives a great mathematical simplification, it deprives from the inner structure of the fluid. Thus, for many concepts like viscosity, thermal conductivity and electrical conductivity, it is needless to depend on the empirical formulation. Recently, some attempts have been made to develop these concepts by starting from the discrete picture, but the clarity is still yet to come.

Basically, fluids can be classified broadly into Newtonian and non-Newtonian. All such fluids which satisfy the Newton’s law of viscosity are termed as Newtonian fluids while others are called non-Newtonian fluids. Non-Newtonian fluids could either be plastic, Bingham plastic, pseudo plastic, dilatant, thixotropic, rheopectic and visco-elastic.

d) Newtonian fluid

A Newtonian fluid (named after Isaac Newton) is a fluid whose stress versus rate of strain curve is linear and passes through the origin. The constant of proportionality is known as the viscosity. A simple equation to describe Newtonian fluid behavior is
\[ \tau = \mu \frac{du}{dy} \]  

(1.1)

where \( \tau \) is the shear stress exerted by the fluid, \( \mu \) is the fluid viscosity, a constant of proportionality, \( \frac{du}{dy} \) is the velocity gradient perpendicular to the direction of shear.

In common terms, this means the fluid continues to flow, regardless of the forces acting on it. For example, water is Newtonian, because it continues to exemplify fluid properties no matter how fast it is stirred or mixed. Contrast this with a non-Newtonian fluid, in which stirring can leave a "hole" behind (that gradually fills up over time - this behavior is seen in materials such as pudding, starch in water or to a less rigorous extent sand), or cause the fluid to become thinner, the drop in viscosity causing it to flow more (this is seen in non-drip paints, which brush on easily but become more viscous when on walls).

For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure (and also the chemical composition of the fluid if the fluid is not a pure substance), not on the forces acting upon it. If the fluid is incompressible and viscosity is constant across the fluid, the equation governing the shear stress, in the Cartesian coordinate system, is

\[ \tau_{ij} = \mu(u_{i,j} + u_{j,i}) \]  

(1.2)

with covariant stress tensor \( P \) (also written as \( \sigma \))

\[ P_{ij} = -P\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \]  

(1.3)
where, by the convention of tensor notation, $\tau_{ij}$ is the shear stress on the $i^{th}$ face of a fluid element in the $j^{th}$ direction, $u_i$ is the velocity in the $i^{th}$ direction, $x_j$ is the $j^{th}$ direction coordinate.

Any fluid which does not satisfy the above relation, is termed a non-Newtonian fluid, of which there are several types, including polymer solutions, molten polymers, many solid suspensions and most highly viscous fluids.

e) Non-Newtonian Fluids

The Newtonian hypothesis has worked very well in explaining many physical phenomena in various branches of fluid dynamics. This tempts us to remark that most of fluids at least in ordinary situations behave like Newtonian fluids. But, in the recent years, especially with the emergence of polymers, it has been found that, there are fluids which show a distinct deviation from Newtonian hypothesis. Such fluids are called non-Newtonian fluids.

There has arisen a strong need to develop new theories to explain the behavior of different types of non-Newtonian fluids. Most of these theories developed in the recent years are formulated purely on the theoretical bases. The non-Newtonian fluids are broadly classified into the following three categories.

I. Purely viscous fluids,

II. Visco-plastic fluids and perfectly plastic materials

III. Visco-elastic fluids.
Well before taking up these fluids, a comment on the stress tensor $T_{ij}$ and strain tensor $\varepsilon_{ij}$ is called for. In Newtonian fluids a linear relationship between $T_{ij}$ and $\varepsilon_{ij}$ has been established. Subsequently, this definition is used in the principle that $T_{ij}$ is isotropic, when the fluid is at rest and obtained the constitutive equation for Newtonian fluids. This principle, that the stress tensor is isotropic when the fluid is at rest, is followed for all fluids whether it is a Newtonian fluid or a non-Newtonian fluid. Therefore it can be stated mathematically as

$$T_{ij} = -p\delta_{ij} + p_{ij}$$

(1.4)

where $p$ is the pressure and $p_{ij}$ is zero when the fluid is at rest, in equation (4) denotes covariant differentiation. When a non-Newtonian fluid undergoes a general deformation, and then $p_{ij}$ may not be a deviatoric stress tensor and so its trace may not be zero. Therefore, the pressure $p$ is only a dynamical variable. The tensor $p_{ij}$ is usually called as shear stress tensor. To preserve the isotropicity of $T_{ij}$ when the fluid is at rest, it is customary to use $p_{ij}$ rather than $T_{ij}$ for formulating the constitutive equation of non-Newtonian fluid.

