Chapter 1

Introduction and Literature Survey

Cosmology is the science of Cosmos - of order that exists in the universe at large. The word “Cosmology” and “Cosmetology” come from the same greek word “Kosmos” meaning harmony or order. Nevertheless, Cosmology deals with questions which are fundamental to human condition. Cosmologists deal with the answers to such questions like: “what is the universe made up of?, is it finite or infinite in spatial extent?, did it have a beginning some time in the past?, will it come to an end some time in the future?” Like other scientists, cosmologists also rely on observations and experiments to make headway in our quest to uncover the details of our cosmic history.

At first sight the universe consists of stars, stars clusters, and galaxies or the nebulae. The galaxies again form clusters and even super clusters. Besides galaxies, the universe also contains radiation consisting of zero rest mass particles —— photons, perhaps some neutrinos, and gravitational waves. This radiation traveling at the speed of light, is not clustered in gravitationally bound clumps as is the matter that makes up the galaxies. The detected radiation with the greatest energy density is the cosmic background radiation which is the electromagnetic radiation left over from the hot big bang. But how this matter and radiation come into origin, can be understood from the evolution of our observable universe.

1.1 Big Bang cosmology

An overview of the history of evolution of our observable universe can be discussed within the Big Bang Theory, according to which, our universe is believed to be some 13.8
billion years old. The hot big bang model assumes that the universe began in a state of very high energy density and high temperature. Immediately following this event, it is believed that the universe experienced a rapid exponential expansion, known as inflation which cooled energy content of the universe. According to Grand Unified Theories (GUTs), there is a full unification between the electromagnetic, weak and strong forces. As the universe cooled to 100 GeV ($t = 10^{-10}$ sec), unification between the electromagnetic and the weak forces also break down and the universe was then filled with a plasma consisting of $W$ and $Z$ bosons as well as all quarks and leptons. At 100 MeV, condensation of plasma started resulting in the formation of hadrons.

Around 1 MeV, reactions involving neutrinos run at a slower rate than expansion rate of the universe and effective decoupling of neutrinos take place. Around this neutrino decoupling time, the quarks (constituents of baryons and mesons, three quarks making a baryon and a quark—antiquark pair making a meson) become confined into light nuclei and these nuclei along with electrons combined to form complete atoms synthesizing the elements. This is commonly known as nucleosynthesis and this happened at a temperature of about 1 eV. Because the interaction rate between photons and atoms is far less than that between photons and nuclei, the photons decoupled from matter during recombination. The atoms could no longer absorb the thermal radiation and the universe became transparent. Since the time of decoupling known as the time of last scattering, the photons freely streamed from the recombination event and a period of large-scale structure formation with continuous decrease in the temperature was started, which lasts upto now.

The decoupled photons from the recombination event approach us from all directions and constitute the “Cosmic Microwave Background” (CMB) radiation that we observe today. The CMB radiation was accidentally discovered by Penzias and Wilson in 1965 [144] and it indicates the existence of an isotropic background of photon radiation, having a black body spectrum with a temperature of about 2.7 K. Such an existence for the background radiation is one of the most compelling evidence in favor of the big bang theory.

### 1.2 Einstein’s general theory of relativity

Einstein’s ”General Theory of Relativity” (GTR) is the fundamental tool for understanding the nature of our expanding universe. This theory teaches us that it is the matter
content as well as energy distribution of our universe that tells us not only the structure and geometry of space-time, but also how that space-time is evolving. Gravity is included in the metric of space-time under consideration. The beauty of GTR lies in the fact that it is observer/coordinate independent and it implies that all the laws of physics are invariant under coordinate transformations. In our 4-dimensional universe (comprising of 3 spatial and 1 time coordinate), the invariant interval between space-time events is written as:

$$ds^2 = g_{ij} dx^i \, dx^j,$$

where the indices $i$ and $j$ run over 0 to 3 corresponding to time coordinate ($dx^0 = dt$) and 3 other spatial coordinates ($dx^1 = dx$, $dx^2 = dy$, $dx^3 = dz$). $g_{ij}$ is the metric of the space-time which can be represented as a symmetric matrix. But if we talk about the special theory of relativity in which there is no gravity, all the laws of physics are same in all inertial frames, i.e., no preferred frame exist and the speed of light in free space has same value in all inertial systems. So, we may place an example of special theory of relativity, which is described by the flat Minkowski space with following form of metric:

$$g_{ij} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.$$

In an expanding universe, the proper distance between two points is scaled by the universe’s expansion i.e. if the universe is scaled up according to a scale factor $a(t)$, the spatial distances will be also multiplied with this factor. If the space for the universe is flat as in the Minkowski space, then the metric for a cartesian coordinate system must have the form

$$g_{ij} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & a^2(t) & 0 & 0 \\
0 & 0 & a^2(t) & 0 \\
0 & 0 & 0 & a^2(t)
\end{pmatrix}.$$

For a curved space-time, there exists a particular form of above metric which will be discussed in the next section 1.3.

Now, we come to Einstein’s original field equations that relate the curvature of space-time (represented by the Einstein tensor) to the matter/energy (represented by the energy-
momentum tensor) content of the universe, expressed as

\[ R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G T_{ij}. \]  

(1.2.4)

(We use conventions in which \( c = 1 \), where \( c \) is the speed of light). Here \( G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} \) is the Einstein tensor, which encodes information about the curvature of the universe, \( R_{ij} \) is the Ricci tensor, \( R \) is the curvature scalar, \( G \) is Newton’s gravitational constant. \( T_{ij} \) is the energy-momentum tensor which relates to the matter content of the universe.

The Ricci tensor and scalar curvature are defined by

\[ R_{ij} = R^l_{ilj}, \]  

and

\[ R = g^{ij} R_{ij} \]  

with \( R^k_{\lambda ij} \) being the Riemann curvature tensor expressed as

\[ R^k_{\lambda ij} = \Gamma^k_{\lambda,j,i} - \Gamma^k_{\lambda,i,j} + \Gamma^l_{\lambda,j} \Gamma^k_{il} - \Gamma^l_{\lambda,i} \Gamma^k_{jl}, \]  

where \( \Gamma^\lambda_{ij} \) are Christoffel symbols related to \( g_{ij} \) by

\[ \Gamma^\lambda_{ij} = \frac{1}{2} g^{\lambda l} (g_{jl,i} + g_{il,j} - g_{ji,l}). \]  

(1.2.7)

Here comma denotes the partial derivative, for example, \( f_i = \partial_i f = \frac{\partial f}{\partial x^i} \).

