CHAPTER-4

INVENTORY MODEL WITH TWO WAREHOUSES FOR THE DECAYING PRODUCTS HAVING CONSTANT DEMAND

4.1 Introduction

In relation to uncertainties, time also plays a role. Time lags which are present in the supply chain can be intercepted by maintaining stock. A certain amount needs to be kept in stock, to use during this ‘lead time’. When something is ordered, it usually takes a while before the goods are actually delivered; during this period the production cannot stand still and therefore the stock will function as a buffer to overcome this period. Time lags in deliveries can lead to very large fluctuations and are exaggerated down the supply chain. Finally it may sometimes be cheaper to keep some stock. Economies of scale for example are a reason why inventories are kept. Buying bigger quantities is often more beneficial than ordering small amounts, due to the related discounts. Additionally ordering one unit at a time that has to be delivered to a specific place every time the user needs it, requires more logistic movements and accordingly raises high costs as well. Also fluctuating prices may form a reason to keep a stock: buying a product at a low price can provide a benefit. That is off course when the total costs of keeping additional goods in stock is cost-
efficient compared to buying at a higher price, otherwise high stocking costs will immediately diminish the intended profit.

The issue of two warehouse inventory has received considerable attention in recent years. Taking into consideration of the relative higher ordering or set up cost, manager usually finds it is economically beneficial by outsourcing warehouse space in addition to the own warehouse. In two warehouse system, it is assumed that the retailer owns a storehouse with predetermined facility and any amount exceeding this should be stocked in warehouse on rent basis which has quite a large space available.

Under that assumption the holding cost for the rented warehouse (RW) is greater than that of OW, customer are served first from RW then from OW.

An organization can accumulate the same stock in different ways to come up with different outcome, ranging as far as even profit and loss. So, storage of stash ultimately turns out to be the essence of profit of the whole inventory managing practice. Warehousing progressions are very important since they provide the increased speed of material flow in the modern concept of logistics management. Warehousing plays a very essential role in all transfers of products from the place of origin to eventual users.
Hartely (1976) initially focused and concentrated on the problem of two warehouses.

Sarma (1983) further extended Hartely by assuming unbounded production rate and having two level of storage space.

Murdesher and Sathi (1985) further extended the above model by considered bulk release model in which inventory in hired storehouse must first be transferred to owner’s storehouse before its release to the customer.

Pakkala and Achary (1992) Model having shortages and allowing replenishment rate as finite.

Pakkala and Achary (1994) considered bulk release model in two warehouse system.

Bhunia (1998) considered a deterministic EOQ model having various levels of deterioration in the concerned warehouse.

Lee and Ma (2000) proposed the two-warehouse models when demand is a function of time either with or without the consideration of deterioration.

Kar (2001) discussed a two warehouse model. It has EOQ replenishment cost. The model has finite time period.

Yang (2004) described the two storehouse model for worsening and decaying substance with inflation.
Zhou and Yang (2005) discussed model for two stockroom having demand rate which is dependent and the stock itself.

Yang (2006) further extended and enriched the model of two warehousing by allowing partial backlogging. The approach used is for minimum costing.

Singh (2008) discussed a model of two warehouses. The model have increasing demand and rate of deterioration is constant. The supply of RW stockroom is used initially and then stock of owner’s storehouse is utilized only after the stock in RW becomes zero.

Lee and Hsu (2009) developed two-warehouse modeling having time dependent rate of demand.

Tripathy, C.K. & Mishra, U. (2010) formulated a deterministic model used for the substance having deterioration. In this model demand remains constant. Shortages were allowed rate in their study.

Kumar et al. (2012) discussed a supply chain inventory model under inflationary environment having very limited facility for storing goods for decaying items.