Further, all future investigations in the present analysis will be confined to incompressible non-Newtonian fluids as a result of which there is no difference between strain-rate tensor and deviatory strain-rate tensor.

i) Purely Viscous Fluids

In literature the Newtonian fluids are also generally termed as viscous fluids. The term “viscous” is due to the internal friction between the two adjacent fluid layers. In Newtonian fluids, the strain-rate depends linearly on the applied stress. Moreover, when the fluid is at rest the stress tensor is isotropic and thus results in pressure only. Therefore, it is
quite but natural to question as why not to consider the stress tensor as a general function of the strain-rate tensor which is isotropic when the strain-rate is purely zero. Well, a fluid in which the stress tensor $p_{i,j}$ is a given function of the strain-rate is called as purely viscous fluid, provided in the absence of strain-rate, the stress tensor is zero. Mathematically, this can be stated as

$$p_{i,j} = f\left(e_{i,j}\right), \quad f\left(0\right) = 0$$

(1.5)

The choice of the function $f$ cannot be completely arbitrary as it has to satisfy certain requirements that the resultant equation is a tensorial equation. Different forms of the function $f$ give different types of viscous fluids. Hereunder are some simple and common types of purely viscous fluids. The pseudo-plastic fluids are more commonly found than the dilatant fluids.

From figure 1, it is seen that, in pseudo-plastic fluids, as the strain-rate increases, the shearing stress increases faster than in the Newtonian fluids. On the other hand, in the dilatant fluids as the strain-rate increases, the shearing stress increases at a slower rate than in Newtonian fluids. But at large strain-rates, it is observed that, slope of the curves both representing the pseudo-plastic fluids and the dilatant fluids are nearly the same as for the Newtonian fluids and so they approach the Newtonian fluids. For pseudo-plastic fluids the apparent viscosity decreases with the increase of the strain-rate, while for dilatant fluids it increases with the increase of strain rate. Power law fluid model has given a remarkable success in discussing the behavior of all such polymers.
ii) **Visco plastics and Perfectly Plastic Materials**

One of the most important observations in the viscous fluids is that, if certain shearing stress is applied on a fluid, however small it may be, it causes a continuous deformation in the fluid. But, in many materials like paints, pastes, etc., it is noticed that, if applied shearing stress is less than a certain quantity, the material does not move at all. But, when such shearing stress exceeds a certain value, the material starts moving and the strain-rate of the material depends upon the applied stress. Such materials are called plastics. For example as mentioned earlier, when a door is painted, it is noticed that a thin layer of the paint sticks to the door even before it gets dried up. Such a thin layer is being continuously subjected to a shear force developed by gravitational force. Thus the paint can sustain a finite shearing stress without being in motion, without which, the paint would had flowed down till the thickness of the layer is reduced to zero. But, if the thickness of the paint layer is quiet comparable then the paint flows down like a viscous fluid till a thin layer is left, indicating that there is a critical shearing stress, normally termed as yield stress, which the material can
sustain without being in motion, and if the shearing stress exceeds this limit, the material moves like a viscous fluid.

iii) Visco-elastic Fluids

In case of elastic materials, the stress depends on the strain only, i.e., the stress is certain function of the strain. Thus, if a certain stress is applied on an elastic material, the material undergoes some deformation and when this stress is removed the material retraces to its original position. In all such cases, it can be said that, the elastic materials have memory rather than distinct memory, i.e., it is capable of recognizing its earlier shape. On other hand, in fluids, the stress depends upon the rate of deformation and when the stress is removed the strain rate becomes zero. But, the deformation it has accumulated persists, that is, it forgets its original position. In other words, it can be said that, fluids have no memory of the past history. But, there are some fluids like solution, polymers, which have some elastic properties, besides having fluids properties. Such fluids are called visco-elastic Fluids.

A visco-elastic model can be illustrated by a spring dashpot assembly. If in a spring a certain force is applied to it, it is seen that, the spring extends by a certain amount which is proportional to the external force applied on it. On the other hand, in a dashpot when a certain force is applied on the piston it is noticed that, the piston moves with some speed which is proportional to the force applied to it. It can be well said that, the elastic properties of a material can be represented by a spring and the viscous properties by a dashpot.

Composite fluid and porous layers also find their applications in the porous journal bearings. The surfaces of the joint in human beings, animals and birds are articulate cartilage, a smooth rubbery material that is attached to the solid bone. The surface of the particular cartilage is rough and porous, and hence, can trap the synovial fluid. The composite systems also exist in numerous other engineering applications such as fibrous and
granular insulation where the insulation occupies only part of the space separating the heated and cooled walls, porous insulation of ducts which permit convective interaction between the duct walls and the ambient air, heat transfer from hair covered skin, grain storage and drying, paper drying, and food preparation and storage. In all of the above mentioned composite systems, fluid motion in one layer is not independent of that in the other, the interaction being strongly dictated by the conditions at the interface between the two layers.

The existence of a fluid layer in fluid-saturated porous medium is a common occurrence in both geophysical and engineering environments. The formation of geothermal reservoirs is generally believed to be associated with the volcanic activities or tectonic movements, which cause magmatic intrusions at shallow depths in the earth’s crust. Meteoric water, percolating down through the permeable formation, is heated directly or indirectly by the intruded magma and is then driven by buoyancy to the top of the aquifer. Modeling of such systems requires an understanding of the convective interaction between the fluid layer and the adjacent permeable systems. On the other hand, many geothermal areas, which consist of volcanic debris confined by the walls of non-fragmented ignimbrite, can be modeled as a fluid layer bounded by vertical porous walls subject to fluxes of heat and brine concentration.