To be consistent with Mach’s principle and with requirement of the fact of small velocities of stars, Einstein in 1917 [59], proposed that our universe is static like a static sphere filled with matter and he introduced the “Cosmological Constant” (\( \Lambda \)) into his field equations in order to get a static and finite cosmological solution. After this modification, Einstein’s original field equations are given by

\[ G_{ij} + \Lambda g_{ij} = \frac{8\pi G}{c^4} T_{ij}. \]  

(1.2.8)

The main feature of this cosmological term \( \Lambda \) is that it has repulsive gravity which cancels the effect of attractive gravity of matter so that a static universe model can be obtained. Moreover, at the same time in 1917, de Sitter [52] obtained a solution in which the matter density \( (\rho_M) \) was negligible compared to the density of cosmological constant \( (\rho_\Lambda) \). Friedman and Lemaitre, in the 1920’s, showed that the cosmological solutions with matter
and $\Lambda$ are involved with expansion or contraction of the universe rather than to be static. On observational grounds, in 1929, the motion of galaxies was studied by Edwin Hubble [79], who pointed out that galaxies are receding away from us. Moreover from his observational data, he demonstrated that speed of recession ($\nu$) of a galaxy is proportional to its distance ($D$) from us and proposed his famous law of expansion of the universe known as Hubble’s Law [79]

$$\nu = H_0 D,$$

where $H_0$ is the constant of proportionality known as “Hubble’s constant”, which measures the current rate of expansion of the universe. After Hubble’s discovery [79], Einstein postulated that $\Lambda$ should be zero and later on, he referred to this mistake as the Biggest Blunder of his life. But later on, inclusion of such remarkable parameter $\Lambda$ was firmly established with the powerful evidences of supernovae and CMB measurements, which is discussed in detail in the section 1.6 of this chapter.

### 1.3 Study of cosmological models

A *Cosmological Model* is a mathematical description of the universe as a whole. This also tries to explain the reasons of its current aspects along with description of nature of evolution with time. In this section, our first motive it to introduce the concept of *Cosmological Principle*, which states that: our universe is homogeneous and isotropic on sufficiently large scales. Moreover, the models which obey this principle of GTR are known as standard cosmological models otherwise these are termed as non-standard. In the following subsections, we wish to introduce the standard cosmological models which are both homogeneous and isotropic, and the spatially homogeneous cosmological models which are anisotropic.

#### 1.3.1 Standard cosmological models

The two testable structural consequences of the cosmological principle are homogeneity which implies that the universe looks same at every point in space and isotropy which implies that the universe looks same in all directions. Under this assumption, space-time
metric can be written in the FRW form, expressed as
\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \] (1.3.1)
where \( a(t) \) is the cosmic scale factor that measures expansion of the universe, \((r, \theta, \phi)\) are comoving spherical polar spatial coordinates and \( t \) is the cosmic time. The quantity \( k \) is a constant and may be chosen to be +1, 0, or −1 for positively curved, flat, negatively curved spatial sections respectively and therefore, it is called the curvature index. The redshift \( z \) measured from the observed wavelength of light \( \lambda_{obs} \), and the wavelength at which it was emitted \( \lambda_{emit} \), is related to the cosmic scale factor \( a(t) \) by
\[ 1 + z = \frac{a_0}{a(t)}, \] (1.3.2)
where \( a_0 \) is the present value of \( a \) and \( z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \) is the fractional change in wavelength.

Again, in concordance with the cosmological principle, distribution of matter in our observable universe is homogeneous and isotropic on sufficiently large scales. Therefore, cosmological fluid of which the universe is composed, can be treated as a perfect fluid which has an energy-momentum tensor of the form
\[ T_{ij} = (\rho + p) u_i u_j + pg_{ij}, \] (1.3.3)
where \( \rho \) and \( p \) are the energy density and isotropic pressure of cosmic fluid respectively and \( u^i \) is the four velocity vector of cosmic fluid. Moreover, \( T_{ij} \) is symmetric tensor with respect to the interchange of \( i \) and \( j \) and it has zero divergence, i.e.,
\[ T_{ij}^{\;\;\;;\;\;k} = 0, \] (1.3.4)
where a semi-colon denotes co-variant derivative of \( T^{ij} \), defined as
\[ T_{ik}^{\;\;\;\;\;\;\;k} = \frac{\partial T^{ij}}{\partial x^k} + \Gamma^i_{kl} T^{lj} + \Gamma^j_{kl} T^{il}. \]
The above equation (1.3.4) yields the following “Energy Conservation Equation”:
\[ \dot{\rho} + 3H (\rho + p) = 0. \] (1.3.5)

For FRW metric, Einstein’s field equations reduce to following two Friedman’s equations
\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2}, \] (1.3.6)
and
\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p),
\] (1.3.7)

where \( H \) is Hubble’s parameter that measures expansion rate of the universe and evolves according to content of the universe.

Further, for modified Einstein’s field equations (1.2.8) with a new cosmological term \( \Lambda g_{ij} \), modified Friedman equations are expressed as
\[
H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2},
\] (1.3.8)

and
\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}.
\] (1.3.9)

The above equations admit a static solution with nonnegative physical parameters \( \rho, p, \) and \( \Lambda \), for a positively curved universe and such a solution represents the Einstein static universe which was eliminated with the discovery of Hubble’s expansion. But in concordance with recent observations, \( \Lambda \) is actually nonzero, positive but small physical quantity indicating that Einstein may not have blundered at all and this will be discussed in detail in the section 1.6 of this chapter.

The critical energy density is basically the value where gravitational effect of material in the universe causes space to become geometrically flat (\( k = 0 \)) and its value may be determined from the Friedman equation (1.3.6) for \( k = 0 \), given by
\[
\rho_c = \frac{3H^2}{8\pi G}.
\] (1.3.10)

Below this density, our universe has an open, hyperbolic geometry and above this critical density, our universe exhibits closed, spherical geometry. With the help of critical energy density, we may define the total density parameter as
\[
\Omega = \frac{\rho}{\rho_c}.
\] (1.3.11)

The above density parameter is directly related to the spatial curvature in following way:

- \( k = +1 \Leftrightarrow \Omega > 1 \)
- \( k = 0 \Leftrightarrow \Omega = 1 \)
In general, the energy density $\rho$ will include contributions from different source components and relevant feature of each component may be expressed by the evolution of its energy density with expansion of the universe. To specify this feature, we, below, define concept of the equation of state:

**Perfect gas equation of state**

We, here, assume that the individual fluid components $i$ obey the equation of state of the form

$$p_i = \omega \rho_i,$$  \hspace{1cm} (1.3.12)

where $\omega$ is termed as the Equation of State (EoS) parameter and it is a constant. For different values of $\omega$, there exist following types of physically relevant models:

- When $\omega = 0$, we obtain matter dominated model or pressureless fluid (dust) model. This is the case of pure gravitation and it excludes non-linear gravitational effects from all fluid dynamical effects.
- When $\omega = \frac{1}{3}$, we obtain radiation dominated model in which relativistic particles exist in the universe.
- When $\omega = -\frac{1}{3}$, we obtain the cosmological model with curvature.
- When $\omega = -1$, we have the degenerate vacuum or false vacuum or $\rho$ vacuum model [44].
- When $\omega = 1$, the fluid distribution corresponds to an equation of state $\rho = p$, which is known as Zel’dovich fluid or a stiff fluid model [236].