Gupta and Singh (2013) described an all inclusive inventory model with Weibull deterioration and variable holding cost. When new
goods are delivered to the location where the stock is kept (i.e. a warehouse mostly), numerous checks will be there. For instance, proper matching of purchase order and the quantity of goods delivered. In ERP there exists a link to information from the purchase department at this point. The quality check and control is also very important before the goods are put to stock. In some special cases the delivered goods even have to stay in quarantine for a while before the quality is being checked. Because the quarantine process is optional, this block is dotted in the picture. Additionally the completeness of the order is often checked and the delivery date is registered. This information is used to keep track of the reliability of suppliers. Additionally, partial deliveries can be monitored using this information. After complete satisfaction of checking they are added and put to the stock. Again information is stored; at least the added number of materials or the amount of material is registered (in ERP for example). Additionally the location where the goods are stored is registered and eventually special characteristics are also registered like value, size or best-before dates for instance. The registered information from storage is linked back to forecast and used to do new forecasts with: based on the present amount of stock it is decided how many new articles should be ordered for instance.
When stock is stored, stock movements actually form the most important activity at this point. Stock movements may be required for several reasons. Goods are needed at another location for example. Each movement is registered and the location is updated in the system. decayed or damaged goods also cause the amount of goods held in stock to mutate; this mutation also has to be registered. Usually the aim is to keep entire stock costs as low as possible. Stock mutations (movement or writing off) cost money and therefore not only the stock levels should be kept low, but also the number of movements as well.

Inventory control can be used to real-time monitor which amount (quantity and value) of a certain material is in stock and at what location(s) it is stored. Also using the information extracted from Inventory control it is possible to analyze which goods have a high cycle time and which products lay ‘still’ in stock and are thus not being used but do cost money/consume space and form a risk. For cheaper items this is no problem, because they represent a smaller value, but for expensive parts this becomes very interesting. Materials that are unnecessary being kept in stock can also be traced using the information stored. Finally Inventory control is used to optimize the safety levels.

Two warehouse model for deteriorating products have been formulated. In this model the demand rate is kept constant. Weibull distribution
having two parameters is used for deterioration rate. Shortages are allowed with complete backlogging. Infinite planning horizon consider in this model. Mathematical example is solved for enhanced presentation of the overall system. I have further used the sensitivity analysis technique for the support of the system. I believe that it would help in further study including this sort of important.

4.2 Assumptions and Notations

4.2.1 Assumptions

The mathematical representation of the inventory replacement problem is based on the following assumptions:

- Deterioration rate is taken as two parameter Weibull distribution.
- Constant demand rate has been taken.
- Shortages are permitted with absolute backlogging.
- Unlimited planning phase is considered.
- During the cycle there shall be no repair of decayed units.
- I have considered single item stock in a given period.
- Zero lead time will be there.

4.2.2 Basic Notations

I have used the following notations:

W: Capacity of the OW
$\alpha_1 \beta_1 t_1^{\beta -1}$: Weibull distribution deterioration where $\alpha_1$, $\alpha_2$ and $\beta_1$, $\beta_2$ are scale parameter and shape parameter respectively and $\alpha_1, \alpha_2 \geq \beta_1, \beta_2$.

$C_{h1}$: In OW, Holding cost per unit time.

$C_{h2}$: In RW, Holding cost per unit time.

$C_D$: Procurement cost.

$C_S$: Shortage Cost.

$C_0$: Replenishment cost.

$C_1$: Set up cost

$t_r$: The time at which the inventory level reaches zero in RW

$t_0$: The time at which the inventory level reaches zero in OW

$t_s$: The time at which the shortage reaches the lowest point in the replenishment cycle

$I_1(t)$: Level of inventory, at time t, in RW.

$I_2(t), I_3(t), I_4(t)$: Level of inventory, at time t, in OW.

$I_5(t)$: Negative inventory level at time t.

$TC_i$: Present value of total cost $i=1,2$.

4.3 Model Formulation and Solution

Case 1: When $\eta_1 < \mu_1$

We discussed the model which is deterministic in nature for deteriorating matter with the conventional two warehouse representation where
shortage arise at the end of the phase at time \( t=0 \), the earlier shortages are backlogged as certain lot size stock gets into the system. Certain quantities are stored in owner’s storehouse and the outstanding commodities are kept in the rented one. First of all, goods in RW are consumed due to cost involved in it. Only after that the goods stored in OW is utilized. For the duration from \( O \) to \( tr \), the stock in RW decreases in a gradual manner according to demand. But in OW, for the period of \((tr,\mu_1)\), due to worsening and also due to demand the inventory shows depletion. Due to constant demand and there is deterioration as well, stock become depleted during time interval of \((\mu_1 \ t_0)\). At time \( to \), both the warehouses that OW and RW become empty and exhausted. At this point in time, shortages can be allowed and there can be complete backlogging. Now, before the next cycle, the quantity which is in shortage is supplied to the concerned and valued customers. At \( ts \), next replacement phase starts.