Viscous fluid flow over wavy wall had attracted the attention of relatively few researchers although the analysis of such flows finds application in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching ablative surfaces and film vaporization in combustion chambers. The problem by considering the permeability of the bounding surface in the reactors assumes greater significance.
f) Equations of the motion if the bounding surface is porous

In many of the chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem by considering the permeability of the bounding surfaces in the reactors attracted the attention of several investigators.

Flow through porous media has been the subject of considerable research activity in recent years because of its several important applications notably in the flow of oil through porous rock, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactor for economical separation or purification of mixtures and so on.

An important application is in the petroleum industry, where crude oil is tapped from natural underground reservoirs in which oil is entrapped. Since the flow behavior of fluids in petroleum reservoir rock depends, to a large extent, on the properties of the rock, techniques that yield new or additional information on the characteristics of the rock would enhance the performance of the petroleum reservoirs. A related biomechanical application is the flow of fluids in the lungs, blood vessels, arteries and so on, where the fluid is bounded by two layers which are held together by a set of fairly regularly spaced tissues.

Flow in a porous medium is an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous
matrix, and therefore it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure, however, is highly complicated and differs from medium to medium. A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries such as a porous rock, for eg., limestone or sponge. When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual path of an individual fluid particle cannot be followed analytically. In that case, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistance force generated by internal friction between the fluid and the pore structure. This resistance force is characterized by Darcy’s semi-empirical law established by Darcy [1]. The simplest model for flow through a porous medium is the one-dimensional model derived by Darcy [1]. Obtained from empirical evidence, Darcy’s law indicates that, for an incompressible fluid flowing through a channel filled with a fixed, uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. In multidimensional flow, the momentum equations for each velocity component derived using the Forchheimer extended Darcy equation is at least speculative. Later, Knupp and Lage [2] analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Forchheimer extended Darcy unidirectional flow model.

Natural convection flow in porous media has application in a broad spectrum of engineering systems. Some of these include building thermal insulation, direct contact heat exchangers, solar heating systems, packed-bed catalytic reactors, nuclear waste disposal
systems and enhanced recovery of petroleum resources. Most of the studies in natural convection in porous media have been based on Darcy’s account for non-Darcian inertial effects. These effects represent the additional pressure drop across the porous media resulting from interpore mixing for fast flows. In this case, the total pressure drop across the porous medium is a quadratic function of the velocity of the fluid flowing in it. In the presence of a heated boundary, heat transfer to the porous medium can be accounted for by employing the conservation of energy law and Brinkman’s extension of Darcy’s law modified to include the thermal buoyancy effect. The boundary and inertia effects on convective flow and heat transfer were analyzed and discussed by Vafai and Tien [3].

A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien [4]. The local volume averaging technique has been used to establish the governing equations. The numerical solution of the governing equations is used to investigate the mass concentration field inside a porous media close to an impermeable boundary. In conjunction with the numerical solution, a transient mass transfer experiment has been conducted to demonstrate the boundary and inertia effects on mass transfer. This was accomplished by measuring the time and space averaged mass flux through a porous medium. The results clearly indicate the presence of these effects on mass transfer through porous media.

If the bounding surface is porous, then the rate of percolation of the fluid is directly proportional to the cross sectional area of the filter bed and the total force, say the sum of pressure gradient and the gravity force. In the sense of Darcy

\[
q = CA \left[ \frac{P_1 - P_2}{H_1 - H_2} + \rho G \right]
\]  

(1.6)
where \( A \) is the cross sectional area of the filter bed, \( C = \frac{K}{\mu} \) in which \( K \) is the permeability of the material and \( \mu \) is the coefficient of viscosity and \( q \) is the flux of the fluid. A straightforward generalization of the above equation yields

\[
V = -\frac{K}{\mu} [\nabla P + \rho G \eta] \quad (1.7)
\]

where \( V \) is the velocity vector and \( \eta \) is the unit vector along the gravitational force taken in the negative direction. If any other external forces are acting on the system, instead of gravitational force, then

\[
V = -\frac{K}{\mu} [\nabla P - \rho F] \quad (1.8)
\]

In the absence of external forces, \( V = -\frac{K}{\mu} \nabla P \) as a result of which \( \nabla P = -\frac{\mu}{K} V \). In view of the above discussion, the equations of motion in the \( X \), \( Y \) and \( Z \) directions can be restated as:

\[
\rho \frac{DU_1}{DT} = \rho F_x + \frac{\partial S_{1x}}{\partial X} + \frac{\partial S_{1y}}{\partial Y} + \frac{\partial S_{1z}}{\partial Z} - \frac{\mu}{K} U_1
\]

\[
\rho \frac{DU_2}{DT} = \rho F_y + \frac{\partial S_{2x}}{\partial X} + \frac{\partial S_{2y}}{\partial Y} + \frac{\partial S_{2z}}{\partial Z} - \frac{\mu}{K} U_2
\]

\[
\rho \frac{DU_3}{DT} = \rho F_z + \frac{\partial S_{3x}}{\partial X} + \frac{\partial S_{3y}}{\partial Y} + \frac{\partial S_{3z}}{\partial Z} - \frac{\mu}{K} U_3
\]

g) Introduction to heat transfer

Heat transfer is the net change in energy as a result of temperature differences. This energy is transferred in the direction of decreasing temperature until thermal equilibrium (equality of temperatures) is achieved. Heat can be transferred from one place to another by
three methods: conduction in solids, convection in fluids (liquids or gases), and radiation through anything that will allow radiation to pass. Heat transfer in fluids can occur at a faster rate, as large masses of a fluid can be displaced and can mix with other fluid masses of different temperatures.

