CHAPTER VII

MODELLING FINANCIAL DATA

7.1 INTRODUCTION

The distributions of stock returns, commodity prices, foreign currency exchange rates and other financial data have attracted the attention of numerous researchers. The first step towards the statistical modelling of stock price changes was taken by Bachelier (1900). His approach was based on three assumptions: independence, identical distribution and finite variance of daily changes. Since the price change over a certain period of time can be regarded as the sum of changes over shorter periods of time (weekly change = sum of daily changes, daily change = sum of changes between of the various transactions etc.), Bachelier arrived at a normal model.

Further studies, however, showed that empirical distributions of stock returns had more kurtosis ('fatter tails'), than was predicted by the normal distribution. Mandelbrot (1963 a,b) and Fama (1965) proposed the symmetric stable distribution as a model for asset returns. The family of stable distributions seemed appropriate, because they could allow independent and identically distributed returns and, at the same time, account for the observed leptokurtosis in the data.
Another important reason for focusing on the stability property, is that only stable distributions have domains of attraction (with respect to the summation scheme), that is, they are the only possible limits of (scaled) sums of i.i.d. random variables. This property provides an asymptotic basis for using stable laws to model uncertainty.

As a consequence of the empirical evidence and the fact that the normal distribution is a special case of a stable law, the stable assumption has enjoyed some popularity among financial modelers. However, a number of empirical studies have provided evidence that is inconsistent with the stable hypothesis (Officer, 1972; Blattberg and Gonedes, 1974; Press, 1982; Akgiray and Booth, 1988). In fact, the studies show that the characteristic exponent does not, as it should, remain constant as the sampling period is increased.

In response to these empirical inconsistencies, alternatives to the stable laws have been proposed for asset returns models. DuMouchel (1973) investigates a mixture of normal and stable distributions. Boness et al. (1974) proposes a mixture of normal distributions. DuMouchel’s model leads to fat tailed distributions. while Boness assumptions imply no fat tails. Another alternative, suggested by Practz (1972) and Blattberg and Godnedes (1974) is that
security returns follow a Student's t-distribution. Practz proposed primarily a normal model, but with a random variance. By choosing an appropriate probability distribution for the variance, he arrived at the Student t-distribution. This hypothesis is very attractive, because it captures the observed increase of the characteristic exponent $\alpha$ of the stable law, as the sampling interval increases.

A drawback of all these alternative distributions is that they are not based on a probabilistic scheme, describing them as limiting laws. As a consequence, they do not have domains of attraction and they are not closed with respect to scaling under certain transformations, that is, they are not stable. Mandelbrot (1977) argued that the class of distributions of stock price changes should be closed with respect to scaling.

Minnik and Rachev (1989, 1991, 1993) have considered various probability schemes and extended the stability concept of Mandelbrot, which arises from one specific (summation) scheme. These lead to a variety of distributions, stable with respect to the underlying scheme. They also fitted these alternative stable distributions to the stock-index data and compared their appropriateness. Their findings were that the (double) Weibull distribution, which arises in the geometric...
summation scheme dominates all other alternative stable laws. One of the implications of the stability property is that stable distributions can be classified according to the one overall parameter: the index of stability, determining the main properties of the distributions.

This chapter is devoted to modelling financial data pertaining to the exchange rate change of certain currencies in terms of Indian Rupees, normal, stable, geometric stable (GS) and negative binomial stable distributions (NBS) are used in the process of modelling these data sets. Comparison between the various models constructed has also been made. The Kolmogrov distance test is used to test the better fit of the data.

Our purpose is to present an NBS model for the foreign exchange rate change that competes successfully with the normal, stable and geometric stable models. We shall study the distribution of Dollar, Pound, Mark and Riyal unit of American, British, German and Saudi Arbian currency exchange rates respectively in relation to Indian Rupees. The data values are daily exchange rate changes collected from Malayala Manorama, a national daily news paper in Malayalam language. The data values are from 1-1-1990 to 31-12-1998 for Dollar and Pound, from 1-3-1992 to 31-12-1998 for Mark.
and Riyal. There are 1641, 1624, 1265, 1267 values for Dollar, Pound, Mark and Riyal respectively. We consider the change in the log (price) from time $t$ to $t+1$. Observations are simulated from the respective distributions, and histograms are drawn for the data values and simulated observations. We fit the Dollar, Mark and Riyal for normal, stable and GS distributions of which the GS distribution is better fit data than normal and stable models. The pound exchange rate change is fitted for normal, stable, GS and NBS models for different values of $r$. While fitting the data values, it is shown that the NBS distribution is found better fit than normal and stable models. $r = 1$ gives the fitted model for the GS distribution. For $r < 1$, the NBS distribution is better fit data than normal, stable, and GS models judged in terms of Kolmogrov distance test.