Further, it is convenient to define the density parameters for different source components by

$$\Omega_i = \frac{\rho_i}{\rho_c} = \left( \frac{8\pi G}{3H^2} \right) \rho_i.$$  \hspace{1cm} (1.3.13)

The ranges for the values of $\Omega_i$’s allowed in principle (as opposed to constrained by observation) will rely on a complete theory of matter fields, but in lack of that and to get a handle on what constitutes sensible values, we may still invoke various energy conditions
irrespective of the relationship between $\rho$ and $p$. Basically, matter composing the universe exhibits a unique quality that its energy density must be (almost) always positive. The energy conditions of general relativity provide us different ways to present this quality of matter in a more precise manner [222]. Physically, these conditions in terms of $\rho$ and $p$, and possible effects from their violation, may be interpreted as follows:

- **Null Energy Condition (NEC)**: $\rho + p \geq 0$. If the NEC is violated, then it results in super acceleration of the scale factor as occurs in phantom models [188] while fulfillment of NEC indicates that dark energy can exist in the form of quintessence.

- **Weak Energy Condition (WEC)**: $\rho + p \geq 0$ and $\rho \geq 0$. WEC is a combination of NEC with an additional fact that the energy density should be positive [222].

- **Strong Energy Condition (SEC)**: $\rho + p \geq 0$ and $\rho + 3p \geq 0$. The violation of SEC arises from negative energy density or a large negative pressure [71] that may possibly occur during inflationary processes such as that in the early universe whereas its fulfillment indicates a decelerating universe.

- **Dominant Energy Condition (DEC)**: $\rho + p \geq 0$ and $|p| \geq |\rho|$. It states that either the energy density is positive and greater in magnitude than the pressure, or the energy density is negative and equal in magnitude to a compensating positive pressure. In terms of the EoS parameter $\omega$, it implies that either $\rho$ is positive and $|\omega| \leq 1$ or $\rho$ is negative and $\omega = -1$, i.e., a negative $\rho$ is allowed only if it is in the form of vacuum energy [39]. This is the most appropriate energy condition stating that energy does not flow faster than the speed of light [71]. If $\rho$ is positive, then we obtain the conventional DEC stating that $\rho \geq 0$ and $-\rho \leq p \leq \rho$ which expresses same conditions as the WEC with an additional fact that pressure should not exceed energy density of the universe.

Additionally, it is also demanding that the speed of sound ($v_s = \frac{dp}{d\rho}$) must obey the relation

$$0 \leq v_s \leq 1,$$

(1.3.14)

or

$$0 \leq \frac{dp}{d\rho} \leq 1.$$

(1.3.15)
Such an inequality is required for the local stability of matter (lower bound) and causality (upper bound), respectively.

No doubt, the standard cosmological models are unique and simple, but, to be able to interpret any observational tests for these models, it is found convenient to investigate structural consequences of some more general cosmological models than the standard FRW models. To fulfill this need, we describe spatially homogeneous cosmological models in the next section of this chapter.

1.3.2 Spatially homogeneous cosmological models

Very firstly, in this section, we give some basic information about the spatially homogeneous cosmological models. There are basically three different types of spatially homogeneous cosmological models with three different possibilities for isotropy at a general point, as discussed below:

1. **Isotropic**: It represents the class of cosmological models having same observations in different directions. In such a case, we have the mean expansion scale factor \( a \). Basically, this is the FRW family of space-time geometries as discussed in the preceding section 1.3.1.

2. **Anisotropic**: It represents the class of spatially homogeneous cosmological models in which observations are different in different directions. In this class, we have three different expansion scale factors known as the “Directional Scale Factors” \( A, B \) and \( C \) along three different directions \( x, y \) and \( z \) respectively.

3. **Locally Rotationally Symmetric (LRS)**: In such a class of spatially homogeneous models, all observations at every general point are rotationally symmetric. This differs from anisotropic cosmological models in the aspect that the observations in two particular different directions are same and different from the observation along third direction. In LRS space-time, out of \( A, B, C \), two of the directional scale factors are equal and different from the third one.

This thesis has included certain investigations into anisotropic and LRS but spatially homogeneous, cosmological models of our universe. One may have a query in mind that *why do we study anisotropic cosmological models?*. To address this issue in detail, some
important facts are summarized below:

(a): The standard models of cosmology are successful in explaining expansion of the universe from a hot big bang origin leading to the observed galactic redshifts and remnant black body radiation, associated remarkably with the element abundance predictions and observations [141]. These models can give large-scale smoothed out features of the observable physical universe but they are not suitable to describe well the real universe. In order to get a picture of realistic (‘almost-FRW’) universe, one needs to perturb standard cosmological models to examine the inhomogeneities and anisotropies arising during the structure formation. To study these issues, it is required to develop an understanding of geometry and dynamics of more general cosmological models specially the spatially homogeneous models with the property of anisotropy.

(b): The standard cosmological models are not sufficient to describe early phase evolution of our universe. Therefore, a question connected with the initial conditions of the universe naturally arise that “In what type of initial physical singularity did the universe originate?”. The approach to the singularity in FRW universe is isotropic and the universe collapses to a point. However, in anisotropic cosmological models, there are mainly four different types of singularities which are distinguished by the behavior of spatial scale factors \((A, B, C)\) as \(a \to 0\):

- **point** means that all three \(A, B, C\) tend to zero,
- **cigar** means that two of the \(A, B, C\) tend to zero and the third increases without bound,
- **barrel** means that two of the \(A, B, C\) tend to zero and the third approaches a finite value,
- **pancake** means that one of the \(A, B, C\) tends to zero and two approach finite values.

The above written names refer to change of shape of a spherical element of the cosmological fluid as the singularity is approached. These were introduced by Thorne (1967) [207] in connection with spatially homogeneous models and can be applied generally.

(c): It is interesting to notice that, a space isotropic around at least two points is always homogeneous while a homogeneous space is not necessarily isotropic. The theoretical argument by Misner [129] indicate the existence of an anisotropic universe in the past. On
observational grounds, the differential microwave radiometers on NASA’s cosmic background explorer have recorded anisotropy in the universe in various angle scales. These anisotropies in the CMB hide in their hearts the entire history of cosmic evolution down to recombination and these are also conceived to be an indication of geometry of universe and matter composing the universe. Also, modern experimental data including the Type Ia Supernovae (SNeIa) and the large scale structure indicate the existence of an anisotropic universe that has gained isotropization with passage of time.

In continuation with the above discussion, for the description of entire evolution of our universe, it is desired to choose some spatially homogeneous models which are initially anisotropic and become more isotropic or almost isotropic with passage of time, asymptotically tending to the FRW models. Bianchi models provide a generic description of homogeneous anisotropic cosmologies that play a significant role in the description of early phase evolution of the universe and they help in finding more general cosmological models than the isotropic FRW models. In recent years, Bianchi universes have been gaining an interest of observational cosmology since the WMAP data [75–77] seem to require an addition to the standard cosmological models with positive $\Lambda$ that resembles the Bianchi morphology [35, 36, 78, 86–88].

The simplest generalization of the spatially homogeneous, isotropic and flat FRW models to allow for different expansion factors in three different orthogonal directions, that we are going to investigate in this thesis, are “BIANCHI TYPE-I” cosmological models with following form of line element [73]:

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2,$$

(1.3.16)

where the metric potentials $A$, $B$ and $C$ are functions of cosmic time $t$ only. In the past, Bianchi type-I space-times have been investigated by Raychaudhuri [166], Heckmann and Schucking [72], Henneaux [74] and several others.