The primary purpose of this representation is to minimize the various costs associated with this inventory system. That is the holding cost, shortage cost, ordering costs and deterioration cost.
Now, I can mathematically represent the system as follows:

\[ I_1'(t) = -D_1, \quad 0 \leq t \leq t_r \text{ or } 0 \leq t \leq \eta_1 \]

\[ I_2(t) = W, \quad 0 \leq t \leq t_r \text{ or } 0 \leq t \leq \eta_1 \]

\[ I_3'(t) + \alpha_1 \beta t^{\beta - 1} I_3(t) = -D_1, \quad t_r \leq t \leq \mu_1 \text{ or } \eta_1 \leq t \leq \mu_1 \]

\[ I_4'(t) + \alpha_1 \beta t^{\beta - 1} I_4(t) = -D_1, \quad \mu_1 \leq t \leq t_0 \]

\[ I_5'(t) = -D_1, \quad t_0 \leq t \leq t_s \]

With the boundary conditions \( I_1(t_r) = 0, \ I_3(t_r) = W, \ I_4(t_0) = 0, \ I_5(t_0) = 0, \)

one can arrive the following equations

\[ I_1(t) = D_1(t_r - t), \quad 0 \leq t \leq t_r \text{ or } 0 \leq t \leq \eta_1 \]

\[ I_2(t) = W, \quad 0 \leq t \leq t_r \text{ or } 0 \leq t \leq \eta_1 \]

\[ I_3(t) = W e^{\alpha_1 (t_r - t)} + \left[ D_1(t_r - t) + \alpha_1 D_1 \left\{ \frac{1}{\beta_1 + 1} (t_r^{\beta_1 + 1} - t^{\beta_1 + 1}) - t^{\beta_1} (t_r - t) \right\} \right] \]
\[ t_r \leq t \leq \mu_1 \text{ or } \eta_1 \leq t \leq \mu_1 \]

\[ I_4(t) = D_1 \left[ (t_0 - t) + \frac{\alpha_i}{(\beta_i + 1)} \left\{ t_0^{\beta_i+1} - t^{\beta_i+1} \right\} - \alpha_i t^{\beta_i} (t_0 - t) \right] , \]

\[ \mu_1 \leq t \leq t_0 \]

\[ I_5(t) = D_1 (t_0 - t) , \quad t_0 \leq t \leq t_s \]

**Present worth Ordering Cost**

The current worth of ordering cost per cycle:

\[ C = C_0 \]

**Present worth Holding Cost**

\[ HC_{rw} = C_{h2} \left[ \int_0^{t_r} I_1(t) dt \right] \]

\[ = C_{h2} \frac{D_1}{2} t_r^2 \]

**Present worth Holding Cost**

\[ HC_{ow} = C_{h1} \left[ \int_0^{t_r} I_2(t) dt + \int_{t_r}^{\mu_1} I_3(t) dt + \int_{\mu_1}^{\eta_1} I_4(t) dt \right] \]

\[ = C_{h1} \left[ W t_r + W (\mu_i - t_r) + W \alpha_i t_r^{\beta_i} (\mu_i - t_r) - \frac{W \alpha_i}{(\beta_i + 1)} \left\{ \mu_i^{\beta_i+1} - t_r^{\beta_i+1} \right\} \right] \]

\[ + D_1 \left\{ \mu_i t_r - \frac{\mu_i^2}{2} - \frac{t_r^2}{2} \right\} + \frac{\alpha_i D_1}{(\beta_i + 1)} t_r^{\beta_i+1} (\mu_i - t_r) \]