i) Conduction

It is the flow of heat through solids and liquids by vibration and collision of molecules and free electrons. The molecules of a portion of a system at a higher temperature vibrate faster than the molecules of other regions of the same or of another system at lower temperature. The molecules with a higher movement collide with the molecules less energized and transfer part of their energy to the less energized molecules of the colder regions of the structure.

Metals are considered to be the best thermal conductors; while non-metals are poor thermal conductors. For comparison, the thermal conductivity \((k)\) of the copper is \(401\text{ W/mK}\), while the thermal conductivity \((k)\) of the air is \(0.0263\text{ W/mK}\). The thermal conductivity of the carbon dioxide CO\(_2\) is \(0.01672\text{ W/mK}\), almost the thermal conductivity of an isolator. In general, the conductivity gradient for a given system can be expressed as \(q = -kA(\Delta T/\Delta n)\), where \(\Delta T/\Delta n\) is the temperature gradient in the direction of area \(A\), and \(k\) is the thermal conductivity constant obtained by experimentation in \(\text{W/mK}\).

ii) Radiation

The basic mechanism involved in heat transfer process includes radiation (the transfer of energy in the form of electromagnetic waves) and conduction (the transfer of kinetic energy). Essentially, radiation is the heat transfer by electromagnetic waves or photons. The
A salient feature of this phenomenon is that, it does not need a propagating medium. Both conduction and convection require matter to transfer heat. Radiation is a method of heat transfer that does not rely upon any contact between the heat source and the heated object. Heat can be transmitted through empty space by thermal radiation. Thermal radiation (often called infrared radiation) is a type of electromagnetic radiation (or light). Radiation is a form of energy transport consisting of electromagnetic waves traveling at the speed of light. No mass is exchanged and no medium is required.

Objects emit radiation when high energy electrons in a higher atomic level fall down to lower energy levels. The energy lost is emitted as light or electromagnetic radiation. Energy, which is absorbed by an atom, causes its electrons to “jump” up to higher energy levels. All objects absorb and emit radiation. When the absorption of energy balances the emission of energy, the temperature of an object stays constant. If the absorption of energy is greater than the emission of energy, the temperature of an object rises. On the other hand, if the absorption of energy is less than the emission of energy, the temperature of an object falls.

When an object is placed under the direct sunbeams, in a little while, then the object will be heated. The exchange of heat between the Sun and the object occurs essentially by radiation. The amount of heat transferred by radiation is \( q = e\sigma A (\Delta T)^4 \) where \( q \) is the heat transferred by radiation, \( e \) is the emissivity of the system, \( \sigma \) is termed as Stephan-Boltzmann constant \( (5.6697 \times 10^{-8} \text{W/m}^2\text{K}^4) \), \( A \) is the area involved in the heat transfer by radiation and \( (\Delta T)^4 \) is the difference of temperature between two systems to the fourth or higher power. For a greater thermal capacity, slower is the rate of dissipation of heat.

The atmosphere and the soil do not maintain a load of heat for longer periods than water because they have a specific heat capacity lower than water, hence water absorbs more
heat for increasing its temperature for a determined interval. For equal volumes ($1Kg$, for example), water absorbs more heat than air or soil. The absorbed heat will be transformed into kinetic and potential energy. A body with a high energy density will lose its inner energy slower than a body with a lower energy density.

Water absorbs the incoming solar infrared radiation from sun; due to the frequency of the internal vibration of the water molecules is the same frequency of the waves of the solar infrared radiation. This form of radiative heat transfer is known like resonance absorption.

Human body feels the heat radiated by the Sun and other systems with a higher temperature because it contains $55−75\%$ of water. The radiative energy inciding on skin is absorbed by the molecules of water in the body by resonance absorption. Just then, the infrared radiation absorbed by the body leads to a more intense internal vibration of the water molecules in the body and thereby it gets warmer. However, living beings in general possess thermoregulatory systems that permit the body to eliminate the excess of heat from it, maintaining a quasi-stable internal temperature (it is one of the homeostatic processes of biosystems).

iii) Convection

If there is a temperature difference in a system, heat always moves from higher to lower temperatures. In many heat transfer processes, radiation and convection or conduction work together, although one is often dominant. The method used to transfer heat is usually the one that is the most efficient.