The other spatially homogeneous and anisotropic cosmological models that we are going to study in this thesis are “BIANCHI TYPE-II” space-times, expressed as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy + xdz)^2 + C^2 dz^2.$$  

(1.3.17)

Asseo and Sol [12] have discussed the importance of Bianchi type-II space-time.

For the anisotropic cosmological models, average expansion scale factor, average volume
scale factor, mean Hubble’s parameter are given by

\[ a = (ABC)^{\frac{1}{3}}, \quad (1.3.18) \]

\[ V = ABC, \quad (1.3.19) \]

\[ H = \frac{\dot{a}}{a} = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{1}{3} (H_1 + H_2 + H_3), \quad (1.3.20) \]

where \( H_1 = \frac{\dot{A}}{A}, \ H_2 = \frac{\dot{B}}{B}, \ H_3 = \frac{\dot{C}}{C} \) in equation (1.3.20) are directional Hubble factors in the directions of \( x, y \) and \( z \) axes respectively.

The usual definitions of dynamical scalars such as the expansion scalar \( \theta \) and the shear scalar \( \sigma \) for an anisotropic space-time are considered to be

\[ \theta = u_{ij} = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (1.3.21) \]

and

\[ \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left[ \left( \frac{\dot{A}}{A} \right)^2 + \left( \frac{\dot{B}}{B} \right)^2 + \left( \frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \theta^2, \quad (1.3.22) \]

where \( \sigma_{ij} \) is the shear tensor, given by

\[ \sigma_{ij} = u_{i;j} + \frac{1}{2} (u_{i;k} u^k u_j + u_{j;k} u^k u_i) + \frac{1}{3} \theta (g_{ij} + u_i u_j). \]

Further, the anisotropy parameter \( A_m \) that predicts anisotropy in the universe at any time \( t \), may be defined as

\[ A_m = \frac{2\sigma^2}{3H^2} = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H}{H} \right)^2. \quad (1.3.23) \]

\[ 1.4 \ \text{Accelerated expansion of the universe} \]

The discovery of cosmic acceleration is one of the most recent and remarkable developments in modern cosmology. The physical origin of cosmic acceleration and its understanding remains a deep mystery. We all know at this stage that our universe is expanding and according to GTR, if two basic constituents of the universe are matter and radiation, the expansion must slow down due to the presence of gravity. The accelerating expansion raises two possibilities, either of which would have profound implications for our understanding of the cosmos and of the laws of physics. The first possibility is that major portion (\( \approx 75\% \)) of energy density of the universe exists in the form of a smooth component with large negative pressure and according to GTR, a fluid with static or almost static
density may cause an accelerating expansion. The other possibility is that there is a break
down of GTR on cosmological scales and cosmic acceleration may arise from new gravit-
tional physics, perhaps involving extra spatial dimensions. Nowadays, the hypothetical
fluid with static density discussed under first possibility is given the name \textit{Dark Energy}.
Different candidates may be suggested for dark energy but the simplest and best candidate
is positive cosmological constant $\Lambda$ [201, 205]. Moreover, cosmological model in which
cosmological constant $\Lambda$ is assumed to be the source of dark energy, is called $\Lambda$CDM
model and more detail study about $\Lambda$ as a candidate of dark energy has been presented
under the section 1.6.

In recent times, accelerated expansion of the universe have been confirmed observation-
ally by several probes, a few of which are discussed below:

\subsection*{1.4.1 Supernovae type Ia}

SNeIa as bright, standardizable candles [112] provided direct evidence for cosmic accelera-
tion and they have strongly constrained the dark energy EoS parameter. If we talk about
the probes of cosmic acceleration, then the SNeIa are most effective and mature one.
Initially, the High-Z Supernova Team [65, 66, 170, 187, 212] and the Supernova Cos-
mology Project [142, 143, 145] started probing cosmic acceleration by collecting large
enough high redshift supernovae by carefully observing deep into small patches of sky
and they were aiming to measure the matter density parameter through the distance red-
shift relation of SNeIa. These two groups compared their high redshift sample ($z \sim 0.5$)
to the Calan-Tololo nearby sample ($z < 0.1$) and concluded that their magnitude-redshift
relation could not be described by matter dominated universe as their observed relation
favored an accelerated expansion at about half of the age of universe [145, 170]. Over the
intervening decade, evidence from the results of other supernovae surveys strengthened
the physical issue of cosmic acceleration. In 2003, Knop et al. [100] provided high qual-
ity light curves from the observations of Hubble space telescope and extended supernovae
measurements to redshift $z \simeq 1.8$ by providing strong evidence for the expected earlier de-
celerating phase of the universe [171–173]. Therefore, supernova measurements provide
a strong physical background for phase transition in the universe from past decelerating
phase to current accelerating scenario.
1.4.2 Cosmic microwave background

During the study [31, 85, 162], it was noticed that a very important probe constraining accelerating expansion of the universe is the CMB. Independent evidence for dark energy from the anisotropies of CMB [85, 162] confirmed the cosmic acceleration. Although various researchers [28, 50, 69] have submitted their contribution in study of the associated cosmological problem including the study on dark energy, but, independent evidence for CMB was first predicted by George Gamow and colleagues. In continuation of section 1.1, it is worth while to mention that the first observational evidence of the CMB was found by Penzias and Wilson in 1965 [144]. Later, the COBE satellite found temperature of the CMB to be 2.7 K with a black body spectrum. It has been also observed from the work of peer researchers that they have developed the subject by studying various cosmological parameters. Basically, analysis of temperature anisotropies in the CMB discovered by the COBE satellite [31] inaugurated a new method for constraining various cosmological parameters such as $\Omega_M$, $\Omega_\Lambda$ and many others.

1.4.3 Age of the universe

The comparison of expansion age of the universe that depends upon expansion history with the independent age estimates can also be used to probe cosmic acceleration or we can say, dark energy. In a matter dominated cosmological model, age of the universe is given by $t_0 \approx H_0^{-1}$ for low matter density, $t_0 \approx 2/3 H_0^{-1}$ for a flat universe, and has more small values for the closed universes. In a matter dominated model of the universe, all densities with $\Omega_M \lesssim 0.1$ yield shorter ages which are incompatible with constraints from galaxy clustering. The constraint on age of the universe derived from the confrontation of star models with real stars is cosmologically relevant since ages of oldest stars in globular clusters is given by $12 < t_0 < 15$ Gyr [103]. Such constraints combined with a weak constraint from dynamical measurements of the matter density, $0.2 < \Omega_M < 0.3$ reveals that a consistent age is possible if $-2 \lesssim \omega \lesssim -0.5$. Age consistency provides an additional strong evidence for large negative pressure in the form of dark energy. In a universe with dark energy, we can have $t_0 \approx 13$ Gyr and low matter density ($\Omega_M \sim 0.3$). So, one may conclude that the present accelerating models had a slower expansion in the past and therefore, models with dark energy predict a larger age than the same models without dark energy. In a flat universe, CMB anisotropy in combination with the large scale
structure measurements yield constraint on age of the universe \( t_0 = 13.8 \pm 0.2 \text{ Gyr} \) [206]. Moreover, for a flat \( \Lambda \text{CDM} \) model, the CMB anisotropies constrained age of universe as \( t_0 = 13.77 \pm 0.13 \) [102].

