CHAPTER I
This thesis deals with some problems of geometric theory of relativity in four and higher dimensions. It was Einstein's great insight that gravity as universal force could be described by a curvature of space time consisting of our time and three spatial dimensions that has led him to formulate the famous Einstein field equations of general relativity. The great mathematician Albert Einstein introduced this theory by extending the special theory of relativity to the non-inertial frame. He believes that all the physical laws in nature are invariant relative to any coordinate transformation i.e., for inertial as well as non inertial systems also. This results the general theory of relativity. Up to the end of the nineteenth century, space and time were brought to be distinct concepts. Albert Einstein unified them into a concept of four-dimensional space-time when he developed the special theory of relativity in 1905. Einstein proceeded further and unified space-time geometry and gravitation in 1915 in his general theory of relativity. According to this theory of Einstein, gravitation is but a manifestation of curvature of space time. For example, the curved space-time due to the Sun's gravity guides the motions of planets round it. After the formulation of general relativistic theory of gravitation, Einstein had made a sustained effort to unify gravity and electromagnetism for the rest of his life till 1955 within the frame of four-dimensional space-time. But he did not succeed. It is thought by many scientists that Einstein failed because he started this time from wrong premises.
The first quarter of the twentieth century saw the birth of quantum mechanics which unified the theories of photons (particles like electrons, protons, atoms, molecules etc. And this happened also within the frame of four-dimensional space-time.

Gravity is particularly observable when objects possess large masses. The electromagnetic force holds together the nucleus and the electrons in an atom. Besides gravity and the electromagnetic force, there are two nuclear forces – the strong force and the weak force. In the nucleus of an atom, there are protons and neutrons. The protons are positively charged and repel one another. Therefore the protons and neutrons cannot hold themselves together in a nucleus. The strong force is responsible for binding together the protons and neutrons in a nucleus. The strong force is responsible for binding together the protons and neutrons in a nucleus. Heavy nuclei, such as uranium are unstable. They automatically break apart, releasing small fragments and debris. We call this phenomenon radioactivity. The weak force is responsible for this phenomenon of radioactivity.

In the late sixties of the twentieth century Abdus Salam and Steven Weinberg worked out the unification of electromagnetic force and the weak force. This is called electro-weak theory. Thereafter, the strong force was unified with the electro-weak theory leading to what is known as GUT, i.e., the General Unified Theory. All this was accomplished within the frame of four-dimensional space-time. Unfortunately, gravity defied unification with the other three forces of nature.

Prior to Einstein, there were two mathematicians who attempted to unify gravity and electromagnetism. The first unification, based on a generalisation of Riemannian geometry in the usual four space time dimensions, was proposed by Herman Weyl in
1918. Inspired by Weyl’s work, another mathematician, Theodor Kaluza was the first to suggest that unification might be achieved by extending space time to a five dimensional manifold. Kaluza did the work in 1921. In recent years, growing interest in higher-dimensional general relativity and black hole solutions within this theory has been influenced by several fields including string theory which contains general relativity and consistency of which requires an appropriate number of dimensions. The present day universe in which we live in is four-dimensional. Recently attempts are being made by researchers to develop the theory of relativity in a higher-dimensional space-time. Now-a-days the studies in cosmology are specially attracting researchers to investigate the origin of the accelerated expansion of the present universe. Astronomical observations tells us that the present day universe has the critical energy density containing the about 70% dark energy, 25% dark matter and 5% baryonic matter and radiation, responsible for cosmic acceleration [1].

