## CHAPTER 4

Chapter 4 MHD effects on Darcy-Forchheimer Mixed Convection in a fluid saturated Porous media with Chemical Reaction and Viscous Dissipation

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CHAPTER 4

4.1 Introduction

A situation where both the forced and free convection effects are of comparable order is called mixed convection. The analysis of mixed convection boundary layer flow along a vertical flat plate in a fluid saturated porous media has received considerable theoretical, practical and numerical interest of technology. A detailed study of convective heat transfer in Darcy porous medium can be discussed in the book by Nield and Bejan [73].

The chemical reaction effects in both heat and mass transfer receive a considerable amount of attention rendered by many authors [4, 64, 93]. The heat and mass transfer from different geometries in porous media has many geophysical applications like, to dry porous solids, enhancing oil recovery, packing-bed catalytic reactors, geothermal reservoirs, thermal insulation, cooling of nuclear reactors and underground energy and species transport [50, 75, 81]. The research efforts e.g.[13, 21, 61, 63, 74, 114] concern on Darcy free convection, which states the volume-averaged velocity is proportional to the pressure gradient.

Shenoy [98] studied the problem of steady state flow, Darcy-Forchheimer, natural, forced and mixed convection passed non-isothermal bodies of arbitrary shape. Rami investigated [85] simultaneous heat and mass transfer by Darcy-Forchheimer mixed convection from a flat plate surrounded in a porous medium. Srinivasacharya [102] have studied MHD, Radiation effect on Non-
Darcy mixed convection flow. Yih [113] studied the forced convection over a wedge with suction/blowing. Hiemenz flow has been studied by Kandaswamy et al [57].

The present paper deals with the effects of MHD on the flow of Darcy-Forchheimer mixed convection and mass transfer of a steady, two-dimensional, laminar boundary layer about an isothermal vertical flat plate embedded in a porous medium in the presence of chemical reaction and viscous dissipation effects. The governing non-linear differential equations are linearized by using the Quasi-linearization technique. The implicit finite difference scheme is used to solve the coupled linear differential equations.

4.2 Mathematical formulation

Consider steady, two-dimensional, Darcy-Forchheimer mixed convection over a vertical flat plate in a fluid saturated porous medium at constant temperature is $T_w$, concentration is $C_w$, ambient temperature is $T_\infty$ and concentration is $C_\infty$. Assumed the properties of the fluid and the porous medium are to be constant, homogenous and isotropic. The $x$ axis is taken as along the surface from its leading edge and the $y$ axis is normal to it, as shown in Figure 4.2.1. A uniform transverse magnetic field of strength $\beta_o$ is applied parallel to the $y$ axis. Considering the Boussinesq approximation and Brownian motion of particles, the governing equations for the boundary-layer flow from the wall to the fluid saturated porous medium are:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\left[ 1 + \frac{\sigma \beta}{\rho v} K_1 \right] \frac{\partial u}{\partial y} + \frac{C_f \sqrt{K_1}}{v} \frac{\partial (u^2)}{\partial y} = \pm \frac{g}{v} \frac{K_1}{\nu} \left[ \beta_T \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y} \right]
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \nu \left( \frac{\partial u}{\partial y} \right)^2
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma (C - C_\infty)
\]

\[
\rho = \rho_\infty \left\{ 1 - \beta_T (T - T_\infty) - \beta_C (C - C_\infty) \right\}
\]

In the above equations

\(u\) is the velocity component in \(x\) direction, \(v\) is the velocity component in \(y\) direction, \(K_1\) the Darcy permeability, \(\nu\) is the kinematic viscosity, \(\gamma\) is the rate of chemical reaction, \(\rho\) is the density, \(\sigma\) conductivity of the fluid, \(\beta_T\) is the coefficient of thermal expansion, \(\beta_C\) is the coefficient of concentration expansion, \(g\) is the acceleration due to gravity, \(C_p\) is the specific heat at constant pressure, \(C_f\) is the Forchheimer coefficient, \(T\) is the temperature, \(C\) is the Concentration, \(\alpha\) is the thermal diffusivity, \(D\) is the Brownian diffusion. The plus sign corresponds to the case where the buoyancy has a component “aiding” the forced flow and minus sign refer to the “opposing” case.
Fig. 4.2.1 Schematic diagram of the problem

\[ v = 0 \quad T = T_w \quad C = C_w \quad \text{as} \quad y = 0 \]
\[ u = u_\infty \quad T = T_\infty \quad C = C_\infty \quad \text{as} \quad y = \infty \]  \hspace{1cm} (4.2.5)

We introduce the dimensionless variables for mixed convection:

\[ \eta = \sqrt{Pe_x} \frac{y}{x}, \quad \psi = \alpha \sqrt{Pe_x} f(\eta), \]
\[ \theta(\eta) = (T - T_\infty) / (T_w - T_\infty), \]
\[ \phi(\eta) = (C - C_\infty) / (C_w - C_\infty) \]  \hspace{1cm} (4.2.6)

Here \( \psi \) is the stream function and \( \eta \) is the dimensionless similarity variable and continuity equation is satisfied by \( \psi \).