### 7.2 REPRESENTATION AND SIMULATION

The most widely used method of simulation of random variables is the inversion method. Let $F$ be the distribution function of the random variable $X$, then

$$X = F^{-1}(u)$$

Where $F^{-1}$ is the inverse function of $F$ and $u$ is a uniform random variable on $(0,1)$. But the method cannot be adopted when $F$
has not a closed form. But now computer algorithms are available for most of the commonly used distributions.

Simulating random variables from stable, geometric stable and NBS distributions has the following steps (Kozubowski (1999), Kozubowski et al. (1995)).

**A standard stable \( S_\alpha (1, \beta, 0) \) generator**

- Generate a standard exponential variate \( W \).

- Generate uniform \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \) variate \( V \), independent of \( W \).

- \[ \text{IF } \alpha = 1 \]
  
  THEN Set \( X \leftarrow \frac{2}{\pi} \left[ \left( \frac{\pi}{2} + \beta V \right) \tan V - \beta \log \left( \frac{W \cos V}{\frac{\pi}{2} + \beta V} \right) \right] \)

  ELSE {
    
    Set \( b \leftarrow \frac{\arctan(\beta \tan(\pi \alpha/2))}{\alpha} \)

    Set \( s \leftarrow \left[ 1 + \beta^2 \tan^2 \frac{\pi \alpha}{2} \right]^{1/\alpha} \)

    Set \( X \leftarrow s \frac{\sin(\alpha(V+b))}{\cos V} \left( \frac{\cos(V - \alpha(V+b))}{W} \right)^{\left( 1 - \alpha \right)/\alpha} \}

}
Generating GS random variable and NBS random variable has the following steps.

- Generate a non-negative random variable $Z$.
- Generate standard stable variate $X \sim S_\alpha (1, \beta, 0)$, independent of $Z$.
- IF $\alpha \neq 1$
  
  THEN Set $Y \leftarrow \mu Z + Z^{1/\alpha} \sigma X$.
  
  ELSE Set $Y \leftarrow \mu Z + Z\sigma X + \sigma Z\beta \left( \frac{2}{\pi} \right) \log(Z\sigma)$.
  
- RETURN $Y$.

The r.v $Y$ is GS and NBS according as $Z$ is exponential and negative binomial respectively.

### 7.3 APPLICATIONS IN MODELLING

Here we are applying NBS distribution to model financial data set. It is assumed that an exchange rate change is the sum of a large
number of small changes. Unlike in $\alpha$-stable model, the summation is taken up to a random time determined by a negative binominal random variable $T(r,p)$. When $r = 1$, $T(r,p)$ is a geometric random variable. If $Y$ represents the daily price change of a particular stock or commodity and $T$ is the random number of transactions in one day, then

$$Y = \sum_{i=1}^{T} X_i,$$

where $X_i$ represent price change between successive transactions and are assumed to be independently and identically distributed.

Kozubowski and Rachev (1994) consider $T$ as geometric random variable so that

$$T(p)$$

stock price change $= \sum_{i=1}^{T(p)} \text{"small changes"}$

$T(p)$ is a geometric random variable representing the moment at which the probabilistic structure governing the returns break down. The parameter $p (0 < p < 1)$ is the probability of small change in the market during each period.