\[ - \frac{\alpha_i D_1}{(\beta_i + 1)(\beta_i + 2)} \left\{ \mu_i^{\beta_i+2} - t_r^{\beta_i+2} \right\} - \alpha_i D_1 \left\{ (t_r - \mu_i) \frac{\mu_i^{\beta_i+1}}{(\beta_i + 1)} \right\} \]
\[ + \frac{\mu_t^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{t_0^{\beta+2}}{(\beta+1)(\beta+2)} + D_1 \left\{ \frac{t_0^2}{2} - t_0 \mu_t + \frac{\mu_t^2}{2} \right\} + \frac{\alpha_t}{(\beta+1)} t_0^{\beta+1} (t_0 - \mu_t) - \frac{\alpha_t}{(\beta+1)(\beta+2)} \left\{ t_0^{\beta+2} - \mu_t^{\beta+2} \right\} + \frac{\alpha_t}{(\beta+1)} (t_0 - \mu_t)(\mu_t - t_r)^{\beta+1} - \frac{\alpha_t}{(\beta+1)(\beta+2)} \left\{ t_0^{\beta+2} - \mu_t^{\beta+2} \right\} \]

Deterioration Cost

The value of the deterioration cost per cycle can be derived as

\[ DC = C_0 \left[ \int_{t_r}^{t} \alpha_t \beta_t^{\beta-1} I_s(t) dt + \int_{t_r}^{t} \alpha_t \beta_t^{\beta-1} I_s(t) dt \right] \]

\[ = \alpha_t \beta_t^{\beta-1} \left[ \frac{W}{\beta_t} \left( \mu_t^{\beta} - t_r^{\beta} \right) + \frac{W \alpha_t}{\beta} (t_r - \gamma) \left( \mu_t^{\beta} - t_r^{\beta} \right) \right. \]

\[ - \frac{W \alpha_t}{2 \beta_t} \left( \mu_t^{2\beta} - t_r^{2\beta} \right) + \frac{D_1}{\beta_t} (t_r - \mu_t) \mu_t^{\beta} + \frac{D_1}{\beta_t} (\mu_t^{\beta+1} - t_r^{\beta+1}) \]

\[ + \frac{\alpha_t D_1}{\beta_t (\beta+1)} t_r^{\beta+1} (\mu_t^{\beta} - t_r^{\beta}) - \frac{\alpha_t D_1}{\beta_t (\beta+1)(2\beta+1)} (\mu_t^{2\beta+1} - t_r^{2\beta+1}) \]

\[ - \frac{\alpha_t D_1}{2 \beta_t} (t_r - \mu_t) \mu_t^{2\beta} - \frac{\alpha_t D_1}{2 \beta_t (2\beta+1)} (\mu_t^{2\beta+1} - t_r^{2\beta+1}) \]

\[ + D_1 \left\{ \mu_t - t_0 \frac{\mu_t^{\beta}}{\beta_t} + \frac{t_0^{\beta+1}}{\beta_t (\beta+1)} - \frac{\mu_t^{\beta+1}}{\beta_t (\beta+1)} + \frac{\alpha_t}{(\beta+1)(2\beta+1)} \right\} \]

\[ \left[ t_0^{\beta} - \mu_t^{\beta} \right] + \frac{\alpha_t}{2 \beta_t} (t_0 - \mu_t) \mu_t^{2\beta} - \frac{\alpha_t}{(\beta+1)(2\beta+1)} \left[ t_0^{2\beta+1} - \mu_t^{2\beta+1} \right] \]
Present worth Shortage Cost

\[ SC = C_s \left[ D_1 \left( \frac{t_r^2}{2} - t_0 t_s + \frac{t_s^2}{2} \right) \right] \]

Present worth Total Cost

\[ TC_1 = \frac{1}{t_s} \left[ C + HC_{ow} + HC_{rw} + DC + SC \right] \]

To reduce whole average cost per unit time \((TC_1)\). We get the most favorable values of \(t_r\), \(t_0\) and \(t_s\) by solving the subsequent equations simultaneously from the equations

\[
\frac{\partial TC_1}{\partial t_r} = 0, \quad \frac{\partial TC_1}{\partial t_0} = 0, \quad \frac{\partial TC_1}{\partial t_s} = 0,
\]