### 1.5 Cosmic acceleration through the deceleration parameter

A traditional measure of evolution of expansion rate of the universe is the deceleration parameter \( q \) which may be defined as

\[
q = -\frac{a \dddot{a}}{a^2} = -\left( \frac{\dot{H} + H^2}{H^2} \right). \tag{1.5.1}
\]

A positive sign of \( q \) (\( q > 0 \)) represents decelerating models of the universe whereas negative sign (particularly \( -1 \leq q < 0 \)) corresponds to accelerating phase or inflationary models of the universe. Recently, observations from the apparent magnitude-redshift data of SNeIa [45, 100, 145, 170, 172, 212] and from the CMB anisotropies [28, 50, 69] provide direct evidence to suspect that expansion of the universe is accelerating at present scenario. In the preliminary analysis, it was found that observations from the supernovae data favor recent acceleration (\( z < 0.5 \)) and past deceleration (\( z > 0.5 \)). Therefore, one may clearly observe that our universe exhibits phase transition from past decelerated phase to accelerating phase at current scenario and accordingly, the deceleration parameter must show signature flipping [5, 136, 171] between positive and negative values. Moreover, at present, one may examine the variation of cosmic acceleration, i.e., deceleration parameter with cosmic time \( t \) instead of knowing only the beginning of cosmic acceleration and present value of \( q \). Therefore, as per recent research findings by several researchers such as Pradhan and Otarod [154], Pradhan et al. [151], Amirhashchi et al. [8], Akarsu and Dereli [1], we may choose the deceleration parameter to be variable quantity and it is now defined as

\[
q = -\frac{a \dddot{a}}{a^2} = b(t) \text{ say}, \tag{1.5.2}
\]

which may be rewritten as

\[
\frac{\dddot{a}}{a} + b \frac{\dot{a}^2}{a^2} = 0. \tag{1.5.3}
\]
In order to solve equation (1.5.3), we assume $b = b(a)$. It is important to notice that one can assume $b = b(t) = b(a(t))$, as $a$ is also a time dependent function. Further, equation (1.5.3) may be rewritten as

$$\frac{\ddot{a}}{\dot{a}} + \frac{b}{a} = 0,$$  \hspace{1cm} (1.5.4)

which on integration, gives

$$\ln \dot{a} + \int \frac{b}{a} \, da = \ln \alpha,$$  \hspace{1cm} (1.5.5)

where $\ln \alpha \, (\alpha > 0)$ is a constant of integration. Taking antilog on both the sides, we have

$$\exp \left( \int \frac{b}{a} \, da \right) \dot{a} = \alpha,$$  \hspace{1cm} (1.5.6)

which on further integration, yields general solution of equation (1.5.3) with the assumption $b = b(a)$, expressed as

$$\int \left( \exp \int \frac{b}{a} \, da \right) \, da = \alpha t + \ell,$$  \hspace{1cm} (1.5.7)

where $\ell$ is an integration constant. One cannot solve the equation (1.5.7) in general as $b$ is variable. So, to solve the problem for exact solution, we have to choose $\int \frac{b}{a} \, da$ in such a manner that equation (1.5.7) is integrable. For the purpose, we may consider

$$\int \frac{b}{a} \, da = \ln f(a),$$  \hspace{1cm} (1.5.8)

which does not affect nature of generality of solution. Hence from (1.5.7) and (1.5.8), we obtain

$$\int f(a) \, da = \alpha t + \ell.$$  \hspace{1cm} (1.5.9)

No doubt the choice of $f(a)$ in above equation is arbitrary, but for the sake of physically viable models of the universe consistent with the observations, we may consider

$$f(a) = \frac{na^{n-1}}{\sqrt{1 + a^{2n}}},$$  \hspace{1cm} (1.5.10)

where $n$ is a positive constant. With the help of above chosen value for $f(a)$, equation (1.5.9) yields

$$\sinh^{-1}(a^n) = \alpha t + \ell.$$  \hspace{1cm} (1.5.11)
Without any loss of generality and for mathematical convenience, we may choose \( \ell = 0 \) and expression for the scale factor \( a \) in terms of cosmic time \( t \) is expressed as

\[
a(t) = (\sinh(\alpha t))^\frac{1}{n}.
\]  

(1.5.12)

Such type of relation (1.5.12) provides a generalization of the value of \( a(t) \) obtained by Pradhan et al. [151] in connection with the study of dark energy models in Bianchi type-VI\(_0\) space-time and the one suggested by Amirhashchi et al. [8] in connection with the study of dark energy models in FRW universe, by considering time dependent deceleration parameter \( q \).

With the help of above derived expression for the average scale factor \( a(t) \), we may express some physical parameters of the universe such as the spatial volume \( V \), the Hubble parameter \( H \) and the expansion scalar \( \theta \) as

\[
V = a^3 = (\sinh(\alpha t))^\frac{3}{n},
\]  

(1.5.13)

\[
H = \frac{\dot{a}}{a} = \frac{\alpha}{n} \coth(\alpha t),
\]  

(1.5.14)

\[
\theta = 3H = 3\frac{\alpha}{n} \coth(\alpha t).
\]  

(1.5.15)

It may be noticed from above equations (1.5.13) – (1.5.15) that at \( t = 0 \), the spatial volume scale factor \( V \) vanishes while the mean Hubble parameter \( H \) and the expansion scalar \( \theta \) are infinite, which is a big bang scenario. With expansion of the universe, as \( t \to \infty \), \( V \) diverges to \( \infty \) whereas \( H \) and \( \theta \) approach to zero. The more detail study about expansion of the universe based upon the deceleration parameter has been elaborated below:

**Expansion of the universe**

For studying expansion of the universe, it is required to elaborate the nature of time varying mean deceleration parameter \( q(t) \), which is given by

\[
q = n \left(1 - \tanh^2(\alpha t)\right) - 1.
\]  

(1.5.16)

For present age of the universe i.e. at \( t = t_0 \), \( q = q_0 \), we may obtain following relationship between the constants \( n \) and \( \alpha \):

\[
\alpha = \frac{1}{t_0} \tanh^{-1} \left[ 1 - \frac{q_0 + 1}{n} \right]^\frac{1}{2},
\]  

(1.5.17)

which is valid only for \( n > 1 + q_0 \). It may be clearly observed from equation (1.5.16) that \( q > 0 \) for \( t < \frac{1}{\alpha} \tanh^{-1} \left(1 - \frac{1}{n}\right)^\frac{1}{2} \) and \( q < 0 \) for \( t > \frac{1}{\alpha} \tanh^{-1} \left(1 - \frac{1}{n}\right)^\frac{1}{2} \). Graphically,
the behavior of $q(t)$ has been shown in Fig. 2.1 for different values of $n$ and $\alpha$ in accordance with relation (1.5.17) with $t_0 = 13.8$ GYr [206] and $q_0 = -0.73$ [49]. This figure shows that for $0 < n \leq 1$, presented models of the universe are in accelerating phase while for $n > 1$, models are evolving from past decelerating phase to accelerating phase at present scenario which is corroborated by the results from recent observations of SNeIa [45, 100, 145, 170, 172, 212] and from the CMB anisotropies [28, 50, 69] exposing that the present universe is accelerating and value of $q$ lies to some place in the range $-1 \leq q < 0$. If $n > 1$, then models of the universe exhibits phase transition from deceleration to acceleration for which the value of transition redshift $z_t$ has to be calculated. In order to derive an expression for the transition redshift, we must firstly obtain the value of $q$ in terms of redshift parameter $z$. So for the purpose, we firstly define relationship between average scale factor $a$ and redshift parameter $z$ as

$$z = -1 + \frac{a_0}{a},$$

(1.5.18)

where $a_0$ is the present value of the scale factor i.e. at $z = 0$ or at $t = t_0$.