Here we are considering $T(r,p)$, the negative binominal random variable instead of $T(p)$, the geometric random variable. $T(r,p)$
represents \((r > 0, \ 0 < p < 1)\) the moment at which the probabilistic structure governing the \(r\)th returns breaks down and \(p\) is the probability of small change in the market during each period. Thus the distribution of the exchange rate change in one day is approximated by NBS laws, and \(T(r,p)\) represents the moment at which the probabilistic structure governing the exchange rate breaks down. This may be due to new information about war, epidemic condition or other such events that affect the fundamentals of the exchange market. Thus the stability property is maintained up to the random time \(T(r,p)\).

### 7.4 NUMERICAL SIMULATION

We shall consider the distribution of Dollar, Mark and Riyal for normal, stable and geometric stable distributions, and the distribution of Pound for normal, stable, geometric stable and NBS distributions. As usual we consider the change in log (price) from \(t\) to \(t+1\), that is each data point

\[ P_t = \ln(X_{t+1}) - \ln(X_t), \]

where \(X_t\) represents the closing price of the commodity on day \(t\). As shown in the histogram of data sets, the peakedness is the characteristic of many financial data sets. We use the maximum likelihood estimators for estimating \(\mu\) and \(\sigma\), the mean and the
standard deviation of the normal distribution. The moment estimators suggested by Press (1972) is used to estimate the parameters of stable distribution, and the same method of estimation by Kozubowski (1998a) is used to estimate the parameters of the geometric stable distribution. Explicit estimators are available for the parameters of NBS distribution when \( r \) is known. Hence for different values of \( r \), the other parameters can be explicitly estimated.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Normal</th>
<th>Stable</th>
<th>GS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dollar</td>
<td>( \mu = 0.0005493 )</td>
<td>( \alpha = 1.8250 )</td>
<td>( \alpha = 1.8256 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.01812 )</td>
<td>( \beta = -0.03373 )</td>
<td>( \beta = 0.001919 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu = 0.0005494 )</td>
<td>( \mu = 0.0005490 )</td>
</tr>
<tr>
<td>German mark</td>
<td>( \mu = 0.000233 )</td>
<td>( \alpha = 1.9812 )</td>
<td>( \alpha = 1.9813 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.036421 )</td>
<td>( \beta = 0.000247 )</td>
<td>( \beta = 0.0002394 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu = 0.0002331 )</td>
<td>( \mu = 0.0002331 )</td>
</tr>
<tr>
<td>Saudi Riyal</td>
<td>( \mu = 0.000197 )</td>
<td>( \alpha = 1.8204 )</td>
<td>( \alpha = 1.8211 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.091875 )</td>
<td>( \beta = 0.001317 )</td>
<td>( \beta = -0.00124 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu = 0.0001974 )</td>
<td>( \mu = 0.0001973 )</td>
</tr>
</tbody>
</table>

Table 7.4.1

Table 7.4.1 gives the estimated values for the parameters of normal, stable and geometric stable models for Dollar, Mark and Riyal respectively.
Table 7.4.2

Table 7.4.2 provides the frequency distribution of the data values and simulated observations from the normal, stable and geometric stable distributions where D.N.S. GS denote the Data Normal Stable and Geometric Stable respectively.
geometric stable distributions where D, N, S, GS denote the Data values and simulated observations from normal, stable and geometric stable distribution respectively.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Kolmogrov Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td>Dollar</td>
<td>0.5283</td>
</tr>
<tr>
<td>Mark</td>
<td>0.2933</td>
</tr>
<tr>
<td>Riyal</td>
<td>0.6487</td>
</tr>
</tbody>
</table>

Table 7.4.3

Table 7.4.3 shows the Kolmogrov distance for the fitted distributions for the exchange rate changes.

The table 7.4.3 shows that the geometric stable distribution is better fit data than the normal and stable distributions for the exchange rate change of Dollar, Mark and Riyal. Figure 7.4.1, 7.4.2 and 7.4.3 shows the frequency distribution of Dollar, Mark and Riyal for the data, normal model, stable model and geometric model respectively.

Among the four exchange rates considered pound alone is fitted for NBS distribution for different values of r where the geometric stable distribution is a particular type (r = 1) of NBS distribution.
Fig. 74.2
For the normal distribution, \( \hat{\mu} = 0.000567 \), \( \hat{\sigma} = 0.1282 \). For stable distribution, \( \hat{\alpha} = 1.5392 \), \( \hat{\sigma} = 0.0299 \), \( \hat{\beta} = 0.0469 \) and \( \hat{\mu} = 0.000594 \). For normal and stable distributions the Kolmogrov distance is 0.4187 and 0.3213 respectively.