If the following conditions are satisfied.

\[
\frac{\partial^2 TC_1}{\partial t_r^2} > 0, \quad \frac{\partial^2 TC_1}{\partial t_0^2} > 0, \text{and} \quad \frac{\partial^2 TC_1}{\partial t_s^2} > 0
\]

Case 2: When \(\eta_1 = \mu_1\)

The inventory level in RW decreases in a gradual manner during \((0, t_r)\) depending on the demand. It vanishes at time \(t_r\). In owned warehousing the inventory level \(W\) remains the same.
The afterwards \((t_r, \mu_1)\), the stock become depleted because of demand. During the time interval \((\mu_1, t_0)\) the inventory level because of the demand which is constant and also due to deterioration. At both the warehouse, that is, RW and OW because fully depleted and becomes empty. Now the shortages occur. These shortages can be completely backlogged. The customer get the shortage quantity before the next cycle.

![Graphical representation of a two warehouse inventory system for model 1 (Case 2, \eta_l = \mu_1)](image)

The model can be represented as under:

\[ I_1'(t) = -D_1, \quad 0 \leq t \leq t_r \]

\[ I_2(t) = W, \quad 0 \leq t \leq t_r \]
\[ I_3'(t) = -D_1, \quad t_r \leq t \leq \mu_1 \text{ or } t_r \leq t \leq \eta_1 \]
\[ I_4'(t) + \alpha_1 \beta_1 t^{\beta_1-1} I_4(t) = -D_1, \quad \mu_1 \leq t \leq t_0 \text{ or } \eta_1 \leq t \leq t_0 \]
\[ I_5'(t) = -D_1, \quad t_0 \leq t \leq t_s \]

With the boundary conditions \( I_1(t_r) = 0, \quad I_3(t_r) = W, \quad I_4(t_0) = 0, \quad I_5(t_0) = 0 \), one can arrived the following equations

\[ I_1(t) = D_1 (t_r - t), \quad 0 \leq t \leq t_r \]
\[ I_2(t) = W, \quad 0 \leq t \leq t_r \]
\[ I_3(t) = W + D_1 (t_r - t) \quad t_r \leq t \leq \mu_1 \text{ or } t_r \leq t \leq \eta_1 \]
\[ I_4(t) = D_1 \left[ (t_0 - t) + \frac{\alpha_1}{(\beta_1 + 1)} \left( t_0^{\beta_1+1} - t^{\beta_1+1} \right) - \alpha_1 t^{\beta_1} \right], \quad \mu_1 \leq t \leq t_0 \text{ or } \eta_1 \leq t \leq t_0 \]
\[ I_5(t) = D_1 (t_0 - t), \quad t_0 \leq t \leq t_s \]

Holding Cost in RW,
\[ HC_{RW} = C_{h2} \left[ \int_0^{t_r} I_1(t)dt \right] \]
\[ = C_{h2} \left[ \frac{D_1 t_r^2}{2} \right] \]

Holding Cost in OW,
\[ HC_{OW} = C_{h1} \left[ \int_0^{t_r} I_1(t)dt + \int_{t_r}^{t_0} I_1(t)dt + \int_{t_0}^{t_0} I_4(t)dt \right] \]
\[
\begin{align*}
&= C_h \left[ W_f + W(\mu - t_f) + D_i \left\{ \mu t_f - \frac{\mu^2}{2} - \frac{t_f^2}{2} \right\} \right] \\
&+ D_i \left\{ \frac{t_0^2}{2} - t_0 \mu + \frac{\mu^2}{2} \right\} + \alpha_1 \frac{\alpha_1}{(\beta + 1)} t_0^{\beta+1} \left( t_0 - \mu \right) \\
&- \frac{\alpha_1}{(\beta + 1)(\beta + 2)} \left( t_0^{\beta+2} - \mu_1^{\beta+2} \right) + \frac{\alpha_1}{(\beta + 1)} \left( t_0 - \mu_1 \right) \mu_1^{\beta+1} \\
&- \frac{\alpha_1}{(\beta + 1)(\beta + 2)} \left( t_0^{\beta+2} - \mu_1^{\beta+2} \right) \right]\end{align*}
\]