The study of higher-dimensional physics is important because of several important results obtained in the development of Super-string Theory. Weinberg [2] studied the unification of the fundamental forces with gravity, which reveals that the space-time should be different from four. Since the concept of higher dimensions is important, the string theories are discussed in 10 dimensional space time. The success of Yang-Mills super-gravity in ten dimensional space time in its field theory limit has given the studies in higher dimensional physics a new degree of importance. Since the finest of the instruments we have developed so far cannot visualize space time having dimensions higher than four. The remaining dimensions are coiled up to a size of $10^{-33}$ cm as in any Kaluza-Klein type theories [3, 4]. Kaluza and Klein gave their best efforts for the unification of gravity with the electromagnetic interaction. For this purpose, they
framed the higher-dimensional model of space time. Kaluza and Klein suggested a five-dimensional model of space time. The introduction of an extra dimension is basically an extension of Einstein relativity in 5D. The functioning of extra dimension also verified from the STM Theory proposed by Wesson et al [5]. There are dimensions of space time higher than five. Since the present day universe is four-dimensional, the extra dimensions are hidden in nature and must be related to the dark matter and dark energy which are not observable. There are several models regarding the nature of dark energy such as tachyon [6], phantom [7], k-essence [8], etc. The importance of extra dimensions in cosmology has been discussed by many authors. Panigrahi and Chatterjee [9] have found that the inflationary scenario is possible for inhomogeneous extra dimensional mode and for the homogeneous case an initially decelerating universe starts accelerating undergoing a flip. Krori et al [10], Gleiser and Diaz [11] have proposed a higher-dimensional anisotropic Bianchi type I Cosmological model to obtain the 4D perfect fluid solutions in flat universe model which are compatible with contraction of all the extra dimensions. In homogeneous 5D space times Panigrahi et al [12] have shown a decelerating expansion in the early era of the universe along with an accelerated universe without introducing any external quintessence like scalar field in the presence of extra dimensions.

We have in General Relativity the problem of solutions of different types of field equations that determines the geometry of space time created by various objects. The solutions obtained are very much in agreement with all the present day experimental data and astronomical observations.

In this thesis we have presented solutions in four-dimensional relativity as well as higher-dimensional relativity. Accordingly the thesis is divided into two parts. Part I
contains four papers in which we present solutions in higher-dimensional space time. Part II contains two papers all worked out in four-dimensional space time. This thesis contains six papers. Four of them are published in different journals and two are communicated. All the problems satisfy the present day observational data. Accordingly the thesis contains seven chapters including the introduction.

The first chapter is introduction. The second chapter deals with higher-dimensional general relativity. In this paper we study the nature of the universe in five dimensions as accelerating in presence of scalar field. Recently there has been appreciable interest in theories with higher dimensional space time, in which extra dimensions are compactified to a size of $10^{-33}$ cm, which is beyond our ability for measurement. This is because developments of superstring theory and super gravity need higher dimensional space time. Kaluza and Klein suggested a higher-dimensional model of space time for attempting unification of gravity with electromagnetic interaction by imposing an additional dimension. This is an extension of Einstein’s general relativity in five dimensions. Recently Wesson and his collaborators proposed Space Time Matter (STM) Theory [5] from which studies in extra dimension originates. Astronomical observations indicate that our universe currently consists of approximately 73% dark energy, 23% dark matter and 4% baryonic matter and radiation. Many cosmological models have been constructed by making introduction of quintessence from string/M-Theory or brane world model . The universe is dominated by a form of matter with negative pressure which is called dark energy. A kind of repulsive force which acts as anti-gravity is responsible for accelerating universe. There may be another candidate for dark energy known as quintessence which is a scalar field $\phi$ with a potential $V(\phi)$. Quintessence exerts negative pressure and is dynamic in
nature. The universe reaches its critical energy due to the effect of the scalar field which leads to accelerate its expansion. We have studied the dynamical behaviour of a higher-dimensional cosmological model in presence of non relativistic matter and scalar field.

The third chapter deals with a higher-dimensional charged singularity free solution. Here we have generalised a singularity free solution in four-dimensional space time to N-dimensions. We have presented a solution for a static charged fluid sphere in higher dimensions. The solution satisfies the physical conditions inside the sphere. It reduces to Krori-Barua [13] solution if the number of dimensions is four. Efinger [14], Kyle and Martin [15] and Wilson [16] have found relativistic internal solutions for static charged spheres charged spheres in general relativity, but none of these solutions is absolutely free from singularities. Junevicus [17] in his investigation observed that the fluid sphere solutions of Kyle and Martin [15], Wilson [16], Kramer and Neugebauer [18] and Krori-Barua [13] are of special interest since, with the application of suitable conditions, they are completely free of metric singularities and satisfy physical considerations. A static spherically symmetric solution of Einstein-Maxwell equations depending on a particular selection of the metric coefficients $g_{00}$ and $g_{11}$ in curvature coordinates was worked out by Krori-Barua [13]. The solutions obtained thus are singularity free. Now the unification of gravity with other fundamental forces in nature is still an unsolved problem. A fifth dimension is introduced by Kaluza and Klein for trying to unify gravity with electromagnetic interaction. Super String Theory and Yang Mills Super gravity Theory need higher dimensional space time. For this purpose, the higher dimensional space time, are of importance. According to Chodos and Detweiler [19], the present four-dimensional stage of the universe could have been preceded by a
higher-dimensional stage, in which extra dimensions are constructed to a very small size, i.e., of planckian length scale due to dynamical contraction. This paper is an extension of Krori-Barua solutions in N-dimensions. It reduces to KB solution if the number of dimensions is four.