<table>
<thead>
<tr>
<th>R</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( \mu )</th>
<th>Kolmogrov distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.5442</td>
<td>0.08617</td>
<td>0.002938</td>
<td>0.1195</td>
</tr>
<tr>
<td>0.25</td>
<td>1.5432</td>
<td>0.07438</td>
<td>0.00236</td>
<td>0.1367</td>
</tr>
<tr>
<td>0.50</td>
<td>1.5412</td>
<td>0.04719</td>
<td>0.00120</td>
<td>0.1915</td>
</tr>
<tr>
<td>1</td>
<td>1.5402</td>
<td>0.0300</td>
<td>0.000593</td>
<td>0.2377</td>
</tr>
<tr>
<td>2</td>
<td>1.5397</td>
<td>0.0191</td>
<td>0.000297</td>
<td>0.2629</td>
</tr>
<tr>
<td>3</td>
<td>1.5395</td>
<td>0.0147</td>
<td>0.000198</td>
<td>0.2783</td>
</tr>
<tr>
<td>4</td>
<td>1.5394</td>
<td>0.0122</td>
<td>0.000149</td>
<td>0.2876</td>
</tr>
<tr>
<td>5</td>
<td>1.5394</td>
<td>0.0105</td>
<td>0.000120</td>
<td>0.2913</td>
</tr>
<tr>
<td>6</td>
<td>1.5393</td>
<td>0.0093</td>
<td>0.000099</td>
<td>0.2925</td>
</tr>
<tr>
<td>7</td>
<td>1.5393</td>
<td>0.00854</td>
<td>8.48797E-05</td>
<td>0.2913</td>
</tr>
<tr>
<td>8</td>
<td>1.5390</td>
<td>0.00775</td>
<td>7.42728E-05</td>
<td>0.2956</td>
</tr>
<tr>
<td>9</td>
<td>1.5392</td>
<td>0.007178</td>
<td>6.60224E-05</td>
<td>0.2974</td>
</tr>
<tr>
<td>10</td>
<td>1.5392</td>
<td>0.006702</td>
<td>5.94217E-05</td>
<td>0.2974</td>
</tr>
<tr>
<td>11</td>
<td>1.5394</td>
<td>0.00629</td>
<td>5.40208E-05</td>
<td>0.2999</td>
</tr>
</tbody>
</table>

Table 7.4.4. Estimates and Kolmogrov distance
Table 7.4.4 shows the estimated values of $\alpha$, $\sigma$, $\beta$ and $\mu$ for different values of $r$ of NBS distribution. The table also shows the Kolmogrov distance for simulated observations for different values of $r$. When $r = 1$, we get the estimators and the Kolmogrov distance of the geometric stable distribution.

After estimating the parameters of the distributions, observations are simulated from respective distributions, and histograms are drawn for the data sets and simulated values (figure 7.4.4-7.4.7). Because of the scaling, the small frequencies are not visible in the histograms.

The Kolmogrov distance test shows that the NBS distribution is better fit data than normal and stable models. For $r = 1$, we get the goodness of fit of the geometric stable distribution. If we consider the integral values of $r$, the better fit is at $r = 1$. If we can allow $r > 0$, the financial data is better, fitted by NBS distribution for small values of $r$.  

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Fig. 7.4:5

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CHAPTER VIII

WAITING TIME PARADOX AND SOME OPEN PROBLEMS

8.1 INTRODUCTION

The chapter discusses how geometric inactivity and waiting time paradoxes are related in a stationary Markov process generated by the random variable X. It also deals with some open problems in the generalization of the Zipf-Mandelbrot distribution to that of the waiting-time paradox and related distributions.

K. Van Haren and F.W. Steutel (1995) in their paper "Divisibility and Waiting Time Paradox" have shown a new insight in the waiting time paradox by solving the equation

\[ X + Y = Z \]

where X is independent and X and Z are geometrically distributed. This equation shows that X is divisible. A similar decomposition for the waiting time W generated by X is also given and proved that