Present worth Deterioration Cost

\[
DC = C_\rho \left[ \int_{t_0}^{t_f} \alpha_1 \beta_1 \beta_1^{\beta-1} I_s(t) dt \right]
\]

\[
= C_\rho \alpha_1 \beta_1 \beta_1 \left[ (\mu - t_0) \frac{\mu_1}{\beta_1} + \frac{1}{(\beta_1 + 1)} \left( t_0^{\beta+1} - \mu_1^{\beta+1} \right) \right] \\
- \frac{\alpha_1}{\beta_1 (\beta_1 + 1)} t_0^{\beta+1} \left( t_0^{\beta} - \mu_1^{\beta} \right) - \frac{\alpha_1}{(\beta_1 + 1)(2 \beta_1 + 1)} \\
\left\{ t_0^{2\beta+1} - \mu_1^{2\beta+1} \right\} + \frac{\alpha_1}{2 \beta_1} \left( t_0 - \mu_1 \right) \mu_1^{2\beta} - \frac{\alpha_1}{2 \beta_1 (2 \beta_1 + 1)} \left\{ t_0^{2\beta+1} - \mu_1^{2\beta+1} \right\} \right] 
\]

Present worth Shortage Cost

\[
SC = C_S \left[ \int_{t_0}^{t_f} \left[ I_s(t) \right] dt \right]
\]

\[
= C_S \left[ D_i \left( \frac{t_0^2}{2} - t_0 t_f + \frac{t_f^2}{2} \right) \right]
\]
Present worth Total Cost

The total cost per unit time for Case-2 is

\[ TC_2 = \frac{1}{t_s} \left[ C + HC_{ow} + HC_{rw} + DC + SC \right] \]

In order to make \( (TC_i) \), \( i=1,2 \) minimum. The optimal value of \( t_r, t_0 \) and \( t_s \) are obtained from the following equations.

\[ \frac{\partial TC_i}{\partial t_r} = 0, \quad \frac{\partial TC_i}{\partial t_0} = 0, \quad \frac{\partial TC_i}{\partial t_s} = 0, \]

If the conditions as following are satisfied

\[ \frac{\partial^2 TC_i}{\partial t_r^2} > 0, \frac{\partial^2 TC_i}{\partial t_0^2} > 0 \text{and} \frac{\partial^2 TC_i}{\partial t_s^2} > 0 \]

I have used classical traditional optimization techniques to calculate minimum value of total cost.

4.4 Numerical Illustrations and Analysis

\( W = 650 \text{ units}, C_0 = $150 \text{ per setup}, C_1 = $170 \text{ per setup}, C_{h1} = $0.2, C_{h2} = $0.4, C_D = $0.45 \text{ per unit}, C_S = $2/ \text{unit/unit time}, a_1 = 160, \alpha_1 = 0.03, \beta_1 = 2, D_1 = 700 \text{ units}, \mu_1 = 10. \)

Table 4.1: Optimal Solution for Case 1: \( \eta_1 < \mu_1 \)

<p>| N | ( t_r ) | ( t_0 ) | ( t_s ) | ( TC_1 ) |</p>
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Variation in $tr$, $t_0$, $ts$
Variation in $TC_1$
Table 4.2: Optimal Solution for Case 2: η₁ = µ₁

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<td>8.4583</td>
<td>13.2272</td>
<td>185.11</td>
</tr>
<tr>
<td>9</td>
<td>6.1682</td>
<td>8.4584</td>
<td>13.2273</td>
<td>185.08</td>
</tr>
<tr>
<td>10</td>
<td>6.1682</td>
<td>8.4585</td>
<td>13.2274</td>
<td>185.05</td>
</tr>
</tbody>
</table>
4.5 Sensitivity Analysis

4.5.1 Model 1:

Table 4.3: Case 1 (When $\eta_1 < \mu_1$)