Figure 1.1: The plot of deceleration parameter ($q$) versus cosmic time ($t$).

Using the value of average scale factor $a(t)$ from equation (1.5.12) into above relation-
ship (1.5.18), we obtain
\[ t = \frac{1}{\alpha} \sinh^{-1} \left( \frac{\sinh(\alpha t_0)}{(z+1)^n} \right). \]  
(1.5.19)

Further from equation (1.5.17), we have
\[ \sinh(\alpha t_0) = \left( \frac{n - q_0 - 1}{q_0 + 1} \right). \]  
(1.5.20)

Substituting above value of \( \sinh(\alpha t_0) \) into (1.5.19), we obtain following expression for the cosmic time \( t \) in terms of the redshift parameter \( z \):
\[ t(z) = \frac{1}{\alpha} \sinh^{-1} \sqrt{\frac{n - q_0 - 1}{(q_0 + 1)(z+1)^{2n}}}. \]  
(1.5.21)

Now, using above value of \( t(z) \) into equation (1.5.16), following expression for \( q(z) \) is obtained:
\[ q(z) = n - 1 - n \left[ \tanh \left( \sinh^{-1} \sqrt{\frac{n - 1 - q_0}{(q_0 + 1)(z+1)^{2n}}} \right) \right]^2. \]  
(1.5.22)

Such type of relation (1.5.22) provides a two-parameter \((n, q_0)\) parametrization for \( q(z) \) just like the following two-parameter linear expansions proposed in recent research findings:

- \( q(z) = q_0 + q_1 z \) [172], where \( q_0 \) is the present value of deceleration parameter and \( q_1 \) is the deviation in redshift evaluated at \( z = 0 \). If we set \( q_0 = -0.73 \) [49], then a positive transition redshift \( z_t \) may be obtained only for positive values of \( q_1 \) since \( q_0 \) is negative and dynamic transition (from deceleration to acceleration) occurs at \( q(z_t) = 0 \), or equivalently, \( z_t = -\frac{q_0}{q_1} \).

- \( q(z) = 1/2 - a/(1 + z)^b \) [229], where \( a \) and \( b \) are the constants that can be determined from current observational constraints. As \( z \to \infty \), \( q \to 1/2 \) which resembles the value of \( q \) at dark matter dominated epoch. In this case, present value of \( q \) is \( 1/2 - a \) and \( z_t = (2a)^{1/b} - 1 \).

- \( q(z) = q_0 + q_1 z (1 + z)^{-1} \) [228], where the parameter \( q_1 \) describes total correction in the distant past \((z \gg 0, q(z) = q_0 + q_1)\). Moreover, a positive \( z_t \) may be obtained for positive values of \( q_1 > |q_0| \) and dynamic transition occurs at \( z_t = -\frac{q_0}{q_0 + q_1} \).
The expression for transition redshift \( z_t \) for our presented models of the universe may be found by setting \( q = 0 \) at \( z = z_t \) in (1.5.22), given by

\[
z_t = -1 + \left( \frac{n - q_0 - 1}{(n-1)(q_0+1)} \right)^{\frac{1}{n}}.
\]  

(1.5.23)

For some particular values of \( n > 1 \) with \( q_0 = -0.73 \) [49], we have calculated values for transition redshift \( z_t \) as listed in the table given below:

<table>
<thead>
<tr>
<th>( n )</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.25</th>
<th>2.50</th>
<th>2.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_t )</td>
<td>( \approx 1.9159 )</td>
<td>( \approx 1.0886 )</td>
<td>( \approx 0.7652 )</td>
<td>( \approx 0.5910 )</td>
<td>( \approx 0.4817 )</td>
<td>( \approx 0.4066 )</td>
<td>( \approx 0.3518 )</td>
</tr>
</tbody>
</table>

The above discussed work regarding the time variation of deceleration parameter \( q \) is our basic concept with the help of which we have constructed physically viable cosmological models for Bianchi type-I and Bianchi type-II space-times. Further, we will use this physical concept in the entire part of this thesis.

### 1.6 The Cosmological constant as dark energy

The cosmological constant \( \Lambda \) was introduced by Einstein in 1917 [59] as the universal repulsion to allow static homogeneous solutions to Einstein’s equations in the presence of matter in accordance with generally accepted picture of that time and was subsequently withdrawn several times before with the discovery of expansion of the universe [79]. In the mean time, particle theorists have realized that cosmological constant can be measured as the energy density of vacuum which is the state of lowest energy and in principle, there is no reason yet for this vacuum energy to be zero. Moreover, the vacuum energy-momentum tensor can be written as \( T_{ij}^{\text{vac}} = -\rho^{\text{vac}} g_{ij} \) and vacuum can also be treated as a perfect fluid with EoS \( p^{\text{vac}} = -\rho^{\text{vac}} \). By setting \( \rho^{\text{vac}} = \rho_{\Lambda} = \Lambda / 8\pi G \) and moving the \( \Lambda g_{ij} \) term to right hand side in equation (1.2.8), one can observe that the effect of energy-momentum tensor for vacuum is equivalent to that of \( \Lambda \) which is the actual origin of \( \Lambda \) with the energy density of vacuum. In GTR, any kind of energy affects the gravitational field, which makes cosmological constant in the form of vacuum energy as a potentially crucial ingredient of the universe.
Moreover, from powerful evidences of nonzero, small but positive $\Lambda$ from the studies of supernovae, clusters of galaxies, large scale structure, and CMB, the inclusion of such remarkable parameter $\Lambda$ was confirmed and firmly established. With this invention, it is essential to formulate a theory which sets a very small nonzero value to the vacuum energy. Therefore, one can differentiate between a “true” vacuum and a “false” vacuum where a true vacuum would be the state of lowest possible energy with nonzero value and a metastable state in which $\Lambda$ might be zero corresponds to false vacuum. Such a metastable state could decay into state of lowest energy, although its life span could be much larger than current age of the universe. Such a scenario point towards time varying vacuum energy or the dynamical cosmological constant $\Lambda$.