Using Eq.(4.2.6) the change of variables, equations (4.2.2)-(4.2.4) are transformed

\[ (1 + Ha^2) f' + 2Af'f' = \pm \left( \frac{Ra_x}{Pe_x} \right) (\theta' + N \phi') \]  \hspace{1cm} (4.2.7)
\[ \theta' + \frac{1}{2} f \theta' + PrEc f'^2 = 0 \]  \hspace{1cm} (4.2.8)
\[ \frac{Pr}{Sc} \phi' + \frac{1}{2} f \phi' - \delta \phi = 0 \]  \hspace{1cm} (4.2.9)
The boundary conditions are

\[ f(\eta) = 0, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \text{ on } \eta = 0 \]
\[ f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \text{ as } \eta \to \infty \]  \hfill (4.2.10)

Where prime denotes the differentiation with respect to \( \eta \),

- \( \text{Pe}_x = u_x x / \alpha \): local Peclet number
- \( \Lambda = c_f \sqrt{K_1 u_x / \nu} \): inertia parameter
- \( N = \beta_c (C_w - C_\infty) / \beta(T_w - T_\infty) \): buoyancy ratio parameter
- \( \text{Ra}_x = (K_1 e \beta_T)(T_w - T_\infty)x / \alpha \nu \): Rayleigh number
- \( \text{Pr} = \nu / \alpha \): Prandtl number
- \( \text{Ec} = u_x^2 / c_p(T_w - T_\infty) \): Eckert number
- \( \delta = \gamma x / u_\infty \): chemical reaction rate parameter
- \( \text{Sc} = \nu / D \): Schmidt number
- \( \text{Ha}^2 = \sigma \beta_0^2 K_1 / \rho \nu \): magnetic parameter

The Nusselt number \( \text{Nu}_x \) is defined by

\[ \text{Nu}_x = \frac{x q_w}{k_f(T_w - T_\infty)} \]

The Sherwood number \( \text{Sh}_x \) is defined by

\[ \text{Sh}_x = \frac{x q_m}{D(C_w - C_\infty)} \]  \hfill (4.2.11)

Where

- \( q_w, q_m \) are the heat and mass transfer rate per unit surface area
- \( k_f \) is the effective thermal conductivity which is defined by:

\[ q_w = -k_f \frac{\partial T}{\partial y} \bigg|_{y = 0} \]
\[ q_m = -D \frac{\partial c}{\partial y} \bigg|_{y = 0} \]  \hfill (4.2.12)

Using Equations (4.2.6), (4.2.11) and (4.2.12)
The local Nusselt number is defined by

\[ Nu_x/\sqrt{Pe_x} = -\frac{\partial \theta}{\partial \eta} \bigg| \eta = 0 \]  \hspace{1cm} (4.2.13)

The Sherwood number is defined by

\[ Sh_x/\sqrt{Pe_x} = -\frac{\partial \phi}{\partial \eta} \bigg| \eta = 0 \]  \hspace{1cm} (4.2.14)

4.3 Numerical solution

Applying the Quasi-linearization technique [14] to the non-linear equation (4.2.7) we obtain as

\[ (1 + H a^2 + 2 \lambda F) \frac{d}{d \eta} f' + 2 \lambda \frac{d}{d \eta} f = \pm \left( \frac{Ra_x}{Pe_x} \right) \left( \theta' + N \phi' \right) + 2 \lambda F' F' \]  \hspace{1cm} (4.3.1)

Where assumed \( F \) is the value of \( f \) at \( n \)-th iteration and \( f \) is at \( (n+1) \)-th iteration. The convergence criterion is fixed as \( |F - f| < 10^{-5} \).