Interest in studying the phenomena of convective heat and mass transfer between an ambient fluid and a body which is immersed in it stems both from fundamental considerations, such as the development of better insights into the nature of the underlying
physical processes which take place, and from practical considerations, such as the fact that, these idealized configurations serve as a launching pad for modeling the analogous transfer processes in more realistic physical systems. Such idealized geometries also provide a test ground for checking the validity of theoretical analysis. Consequently, an immense research effort has been expended in exploring and understanding the convective heat and mass transfer processes between a fluid and submerged objects of various shapes. Among several geometries which have received considerable attention are plates, circular and elliptical cylinders, and spheres, although much information is also available for some other bodies, such as corrugated surfaces or bodies of relatively complicated shapes.

Convection is a movement of liquid or gaseous volumes. When a mass of a fluid heats up by being upon a hot surface, its molecules are carried away and scattered causing that the mass of that fluid becomes less dense. Given that, the volume of the hot fluid becomes less dense, it will be displaced vertically and or horizontally, while the cold but denser volume of fluid sinks (the low-kinetic-energy molecules displace to the molecules with high-kinetic-energy). Through this process, the molecules of the hot fluid continuously transfer heat to the volumes of colder fluid.

In general, the way in which the convection occurs can be given by the relation

\[ q = hA(T_s - T_w) \]

where \( h \) is for convective heat transfer coefficient, \( A \) is the area implied in the heat transfer process, \( T_s \) is for the temperature of the system and \( T_w \) is a reference temperature.

**h) Forced Convection**

The basic question in heat transfer engineering is to determine the fundamental relationship between the rate of heat transfer and the driving temperature difference. In many
saturated porous media interact thermally with one another and solid surfaces that confine them are in them. Perhaps the simplest and most common heat transfer arrangement is the flow parallel to a flat surface that borders the fluid saturated porous medium. A remarkable and significant similarity between local Nusselt number and the Nusselt number for the isothermal flat wall is found to be successful as both are related to Peclet number which is aligned with the direction of the flow.

Most of the work on forced convection in fluid saturated porous media is concerned under steady state conditions. Notable exceptions are the papers on time-dependent forced convection heat transfer from an isothermal cylinder Kimura [5], and from a cylinder with uniform heat flux Kimura [6]. Nakayama and Ebinuma [7] studied the forced convection heat transfer between a suddenly heated plate and a non-Darcy flow that starts initially from rest. These three papers show that, the simplest and perhaps most important forced convection configuration has been overlooked. In such a configuration, the flow through the saturated porous medium is steady, parallel and uniform. The flow is driven by a pressure difference that is applied in the $x$-direction can be either a Darcy flow or a non-Darcy flow in which the quadratic drag (Forchheimer effect) plays a significant role in the overall flow resistance. What distinguishes the present configuration from the one analyzed by Nakayama and Ebinuma [7] is that, the flow remains steady as the embedded plate is suddenly heated or cooled to different temperature.

Vafai and Thiyagaraja [8] have compared the analytical results with numerical results. They found that, the low Prandtl number analytical solution accurately predicates the temperature distribution for a Prandtl number $Pr$ as high as 8, while the high Prandtl number analytical solution is valid for $Pr$ as low as 100 and possibly for somewhat lower values. The combined effects of inertia and boundary friction were considered by Kaviany [9].
Vafai et al. [10] experimentally and numerically investigated the effects of boundary friction and variable porosity. Their experimental bed consisted of glass beds of 5 mm and 8 mm diameter saturated with water. They found good agreement between observation of the average Nusselt number and numerical predictions when the effect of variable porosity was included (but not otherwise). Cheng [11] noted that, since their experiments were conducted in the range \(100 < \text{Re}_a < 900\) thermal dispersion effects should have been important, and in fact they neglected these. He pointed out that, in their numerical work Vafai et al. [10] used a value of thermal conductivity about three times larger than was warranted and by doing so they had fortuitously approximated the effect of transverse thermal dispersion.

Further, experimental work was undertaken by Renken and Poulikakos [12]. They reported details of thermal boundary layer thickness, temperature field and local Nusselt number. Good agreement was found with the numerical results of Vafai et al. [10] with the effects of flow inertia and porosity variation accounted.

Some further details on this extent of this section can be found in the review by Lauriat and Vafai [13]. Vafai and Kim [14] have analyzed flow in a composite medium consisting of fluid layer overlaying a porous substrate which is attached to the surface.

i) External natural convection

Numerical calculation from the full differential equations for convection in an unbounded region is expensive and hence approximate solutions are important. For small values of the Rayleigh number \(Ra\), perturbation methods are appropriate. At large values of \(Ra\), thermal boundary layers are formed and boundary layer theory is the obvious method of investigation. The boundary layer theory arises as a first approximation for large values of Rayleigh number, when expansions are made in terms of the inverse one-half power of
Rayleigh number. At this order, the effects of entrainment from the edge of the boundary layer, the axial heat conduction and the normal pressure gradient are all neglected.