<table>
<thead>
<tr>
<th></th>
<th>-20%</th>
<th>-10%</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of the parameter ‘$\alpha_1$’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>267.52</td>
<td>267.43</td>
<td>267.31</td>
<td>267.26</td>
</tr>
<tr>
<td>Effect of the parameter ‘W’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>269.68</td>
<td>268.33</td>
<td>267.04</td>
<td>266.79</td>
</tr>
<tr>
<td>Effect of the parameter ‘$D_1$’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>261.21</td>
<td>265.33</td>
<td>269.36</td>
<td>272.15</td>
</tr>
<tr>
<td>Effect of the parameter ‘$\beta_1$’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>267.40</td>
<td>267.39</td>
<td>267.36</td>
<td>267.35</td>
</tr>
<tr>
<td>Effect of the parameter ‘$C_{h1}$’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>267.30</td>
<td>267.34</td>
<td>267.40</td>
<td>267.43</td>
</tr>
</tbody>
</table>
Variation in TC w.r.t. 'β₁'

Variation in TC w.r.t. 'C_{h1}'
Table 4.4: Case 2 (When \( \eta_1 = \mu_1 \))

<table>
<thead>
<tr>
<th>%</th>
<th>-20%</th>
<th>-10%</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>180.60</td>
<td>182.29</td>
<td>187.34</td>
<td>189.78</td>
</tr>
</tbody>
</table>

Effect of the parameter ‘\( \alpha_1 \)’

| TC | 181.28| 183.01| 188.19| 192.37|

Effect of the parameter ‘W’

| TC | 176.33| 181.27| 189.20| 194.63|

Effect of the parameter ‘\( D_1 \)’

| TC | 185.34| 185.30| 185.21| 185.19|

Effect of the parameter ‘\( \beta_1 \)’

| TC | 184.68| 185.10| 185.59| 186.34|

Effect of the parameter ‘\( C_{h1} \)’

<table>
<thead>
<tr>
<th>Variation in TC w.r.t. ‘( \alpha_1 )’</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
</tr>
<tr>
<td>-30% -20% -10% 0% 10% 20% 30%</td>
</tr>
<tr>
<td>180 182 184 186 188 190 192</td>
</tr>
</tbody>
</table>

Variation in TC w.r.t. ‘\( \alpha_1 \)’
Variation in TC w.r.t. 'W'

Variation in TC w.r.t. 'D_1'
Variation in TC w.r.t. 'β₁'

Variation in TC w.r.t. 'Cₜₙ₁'
4.6 Conclusion

The most determining factor is the corporate strategy. First the corporate strategy is set by the management. Accordingly the choices made in inventory management have to fit within the corporate strategy to reach those goals. The pursued service level is an example of the choices that are made by the management. In some cases this is also partly determined by what the market demands. Inventory management is used as a means to achieve those goals. Operational excellence, companies following this strategy aim to offer good quality products against the lowest possible prices. Dell computers, for example is following this strategy. Customer intimacy; firms operating according to this principle, constantly adjust their products to meet the requirements of their clients. These types of firms try to build up a good relation with their clients and aim to have more than just one transaction with a client. High service levels are often an important (sub) goal within this strategy. Product leadership; is a strategy aimed at innovation. Enterprises following this tactic try to stand out due to their new and innovative products; Apple is a good example of this principle. In this study, we have endeavored to develop a two-warehouse inventory system with a very realistic and practical deterioration rate.
This deterioration rate is suitable for items with and without life-period. The two storehouse stock problem is an captivating yet practical topic of decision making and scientific judgement. The two-warehouse modeling is beneficial in many practical situations. Due to liberalization and globalization there is possibility of very high cut throat business competition.

It helps to occupy maximum possible market share. It also helps in strategic decision-making by top management for efficient working of departmental stores. So to store items which in excess and needs storage has to be stored at a separate warehouse which is rented and hired at a different place. Complete backlogged shortages are permitted in this study. Finally the total cost minimization techniques are followed. The numerical example is solved using MATHEMATICA 8.0. The method of sensitivity analysis is carried out to check the stability of the system. The graphical representation is also performed.
REFERENCES


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