The equation (4.3.1), (4.2.8) and (4.2.9) become with the implementation of implicit finite difference scheme

\[ a[i][i-1]+b[i][i][i]+c[i][i][i+1]=d[i] \]  \hspace{1cm} (4.3.2)

\[ a_1[i][i-1]+b_1[i][i]+c_1[i][i+1]=d_1[i] \]  \hspace{1cm} (4.3.3)

\[ a_2[i][i-1]+b_2[i][i]+c_2[i][i+1]=0 \]  \hspace{1cm} (4.3.4)

where

\[ a[i] = 1 + Ha^2 + 2 \lambda F_1[i] - 0.5*h*2 \lambda F_2[i] \]

\[ b[i] = -2*(1 + Ha^2 + 2 \lambda F_1[i]) \]

\[ c[i] = 1 + Ha^2 + 2 \lambda F_1[i] + 0.5*h*2 \lambda F_2[i] \]

\[ d[i] = h*h* \left( \frac{Ra_x}{Pe_x} \right) (\theta_1 + N \phi_1) + 2 \lambda F_2[i]F_1[i] \]

\[ a_1[i] = 1 - 0.5*h*0.5*f[i] \]

\[ b_1[i] = -2 \]

\[ c_1[i] = 1 + 0.5*h*0.5*f[i] \]
\[ d_1[i] = h^*h^* - PrEc_2[i]^*f_2[i] \]
\[ a_2[i] = (Pr / Sc) - 0.5*h* 0.5*f[i] \]
\[ b_2[i] = -2( Pr / Sc) - h^*h*\delta \]
\[ c_2[i] = (Pr / Sc) + 0.5*h*0.5*f[i] \] and
\[ A[i] = 1 + Ha^2 + 2 \lambda F_1[i] \]
\[ B[i] = 2 \lambda F_2[i] \]
\[ D[i] = \pm \frac{Ra_x}{Pe_x} (\theta_1 + N\phi_1) + 2 \lambda F_2[i]F_1[i] \]
\[ A_1[i] = 1, \quad B_1[i] = 0.5*f[i] \]
\[ D_1[i] = - Pr Ec f_2[i]^*f_2[i], \quad A_2[i] = Pr / Sc \]
\[ B_2[i] = 0.5*f[i], \quad C_2[i] = -\delta \]

4.4 Results and discussion

The system of nonlinear ordinary differential equations (4.2.7)-(4.2.9) together with boundary conditions (4.2.10) are locally similar and solved numerically by using implicit finite difference scheme. The coupled nonlinear differential equations, first linearized by using the Quasi-linearization technique. The implicit finite difference scheme is applied to the coupled differential equation. The resulting algebraic system is solved by Gauss- Seidel method. To compute the numerical values we have used the C programming code.

To get the physical insight of the problem, this method is adequate and give accurate result for boundary layer equations. A uniform grid was adopted to concentrate towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped \(|F\cdot f| \leq 10^{-5}\). In the present study,
the boundary conditions for $\eta$ at $\infty$ are replaced by a sufficient large value of $\eta$ where the velocity approaches 1, at temperature and concentration approaches zero. In order to see the effects of step size $h$ run the code for our model with two different step sizes as $h = 0.001$ and 0.05 and in each case we found very good agreement between them on different profiles. A parameter study of physical parameter is performed to illustrate interesting features of numerical solutions.

The results of parametric study are shown graphically in figures 4.6.2 to 4.6.11 and discussed. In the present study we have adopted the following default parameter values for numerical computation $Pr=0.73$, $Ra_x/Pe_x=1$, $\delta=0.4$, $N=2$, $\Lambda=0$, $Sc=1$, $Ec=0.5$ and $Ha =0$. In figures 4.6.2(a)-(c) depicts the effects of buoyancy ratio parameter $N$ on the fluid velocity, temperature and concentrations distributions respectively. As $N$ increases, it can be observed from figure 4.6.2(a) that the maximum velocity increases. From figures 4.6.2(b) and 4.6.2(c) it is observed that the temperature and concentration profiles decrease with the increase of $N$.

Figures 4.6.3(a)-(c) the graphs are drawn for the buoyancy ratio parameter for $N<0$. From figure 4.6.3(a) the velocity profile decreases as $N$ values decreases. From figures 4.6.3(b) and 4.6.3(c) conclude that the temperature and concentration gradients are increased with the decrease of buoyancy ratio parameter. This can be attributed to the fact that increasing $|N|$ increases the vertical velocity and decrease the thickness of the temperature and concentration boundary layers.
The effect of magnetic field parameter $H_a$ on velocity, temperature and concentration profiles are plotted in figures 4.6.4(a)-(c) respectively. It is clear from 4.6.4(a) that the velocity of the fluid decreases with the increase of magnetic parameter $H_a$, while the temperature of the fluid and concentration of the fluid increases with the increase of magnetic field parameter are shown in figures 4.6.4(b)-(c). A magnetic strength increases the Lorentz force, which opposes the flow, also increase and leads to enhance the deceleration of the flow. This results qualitatively agrees with the expectation since the magnetic field exerts retarding force on the mixed convection flow.