Through recent studies, it is firmly established that the hypothetical fluid (dark energy) is tied to Einstein’s cosmological constant $\Lambda$. It is also speculated that the most plausible and most puzzling candidate of dark energy is the Vacuum Energy and the cosmological model in which $\Lambda$ is assumed to be the source of dark energy, is called Lambda Cold Dark Matter (ΛCDM) model. The remarkable feature of these models is that, density of dark energy evolves with expansion of the universe and commonly described by the EoS parameter $\omega$ relating pressure and density as $p = \omega \rho$ which provides a useful phenomenological description [213] of dark energy. As described earlier, the value of EoS parameter for a fluid like cosmological constant, is $-1$.

The idea of accelerated expansion in a matter dominated universe was quiet surprising but a cosmological model composed with a mixture of matter and cosmological constant could describe these observations very well [145, 170]. In such a universe with both matter and cosmological constant or vacuum energy, there is always a competition between the tendency of matter to cause deceleration, the tendency of $\Lambda$ to cause acceleration and ultimate fate of the universe depends upon precise amounts of these components. The physical nature of matter and cosmological constant is generally predicted with the measures of $\Omega_M$ and $\Omega_\Lambda$ respectively. Various probes of cosmic acceleration such as SNeIa, CMB etc. have constrained these density parameters but analysis of temperature anisotropies in the CMB is the best method for constraining these density parameters while the most direct way to measure cosmological constant is the Hubble diagram i.e., by determining relationship between redshifts and distances of faraway galaxies. The supernovae results are consistent with a spatially flat universe if a huge fraction of total
energy density is due to positive cosmological constant. Nevertheless, the supernovae results, observations from the CMB, and dynamical measurements of the matter density are impressively consistent and constrained $\Omega_M$ close to 0.3 and $\Omega_\Lambda$ close to 0.7. Spergel et al. [202] have constrained $\Omega_\Lambda = 0.74$ and $\Omega_M = 0.26$ as favored by three years WMAP collaboration. Recently, with the assumption that our universe is spatially flat and cosmological constant is a candidate of dark energy, Komatsu et al. [102] constrained $\Omega_\Lambda = 0.727 \pm 0.030$.

In modern cosmology, the *Cosmological Constant Problem* may be referred to as the large discrepancy between the observed value of energy density of vacuum which is very small and the value which is derived by particle physicists. Moreover, difference is of the order of $10^{120}$. Therefore, to understand the smallness of cosmological constant is one of the fundamental goal in string theory and other approaches of quantum gravity. So, a new problem that arises in this approach is to determine the right dependence of $\Lambda$ upon cosmic time $t$ which we have tried to solve out through our investigations presented in this thesis.

### 1.7 Time varying gravitational constant

The Newtonian constant of gravitation $G$ plays the role of a coupling constant between geometry and matter in Einstein’s field equations. In physical literature [4, 18, 29, 37, 60, 80, 81], many extensions of Einstein’s GTR with time varying $G$ have been proposed for the purpose either to incorporate Mach’s principle in general relativity or to achieve a possible unification of gravitation along with elementary particle physics. The possibility that the Newtonian coupling parameter $G$ may experience macroscopic space-time variations ranging from laboratory to cosmological scales has been investigated both theoretically [214–216] and observationally [31, 83, 115, 216]. Numerous modifications of general relativity to allow for a variable $G$ based on different arguments have been proposed [227]. Canuto and Narlikar [38] have shown that $G$-varying cosmology is consistent with whatsoever cosmological observations available at present.

During the investigations regarding evidences for variable $G$, it has been noticed that there are mainly three different approaches leading towards a variable constant of gravitation.
**First Approach:** First approach is known as the Dirac large number hypothesis [54, 55]. The ratio of gravitational to electrostatic force is of the order $10^{-40}$. There is no convincing explanation of why such a small dimensionless number appears in the fundamental laws of physics. Dirac pointed out that a dimensionless number of the order of $10^{-40}$ may be constructed with $G$, $h$, $c$ and Hubble’s constant $H$. It means if $H$ is not a constant due to expansion of the universe then the constant $G$ may also vary with time.

**Second Approach:** The second approach is based upon Mach’s principle, which states that in a particular region of space, inertial frame is determined by the matter distribution around that region as pointed out by Lord [120]. In this context, Jordan has introduced coupling parameter of the scalar field, to change energy conservation, as suggested by Dirac [56]. Following the conform equivalence theory, multidimensional theories of gravity are conform equivalent to the theories of usual general relativity in four dimensions with an additional scalar field. One case of this is given by Jordan’s theory [91], which, without breaking energy conservation is equivalent to the theory of C. Brans and R. Dicke of 1961 [29]. The Brans-Dicke theory follows the idea of modifying Hilbert-Einstein theory to be compatible with Mach’s principle. For this, Newton’s gravitational constant has to be variable, dependent on mass distribution in the universe, as a function of a scalar variable, coupled as a field in Lagrangian.

To incorporate Mach’s principle in the theory of Relativity, Brans-Dicke [29, 53] proposed the existence of a long range scalar field which is related to the constant of gravitation $G$.

**Third Approach:** The third approach is based upon the strong gravity theory proposed by Isham et al. [82].

In the theory proposed by Dirac [54], $G$ is a decreasing function of time given by $G \propto t^{-1}$ but Beesham [20] pointed out that such a variable $G$ cosmology does not necessarily imply creation. Also, he suggested a power law variation of $G$ as $G \propto t^s$, $s = \text{constant}$ and such a variation has been used by Kalligas et al. [93] to solve FRW cosmological models. Further, a new variation of the form $G \propto a^s$ has been proposed by Sistero [200]. Motivated from this theory, we have investigated some cosmological models presented under this thesis, with time varying $G$ of the form suggested by Sistero [200].
1.8 Role of viscosity in an expanding universe

The effects of dissipation including the most essential ones which are, bulk viscosity and shear viscosity, are supposed to play an important role during early phase evolution of the universe. With generally accepted picture of cosmological principle, it may be emphasized that nature of irreversible processes that may have occurred in the universe at an age of about one second, is scalar and involve either viscosity or chemical reactions including the creation of matter. The physical processes such as decoupling of neutrinos during the radiation era and decoupling of radiation and matter during the recombination era are expected to give birth to the viscous effects. Misner [128] pointed out that observed isotropy of the universe is due to the action of neutrino viscosity.

To create the relativistic theory of dissipative fluids with first-order deviation from equilibrium, the first attempt was made by Eckart [57] and, Landau and Lifshitz [111]. The second-order relativistic theory which is termed as transient or extended irreversible thermodynamics, was fully developed by Israel and Stewart [84] and Pavon [139]. Weinberg [225, 226] have derived the general formulae for bulk viscosity and shear viscosity and further, evaluated the cosmological entropy production and pointed out that such a viscosity mechanism can account for high entropy of the present day universe. Moreover, for a universe described by the FRW metric, Weinberg [225, 226] suggested that effect of dissipation is to replace the thermodynamic isotropic pressure $p$ with an effective pressure $P$, expressed as

$$P = p - \xi \theta,$$

(1.8.1)

where $\xi$ is the bulk viscosity coefficient and $\theta$ is the expansion scalar. Guth [68] from his inflationary cosmological model, suggested that viscous effects were present in the universe at the beginning of inflationary phase transition. Also according to the Grand Unified Theory (GUT), it is believed that viscous effects were involved during phase transition and string creation. Moreover several cosmologists [239–241] have described the particle creation in terms of effective bulk viscosity coefficient.