The magnitudes of these effects have been investigated using higher order asymptotic analysis by Cheng and Chang [15], Chang and Cheng [16], Cheng and Hsu [17] and Joshi and Gebhart [18]. It is noticed that, the ordering of the Eigen function terms in the perturbation series was dependent on the wall temperature parameter $\lambda$ (exponent in power law variation). It also observed that, the coefficients of the Eigen functions cannot be determined without a detailed analysis of the leading edge effect. Therefore, they truncated the perturbation series at the term where the leading edge effect first appeared. Further, it is found that, the effect of entrainments from the edge of thermal boundary layer was of second order while those of axial heat conduction and normal pressure gradient were of third order.

For the case of the isothermal vertical plate with $\lambda = 0$, the second order corrections for both the Nusselt number and the vertical velocity are zero and the leading edge effect appears in the third order terms. For other values of $\lambda$, both the second order and third order corrections in the Nusselt number are positive and the leading edge effect appears in the fourth order terms.

The slight increase in the surface heat flux in the higher order theories is mainly due to the fact that, entrainments from the outer flow induce a flow parallel to the heated surface. The higher theory has profound effect on the velocity profiles but has relatively small effect on the temperature distribution and hence on the surface heat flux.

Pop, Cheng and Le [19] have shown that, in the case of uniform wall heat flux, the leading edge effects enter the second and subsequent order problems. They cause an increase of the stream wise vertical velocity near the outer edge of the boundary layer and a
consequent increase in heat transfer rate by an amount comparable with entrainment effects, the combination producing a 10% increase at $Ra_x = 100$ and a greater amount at smaller $Ra_x$.

In all above discussions it has been assumed that, Darcy’s law is applicable and the effects of no slip boundary conditions, inertial terms and thermal dispersion are negligible. In fact, however minute may be their contribution, these effects are important at high Rayleigh numbers. The effects of boundary friction and inertia tend to decrease the heat transfer rate while the reverse effect is noticed in case of thermal dispersion. The case of a non-Newtonian power law fluid has been analyzed by Chen and Chen [20]. The cases of arbitrary wall temperature and arbitrary heat flux have been treated using a Merk series technique by Gorla and Zinalbedini [21] and Gorla and Tornabene [22]. Merkin and Needham [23] have discussed the situation where the wall is of finite height and the boundary layers on each side of the wall merge to form a buoyant wake. Singh et al. [24] have studied the problem when the prescribed wall temperature is oscillating with time about a non-zero mean. Zaturska and Banks [25] have shown that, the boundary layer flow is stable spatially.

**j) Internal natural convection**

If the study restriction is dropped and examined the time scales and evaluation of buoyancy driven flow the equations that govern the conservation mass momentum also satisfy the equations of continuity and Darcy’s principle.

By focusing in the vertical boundary layer that develops along the boundary it is noted that, initially the time dependent thickness of the boundary layer $\delta t$ grows by pure conduction with respect to the region of thickness $\delta$ and height $H$ the energy equation in the system dictates the balance between the side heating effect and the thermal inertia of the saturated porous medium equation.
The growth of the conduction layer gives rise to a horizontal temperature gradient of order $\frac{\partial T}{\partial x} \approx \frac{\Delta T}{\delta}$. This development makes the buoyancy term in momentum balance to be finite.

An interesting feature of the transient flow is that, the vertical velocity scale is independent of time. The vertical flow rate however, $\nu \delta$, grows in time as $t^{0.5}$. As soon as fluid motion is present, the energy equation is ruled by the competition among three different scales

\[
\left( \frac{\sigma \Delta T}{t}, \nu \frac{\Delta T}{H} \right) \sim \frac{\alpha_m \Delta T}{\delta^2}
\]

Inertia Convection Conduction

\[
\begin{pmatrix}
(t^{-1}) \\
(t^0) \\
(t^{-1})
\end{pmatrix}
\]

The time dependent of each scale is shown. Since the lateral conduction effect is always present, the convection scale eventually overtakes inertia on the left hand side of equation (17). The time $t$ when this change over takes place, i.e., when the vertical boundary layer becomes convective, given by

\[
\frac{\sigma \Delta T}{t} \approx \nu \frac{\Delta T}{H}
\]

The behavior of flow and heat transfer changes substantially as flow regime departs from the Darcy’s effect. The effect of quadratic drag on the heat transfer through the most basic configuration was demonstrated by Poulikakos and Bejan [26]. In place of the momentum equation, they used the Forchheimer modification of Darcy’s law.

The important heat transfer conclusion of the scale analysis is the overall Nusselt number scales as $(L/H) Ra_{\infty}^{1/4}$ in the limit in which the effect of inertia dominates. A more
accurate estimate was provided by an analytical solution in which Oseen linearized solutions for the two vertical boundary layers were matched to the same stratified core (Poulikakos and Bejan [26]) equation

\[
Nu = 0.899 \frac{L}{H} Ra \ln \left( \frac{Nu}{Ra} \right). \quad (G << 1) \tag{1.11}
\]

This solution is the Forchheimer region counterpart of Oseen linearized solution as derived by Weber [27] for the Darcy limit.