Figures 4.6.5(a)-(c) illustrates the effect of inertia parameter $\Lambda$ on various profiles. Fig. 4.6.5(a) the velocity profile decreases, inertia parameter $\Lambda$ increases. This decrease in the fluid velocity take place because when the porous medium inertia effect increase, the form drag of the porous medium increases. It can be observed that reverse phenomena is noticed on the temperature and concentration distributions in the boundary layer increases owing to the increase in the value of inertia parameter $\Lambda$.

Figures 4.6.6(a)-(c) represents the effect of Schmidt number $S_c$ on profiles such as velocity, temperature and concentration. An increase in the value of $S_c$ cause the species concentration and its boundary layers thickness to increase significantly with a slight increase in the fluid temperature, whereas the velocity and concentration profiles decrease with the increase of $S_c$. 
Figures 4.6.7(a)–(c) present the trend of the velocity, temperature and concentration for the various values of mixed convection parameter respectively. When $Ra_x/Pe_x \gg 1$, flow is dominated by natural convection. $Ra_x/Pe_x \ll 1$ the flow takes to lead forced convection. When $Ra_x/Pe_x = 1$, flow is under mixed convection conditions. Since the buoyancy is aiding the flow, the mixed convection flow is taken positive value. For the opposing flow, the value of mixed convection flow is taken negative value. Figure 4.6.7(a) shows that for aiding flow the fluid velocity in the boundary layer increases with the increase of mixed convection parameter $Ra_x/Pe_x$. The velocity profiles is to increase velocity profiles with increasing mixed convection parameter. Figures 4.6.7(b) and 4.6.7(c) show that temperature and concentration profiles as well as thermal and solutal boundary layer thickness decrease with the increase value of mixed convection parameter $Ra_x/Pe_x$.

Figures 4.6.8(a)–(c) depict the influence of chemical reaction parameter $\delta$ on the fluid velocity, temperature and concentration profiles respectively. It is noticed that increase in the chemical reaction parameter $\delta$ is to reduce the velocity and concentration profiles whereas there is a slight increase in the temperature profile.

In figures 4.6.9(a)-(c) the effect of viscous dissipation $Ec$ on velocity, temperature and concentration are presented. As Eckert number $Ec$ increases, the velocity $f$ profile increases. Due to the effect of $Ec$, the temperature profiles increases is observed from figure
9(b). It is also noticed from figure 4.6.9(c), the effect of viscous dissipation there is no significant changes on concentration profiles.

The heat and mass transfer results in terms of $Nu_x/\sqrt{Pe_x}$ and $Sh_x/\sqrt{Pe_x}$ as a function of the mixed convection parameter $Ra_x/Pe_x$ for different values of chemical reaction $\delta$, Eckert number and buoyancy are depicted in figures 4.6.10 – 4.6.11. It is clear from the figures that $Ra_x/Pe_x$ effects tend to slow down the buoyancy individual flow in the boundary layer and so retard the heat and mass transfer rates. It is also observed that the chemical reaction effect reduces values of $Nu_x$ slightly and increases $Sh_x$. The influence of chemical reaction is more on Sherwood number when compared with Nusselt number. The viscous dissipation effects on $Nu_x$ and $Sh_x$ are shown in figures 4.6.10(b) and 4.6.11(b). This is clear from the figures the effect of Eckert number $Ec$ is to reduce the Nusselt number whereas it increases in Sherwood number. The viscous dissipation effect is more on Nusselt number when compare with the Sherwood number. The effect of buoyancy ratio $N$ on the heat and mass transfer results in terms of $Nu_x/\sqrt{Pe_x}$ and $Sh_x/\sqrt{Pe_x}$ are displaced in figures 4.6.10(c) and 4.6.11(c) respectively. The effect of buoyancy ratio parameter $N$ is to reduce the value of Nusselt number and increases the Sherwood number. Again, it is clear from figures 4.6.10–4.6.11 that increasing $Ra_x/Pe_x$ leads to increases of the momentum transport in the boundary layer, thus decreasing the thickness of the thermal and
concentration boundary and hence increasing the heat and mass transfer rates.