Murphy [132] have obtained an exactly soluble cosmological model of zero curvature Friedmann model in the presence of bulk viscosity alone. Bulk viscosity associated with the GUT (phase transitions) may lead to inflationary cosmology which is used to overcome several important problems in the standard big bang cosmology. Matzner and Mis-
ner [126] have considered dissipative processes in an anisotropic homogeneous world models, and showed that dissipation reduces the anisotropy. Gron [67] studied the inflationary cosmological models of Bianchi type-I with shear, bulk and non-linear viscosity and deduced that viscosity plays significant role in the process of isotropization of the universe. Brevik and Heen [30] have also analyzed viscous Bianchi type-I models to explore the reason behind large entropy per baryon in the universe. Belinskii and Khalatnikov [27] have investigated Bianchi type-I models with an equation of state $p = \omega \rho$, $0 \leq \omega \leq 1$ in the presence of bulk and shear viscosity and concluded that viscosity does not effect the initial big bang singularity while it may be the cause of new behavior of cosmological solutions near the singularity and Belinskii et al. [26] have analyzed Bianchi type-I cosmological models using the Israel-Stewart [84] theory of irreversible thermodynamics.

In context of viscous Bianchi type-I models, Banerjee and Sanyal [15] have obtained a constant ratio between shear and expansion, and Harko [70] discussed the effect of bulk viscosity for a Zel’dovich fluid. Mak et al. [124] have extended the study of Harko [70] for a fluid obeying the equation of state $p = \omega \rho$, $0 \leq \omega \leq 1$ in the presence of constant and time varying bulk and shear viscosity coefficients. In literature, several cosmologists [10, 11, 16, 19, 21, 51, 104, 107, 146, 191] have studied viscous fluid cosmological models in different physical context.

1.9 Presence of cosmic strings in early universe

In order to study the exact physical situation at very early stages of the evolution of our universe, it is desired to know about the origin of topologically stable objects especially the cosmic strings. It is interesting to notice that possibility of transient cosmic acceleration can also be predicted by string theory [40, 63]. It is believed that after big bang explosion and before creation of particles, our universe may have undergone a series of phase transitions as the temperature goes below some critical point related to spontaneous breaking of internal symmetry [219, 238], and this can give rise to various topologically stable defects such as domain walls, strings and monopoles [217]. Out of these cosmological structures, cosmic strings whose word sheets are two dimensional time-like surfaces [97] have been studied in more details [218]. Moreover, existence of large scale network of strings in the early universe does not contradict present day observations of the uni-
verse [97] and their presence could be explained through the GUT [61, 97, 98, 217, 238]. Further, these strings give rise to density perturbations which lead to the formation of galaxies [237] and string tension gives rise to an effective anisotropic pressure. These cosmic strings have stress-energy and they couple to gravitational field leading to an interest in studying the gravitational effects that arise from strings. The general relativistic treatment of strings was initiated by Letelier [113, 114] and Stachel [203]. Letelier [114] investigated some massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-time. The pioneering work in the formulation of energy-momentum tensor for classical massive strings was done by Letelier [113], who considered massive strings to be formed by geometric (massless) strings with particles attached along its extension. Therefore, total energy momentum tensor $T_{ij}$ for a cloud of massive strings can be expressed as:

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j,$$

(1.9.1)

where $\rho = \rho_p + \lambda$ is the rest energy density for a cloud of strings with particles attached to them with $\lambda$ being the string tension density and $\rho_p$ being the particle density of configuration. As pointed out by Letelier [114], the energy densities ($\rho$ and $\rho_p$) for the coupled system are restricted to be positive while the string tension density $\lambda$ may be positive or negative. Further, $u^i = (0, 0, 0, 1)$ is the four velocity vector for cloud of particles (the choice of $u^i$ for the cosmic strings is arbitrary and is chosen for convenience) and $x^i$ is a unit space-like vector representing direction of string especially in the direction of anisotropy. The vectors $u^i$ and $x^i$ satisfy the conditions $u_i u^i = -x_i x^i = -1, u^i x_i = 0$.

In context of Bianchi space-times, string cosmological models have been studied by Kröri et al. [106], Banerjee et al. [17], Tikekar and Patel [208], Pant and Oli [137]. Wang [223] have presented cosmological solutions for a cloud string in Bianchi type II, VIII and IX space-times. Recently, Rikhvitsky et al. [175] have studied Bianchi type-II string cosmological models with magnetic field in loop quantum cosmology and Reddy et al. [167, 168] and Naidu et al. [134] have obtained Bainchi type-II string cosmological models with bulk viscosity in alternative theories of gravitation. Moreover, Singh [189] have discussed the evolution of Bianchi type-I universe with magnetized bulk viscous fluid in string cosmology and Pradhan et al. [157] have studied LRS Bianchi type-II massive string cosmological model in general relativity.
1.10 Alternative theories of gravitation

The GTR by Einstein is the best one to describe gravitational phenomena and it is the sophisticated theory of gravitation. Further, GTR is basic theory for the construction of cosmological models of the universe. Further, cosmological models based on the cosmological principle of general relativity displays a picture of observed large scale properties of the universe. However, some of the desirable features lacking in the theory need to be satisfied. One of such important lacking feature is the violation of Mach’s principle pointed out by Einstein himself. Therefore, after publication of Einstein’s theory, to incorporate the phenomena that lack behind, many modifications to the GTR have been proposed from time to time in physical literature.

The alternative theories of gravitation, especially scalar-tensor theories proposed by Brans and Dicke [29], Barber [18], Saez and Ballester [176] etc. are of considerable interest. The latest inflationary models by Mathiazhagan and Johri [125], extended inflation models by La and Steinhardt [110] and extended chaotic inflation models by Linde [118], are based on these scalar-tensor theories of gravitation. Brans and Dicke [29] scalar-tensor theory of gravitation introduces an additional scalar field $\phi$ beside the metric tensor and a dimensionless coupling constant $w$. This theory tends to general relativity for large value of coupling constant ($w > 500$). Saez and Ballester [176] have formulated a scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of scalar field, an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

Recently, several researchers such as Reddy et al. [169] Rao et al. [163–165], Kumar and Singh [109], Pradhan et al. [159], Naidu et al. [133, 134], Samanta et al. [185] have investigated the evolution of Bianchi type-I and Bianchi type-II space-times in Saez-Ballester theory. Motivated from literature, in this thesis, we have also presented our investigations regarding Bianchi type-I and Bianchi type-II space-times in Saez-Ballester theory of gravitation.

After discussing all the necessary physical aspects from literature, we may say that, importance of the subject is self explaining and fascinating for human to know about the cosmos and cosmology. Under this motivation, we have studied “Certain Aspects of the Expansion of Bianchi I and II Universes with Time Varying Cosmological Constant ‘$\Lambda$’
and Gravitational Constant ‘G’ in the present thesis and detail outcome has been presented in all the foregoing chapters.