Several authors using the Brinkman equation have calculated the flow in a laterally heated rectangular container partly filled by clear fluid and partly with a porous medium saturated by such a fluid. In most of these studies the porous medium forms a vertical layer; the interface can be either impermeable to fluid or impermeable. Sathe et al. [28] reported experimental results, for a box divided in two with a vertical impermeable partition bounding the porous medium, which agreed with calculations made by Tong and Subramanian [29]. Sathe and Tong [30] compared these results with those of as reported by Sathe et al. [31] for the same problem with a permeable interface and with results for a cavity completely filled with porous medium and with a partitioned cavity containing solely clear fluid. Heat transfer is reduced by the presence of porous material having the same thermal conductivity as the fluid, and by the presence of a partition. At low Darcy’s number, \( Da(\leq 10^{-4}) \) the first mechanism is more prominent while for high values of \( Da \) the second produces a greater insulating effect. The differences become accentuated at large \( Ra \). Experiments by Sathe and Tong [32] confirmed that, partly filling an enclosure with porous medium may reduce the heat transfer more than totally filling it.

The case of rectangular cavity with a porous medium occupying the lower half, the interface being permeable, was studied numerically by Nishimura et al. [33]. The results
agreed well with previous experiments by those authors. As one would expect, most of the flow and the heat transfer occurs in the fluid region.

**k) Mixed Convection**

Heat transfer in porous medium has been gaining utmost importance due to its wider applicability in general and acceptability in particular in the emerging areas viz; in soil physics, geothermal energy extraction, chemical engineering, oil reservoir modeling and in biological systems. Radiation in heat transfer accounts in high temperature applications viz., plasma physics, nuclear reactions, nuclear waste disposal, liquid metal flows, and magneto hydrodynamic accelerators and in power generation systems. Further, the problem assumes greater significance in studying MHD combined dispersion and dissipation effects in porous medium. Understanding the boundary layer development and convective heat transfer characteristics of mixed convection in porous media are of primary requirement.

**l) Inclined or Vertical plane wall**

This concept is guided exhaustively and reviewed intensively by Lai et al. [34]. Under the treatment of boundary layer flow on heated plane walls inclined at some non zero angle to the horizontal under the steady state conditions the governing equations are

\[
\frac{\partial^2 \psi}{\partial y^2} = \pm \frac{g \beta K}{\nu} \frac{\partial T}{\partial y} \quad (1.12)
\]

\[
\frac{\partial \psi \frac{\partial T}{\partial y}}{\partial y} - \frac{\partial \psi \frac{\partial T}{\partial x}}{\partial x} = \frac{\partial}{\partial y} \left( \alpha_m \frac{\partial T}{\partial y} \right) \quad (1.13)
\]

where \(x\) and \(y\) are the reference coordinate system in the Cartesian form, \(\psi\) indicates the stream function, \(T\) is the temperature, \(K\) is the permeability, \(\beta\) is the thermal expansion coefficient, \(\alpha_m\) is the thermal diffusivity of the porous medium, \(\nu\) is the kinematic viscosity.
Here $\pm g_x$ is the component of $g$ in the positive $x$ direction, i.e. the direction of the stream velocity $U_\infty$ at infinity. The $+$ sign corresponds to the case where the buoyancy force has a component “aiding” the general flow, and the $-$ sign to the “opposing” case. A similarity solution is being sought and allowed for suction/injection at the wall. Hence, the boundary conditions are

$$
at \ y = 0: \quad T = T_\infty \pm Ax^\lambda, \quad \nu = -\frac{\partial \psi}{\partial x} = ax^n$$  \hspace{1cm} (1.14)

$$\text{as } y \to \infty: \quad T = T_\infty, \quad u = \frac{\partial \psi}{\partial y} = U_\infty = Bx^m$$  \hspace{1cm} (1.15)

where $A, \alpha$ and $B$ are constants. The exponent $m$ is related to the angle of inclination $\gamma \pi / 2$ (to the incident free stream velocity) by the relation $\gamma = 2m/(m+1)$. It is noticed that, a similarity solution exists if $\lambda = m$ and $n = (m-1)/2$. The range of possibilities includes the cases

$\lambda = m = 0, \quad n = -1/2$ \quad (Vertical isothermal wall, injection $\propto x^{-1/2}$)

$\lambda = m = 1/3, \quad n = -1/3$ \quad (Wall at 45° inclination, constant heat flux)

$\lambda = m = 1, \quad n = 0$ \quad (Stagnation flow normal to vertical wall, linear temperature variation, uniform injection)

Introducing the similarity variables as

$$\eta = \left(\frac{U_\infty x}{\alpha_m}\right)^{1/2} \frac{y}{x}, \quad f(\eta) = \frac{\psi}{\left(\alpha_m U_\infty x\right)^{1/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}$$  \hspace{1cm} (1.16)

and the wall suction parameter as

$$f_w = -2a/\left(\alpha_m B\right)^{1/2}$$  \hspace{1cm} (1.17)
It follows that

\[ f' = \pm \frac{Ra_x}{Pe_x} \theta \]  

(1.18)

\[ \theta' = -\frac{\lambda + 1}{2} f \theta + \lambda f' \theta \]  