4.5 Conclusions

The numerical calculations are obtained for the profiles such as velocity, temperature and concentration and also presented the local Nusselt number and local Sherwood number. We conclude the following results:

- The effect of magnetic field parameter \( Ha \) is to decrease the velocity profile, whereas the temperature and concentration profiles increases with the increase of magnetic field parameter \( Ha \).
- It is observed that with the increase of buoyancy ratio \( N \) that the velocity increases. With the increase of \( N \) the temperature and concentration profile decreases.
- The velocity profile decrease with the increase of inertia parameter \( \Lambda \), while the temperature and concentration profiles increases with the increase of inertia parameter \( \Lambda \).
- The local Nusselt number decreases, when chemical reaction parameter \( \delta \) and Eckert number \( Ec \) increase. On the other hand, Sherwood number \( Sh_x \) increase with the increase of chemical reaction parameter \( \delta \) and Eckert number \( Ec \).
- The effect of buoyancy ratio parameter is to decrease the Nusselt number whereas it increases the Sherwood number.
Fig. 4.6.2. Effect of buoyancy ratio parameter $N$ on profiles of 
(a) Velocity (b) Temperature (c) Concentration
Pr=0.73, $\Lambda=0$, Sc=1, $Ra_{x}/Pe_{x}=1$, Ha=0, Ec=0.5, $\delta=0.4$
**Fig. 4.6.3.** Effect of buoyancy ratio $N$ (negative) on a) Velocity profile (b) Temperature profile (c) concentration profile

$Pr = 0.73$, $\Lambda = 0$, $Sc = 1$, $Ra_x/Pe_x = 1$, $Ha = 0$, $Ec = 0.5$, $\delta = 0.4$
Fig. 4.6.4. Effect of magnetic parameter $Ha$ on profiles of
(a) Velocity (b) Temperature (c) Concentration
$Pr = 0.73, \Lambda = 0, Sc = 1, Ra_x/Pe_x = 1, N = 2, Ec = 0.5, \delta = 0.4$
Fig. 4.6.5. Effect of inertia parameter $\Lambda$ on profiles of
(a) Velocity  (b) Temperature  (c) concentration
$Pr = 0.73, Ha = 0, Sc = 1, Ra_x/Pe_x = 1, N = 2, Ec = 0.5, \delta = 0.4$
Fig. 4.6.6 Effect of Schmidt number Sc on profiles of
(a) Velocity (b) Temperature (c) concentration
Pr = 0.73, Ha = 0, Λ = 0, \( \text{Ra}_x/\text{Pe}_x = 1 \), N = 2, Ec = 0.5, δ = 0.4
Fig. 4.6.7(c). Effect of mixed convection $Ra_x/Pe_x$ on profiles of (a) Velocity (b) Temperature (c) Concentration

$Pr = 0.73$, $Ha = 0$, $A = 0$, $Sc = 1$, $N = 2$, $Ec = 0.5$, $d = 0.4$
Fig. 4.6. 8 Effect of chemical reaction parameter $\delta$ on profiles of 
(a) Velocity (b) Temperature (c) Concentration 
$Pr=0.73$, $Ha=0$, $\Lambda=0$, $Sc=1$, $N=2$, $Ec=0.5$, $Ra_x/Pe_x=1$
Fig. 4.6.9 Effect of Eckert number $E_c$, profiles of
(a) Velocity  (b)Temperature  (c) concentration
$Pr = 0.73$, $Ha = 0$, $\Lambda = 0$, $Sc = 1$, $N = 2$, $\delta = 0.4$, $Ra_x/Pe_x = 1$
Fig. 4.6.10(a) Effect of Nusselt number for various values of
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Fig. 4.6.10(b) Effect of Nusselt number for various values of
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$Sc=1$, $N=2$, $\delta=0.4$

Fig. 4.6.10(c) Effect of Nusselt number for various positive
values of buoyancy ratio, $Pr=0.73$, $Ha=0$, $\Lambda=0$,
$Sc=1$, $Ec=0.5$, $\delta=0.4$
Fig. 4.6.11(a) Effect of Sherwood number on chemical reaction
Parameter $\delta$, $Pr = 0.73$, $Ha = 0$, $\Lambda = 0$, $Sc = 1$, $N = 2$, $Ec = 0.5$

Fig. 4.6.11(b) Effect of Sherwood number for various values of Eckert number $Pr = 0.73$, $Ha = 0$, $\Lambda = 0$, $Sc = 1$, $N = 2$, $\delta = 0.4$

Fig. 4.6.11(c) Effect of Sherwood number for various positive values of buoyancy ratio, $Pr = 0.73$, $Ha = 0$, $\Lambda = 0$, $Sc = 1$, $Ec = 0.5$, $\delta = 0.4$