The fourth chapter deals with a higher-dimensional cosmological model in Lyra geometry. This model is suitable for early stage of the universe. The geometrization of gravitation by Einstein in his general theory of relativity inspired several authors to geometrize other physical fields. Weyl [20] proposed a unified theory to geometrize gravitation and electromagnetism. But due to the non-integrability of length transfer this theory was never considered seriously. This theory inspired Gehard Lyra to develop what is called Lyra geometry. Lyra [21] proposed a new modification of Riemannian geometry by using a gauge function to remove the non-integrability of the length of a vector under parallel transport. Halford [22] has showed specifically that the displacement field in Lyra geometry acts the role of cosmological constant in normal general relativistic treatment. Rahman et al [23], Singh et al [24] and Mohanty et al [25] have constructed five dimensional models (cosmological)sssss in Lyra’s manifold. Singh and Desikan[26] found solutions for four-dimensional FRW model in Lyra geometry. Exact solutions of the field equations are obtained. Our model is suitable for early stage of the universe.

The fifth chapter deals with a five-dimensional cosmological model with Chaplygin equation of state in Lyra geometry. A type of dark energy, the pure Chaplygin gas model is taken for study as it possesses negative pressure which is responsible for accelerating the universe. We have observed that our universe, after the big bang, rapidly expands with the Chaplygin gas and the Lyra field quickly dying out.
as the displacement field decreases with the increase of time. Our model represents the early era of the universe after birth. The idea of higher-dimensional theory was originated in superstring and supergravity theories with the other fundamental forces in nature. The unification of gravitational forces with other forces in nature is not possible in the usual four dimensional space time. Higher dimensional theory might be useful at very early stages of the evolution of the universe. As time evolves, the standard dimensions expand while the extra dimensions shrink to the Planckian dimension, which is beyond our ability to detect [27]. Five dimensional cosmology has attracted interest in recent time. P. S. Wesson [28-32], singly as well as with his collaborators, has made considerable contribution to this field. In recent times, considerable works with great interest are going on for the study of the chaplygin gas equation of state to explain the accelerating phase of the present universe. Also the universe is dominated by a form of matter with negative pressure which is widely known as dark energy today. It is interesting to obtain a five dimensional accelerating model with chaplygin gas for dark energy in Lyra geometry.

The sixth chapter deals with a dark energy model in Bianchi type V string space time in Lyra geometry. The idea of string theory was established to describe the initial stage of the evolution of the universe. The recent observations of the universe point out the existence of a large amount of strings in the early stage of the universe [33, 34]. The study of cosmic strings in relativistic framework was done by Stachel [35], Letelier [36]. Recently a large number of observations of the luminosity of type Ia Supernovae [37-39] show towards an accelerated expansion of the universe. This implies that the pressure and energy density condition of the universe is $\rho + 3p < 0$, where $p$ and $\rho$ are the pressure and density respectively. This indicates that the universe is dominated by a
form of matter with negative pressure which drives the universe to undergo an
accelerating expansion. We have studied the Bianchi type v string cosmological model
in Lyra geometry taking negative pressure, which is a criteria of dark energy and also
taking variable deceleration parameter. We have observed that the universe is
accelerating.

The seventh and last chapter deals with a research paper where we have derived
Kerr-de Sitter metric in a magnetic field using null tetrad formalism [40]. Now Melvin’s
Magnetic Universe is a solution of the Einstein’s equation that describes a matter-free
universe, endowed with magnetic field [41, 42]. Recently Castello-Branco et al.[43]
investigated the distribution of gravitational energy in the space time of a Schwarzschild
black hole immersed in a cosmic magnetic field. We have derived the magnetized Kerr-
de Sitter metric in a magnetic field. In this paper we have examined three types of
geometries of the metric. Firstly, it is observed that the dragging effect on a particle
moving in the field of a magnetized Kerr-de Sitter object is affected by magnetic field.
Also we have found that a coriolis-like dragging is experienced by a particle moving in
the field of a magnetized Kerr-de Sitter object in de Sitter space time. Secondly,
magnetized Ker-de Sitter black hole is discussed. It is found that there are two distinct
infinite red shift surfaces for this space time.
REFERENCES


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