(1.19)

\[ \theta(0) = 1, \ f(0) = f_u, \ \theta(\infty) = 0, \ f'(\infty) = 1 \]  

(1.20)

where

\[ F_0 = \frac{c_x K^{1/2}}{x}, \ \text{Re}_x = \frac{U_x}{v}, \ \text{Pe}_x = \frac{U_x}{\alpha_m} \]

\[ \rho d_p \alpha = \frac{g \beta K x (T_w - T_\infty)}{v \alpha_m}, \ \text{Ra}_d = \frac{g \beta K d_p (T_w - T_\infty)}{v \alpha_m} \]  

(1.21)

A positive \( f_w \) indicates withdrawal of fluid. The case of forced convection corresponds to letting \( Ra_x \to \infty \). The case of natural convection requires a different similarity variable. Lai and Kulacki [35] noticed that, the effect of increasing \( \lambda \) is to raise the Nusselt number slightly.

The effects of flow inertia and thermal dispersion were studied by Lai and Kulacki [36]. In which equation (1.12) is replaced by

\[ \frac{\partial^2 \psi}{\partial y^2} + \frac{\chi}{v} \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2 = \pm \frac{g \beta K}{v} \frac{\partial T}{\partial y} \]  

(1.22)

where \( \chi = C_p K^{1/2} \) and in equation (1.13) \( \alpha_0 \) is replaced by \( \alpha_x \), the sum of a molecular diffusivity \( \alpha_0 \) and a dispersive term \( \alpha' = \rho \alpha d_p \), where \( d_p \) is the mean pore diameter and \( C \) is a constant. When the bounding surface is held vertical, isothermal and under the assumption that, there is no suction at any of the boundaries, the Equations (1.18) – (1.20) can be replaced by:
\[ f^\prime + F_0 \operatorname{Re}_x \left( f^\prime \right)^2 = \pm \frac{Ra_c}{Pe_e} \theta^\prime \] (1.23)

\[ \theta^\prime + \frac{1}{2} f \theta^\prime + CPe_x \left( f^\prime \theta^\prime + f \theta^\prime \right) = 0 \] (1.24)

\[ \theta(0) = 1, \quad f(0) = 0, \quad \theta(\infty) = 0, \quad f^\prime(\infty) = 1 \] (1.25)

The local Nusselt number \( Nu_x \) is given by

\[ \frac{Nu_x}{Pe_x^{1/2}} = \left( \frac{Ra_c}{Pe_e} \right)^{1/2} \left[ -\left[ 1 + CRa_d f^\prime(0) \right] \theta(0) \right] \] (1.26)

It is noticed that, the effect of quadratic drag is to reduce the aiding or opposing effect of buoyancy in increasing \( Nu_x / Pe_x^{1/2} \), while that of thermal dispersion is (as expected) to increase the heat transfer.

For a vertical surface, higher order boundary layer theory (for Darcy flow) has been developed by Merkin [37] and Joshi and Gebhart [38]. Merkin pointed out that, in the case of opposing flow there is separation of the boundary layer downstream of the leading edge. Ranganathan and Viskanta [39] included the effects of inertia, porosity variation and blowing at the surface. It has been reported that, an unexpected result that porosity variation affected the Nusselt number by not more than 1%. Chandrasekhara and Namboodiri [40] have studied the effect of variation of permeability and conductivity. Lai and Kulacki [41] have examined the effect of viscosity variation with temperature. It is noticed that, for liquids the Nusselt number values are greater than those for the constant viscosity case and for gases the reverse holds. Ramanaiah and Malarvizhi [42] have obtained a similarity solution for the
combination of lateral mass flux and inertia when the linear Darcy drag term is negligible in comparison with the quadratic drag.

Chen and Chen [43] have studied the combined effects of quadratic drag, boundary friction, thermal dispersion and non-uniform porosity and the consequent non-uniform conductivity, for the case of aiding flow on a vertical surface. As expected, boundary friction reduces the velocity at the wall, inertia generally reduces the velocity, thermal dispersion has negligible effect on the velocity and non-uniform porosity substantially increases the velocity just out from the wall. The temperature gradient at the wall is reduced by boundary friction and quadratic drag increased by variable porosity, the overall effect is reduction. Consequently, the local Nusselt number is reduced by boundary friction and quadratic drag and increased by variable porosity, the overall effect is little change. The local Nusselt number $\bar{Nu}$ is increased about threefold by thermal dispersion. The effect of increase of $Ra Pe^m$ is to increase $\bar{Nu}$ and increase the amount of channeling.

m) Disposition

In view of all above applications an attempt has been made to study the behaviour of various fluid parameters that are encountered in different situations. This thesis mainly deals with the extensive analysis of mixed convective fluid flows in porous media and the effects of dimensionless velocity, temperature, radiation effects and mixed convective terms along with other participating parameters had been discussed qualitatively and illustrated graphically.

All problems discussed in this thesis present a classical display of different problems examined by considering purely thermal and viscous terms alone. Also, an interesting feature
that has been noticed is highlighted wherever possible. In this thesis, each chapter has been prefixed with a nomenclature for easy reference and readability.

n) REFERENCES


