CHAPTER V
5. Five-Dimensional cosmological model with chaplygin equation of state in Lyra geometry

According to the Superstring theory, the Universe was born in higher dimensions. The present four-dimensional universe is accelerating according to recent observations. We investigate in this paper a five-dimensional accelerating model with chaplygin gas in Lyra geometry. The model reduces to four dimensions in due course. The physical and geometrical properties of the model are briefly discussed.

Keywords: Chaplygin gas, Lyra geometry, Superstring theory, Higher-dimension.

5.1 INTRODUCTION

Recently researches are going on for finding out the origin of accelerated expansion of the present universe. Observational evidences point towards an accelerated expansion of the universe. The dark energy occupies about 73% of the energy of the universe, while dark matter occupies about 23% and the usual baryonic matter 4%. The dark energy is responsible for cosmic acceleration (Turner MS) [1]. The idea of higher-dimensional theory was originated in superstring and super gravity theories with the other fundamental forces in nature. The unification of gravitational forces with other

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forces in nature is not possible in the usual four-dimensional space-times. So, higher dimensional theory might be useful at very early stages of the evolution of the universe. In fact, as time evolves, the standard dimensions expand while the extra dimensions shrink to the Planckian dimension, which is beyond our ability to detect (Chatterjee et al.[2]). Appelquist et al.[3], Chodos and Detweller [4] constructed cosmological model in higher-dimensional homogeneous space time in general relativity. Five-dimensional cosmology has attracted much interest in recent literature. Particularly, P. S. Wesson [5-9], singly as well as with his collaborators, has made considerable contribution to this field. Besides, there are others also who have made a number of publications in this cosmology (Samanta et al., Das et al., Mahanty et al.[10-14]). The geometrization of gravitation by Einstein in his general theory of relativity inspired several authors to geometrize other physical fields. Weyl [15] proposed a unified theory to geometrize gravitation and electromagnetism. But due to the non-integrability of length transfer, this theory was never considered seriously. However, this theory inspired Gehard Lyra to develop what is called Lyra geometry. Lyra [16] proposed a new modification of Riemannian geometry by introducing a gauge function to remove the non-integrability of the length of a vector under parallel transport. Five-dimensional cosmological model in Lyra’s manifold are constructed by Rahaman et al [19], Singh et.al. [20] and Mohanty et al [21-23]. Beesham [24] considered four-dimensional FRW cosmological model in Lyra’s geometry with time dependent field. Recently, scientists show great interest on the chaplygin gas equation of state in order to explain the accelerating phase of the present universe as well as to unify the dark energy and dark matter. The universe is dominated by a form of matter with negative pressure which is widely referred to as
dark energy today. It is interesting to obtain a five-dimensional accelerating model with chaplygin gas for dark energy in Lyra geometry. This model awaits investigation.

We take the equation of state for the chaplygin gas for dark energy in the form

$$p = -\frac{A}{\rho}$$

(1)

where $p$ is the pressure and $\rho$ is the energy density of the gas and $A$ is a positive constant.

Sen [17], Sen and Dunn [18] suggested a new scalar tensor theory of gravitation and constructed an analogue of the Einstein’s field equations based on Lyra geometry which may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_{ij} \phi - \frac{3}{4} g_{ij} \phi \phi^k = -\chi T_{ij},$$

(2)

where $\chi = 8\pi G$ and $\phi_i$ is the displacement vector according to the Lyra geometry and other symbol have their usual meaning in the Riemannian geometry.

This paper is organised as follows: In section 2, the metric and the field equations are presented. In section 3, we deal with the solutions of field equations. Section 4 describes some physical and geometric properties of the model. The paper ends with a conclusion in section 5.

5.2 THE METRIC AND FIELD EQUATIONS
We consider a five-dimensional space-time model described by the line-element

\[
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2(\phi) d\phi^2 \right] + b^2(t) dy^2
\]

(3)

where \( a(t) \) and \( b(t) \) are scale factors for the four-dimensional space-time and the fifth dimension respectively. \( y \) is the fifth dimensional coordinate. The five-dimensional time-like displacement vector \( \phi_i \) in (1) is defined as

\[
\phi_i = (\beta(t), 0, 0, 0, 0)
\]

(4)

The energy momentum tensor is taken as

\[
T_{ij} = (\rho + p) u_i u_j - p g_{ij}
\]

(5)

together with the coordinates satisfying

\[
g_{ij} u^i u^j = -1
\]

(6)

where \( p, \rho \) and \( u^i \) are pressure, energy density and five dimensional velocity vector of the fluid distribution respectively.

The field equations (2) for the metric (3) using the equations (4) and (5) are

\[
3 \frac{\dot{a}}{a} + 3 \left( \frac{\dot{\beta}}{\beta} \right)^2 = \chi \rho + \frac{3}{4} \beta^2
\]

(7)

\[
2 \frac{\dot{a}}{ab} \left( \frac{\dot{a}}{a} \right)^2 + \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} = -\chi \rho - \frac{3}{4} \beta^2
\]

(8)

\[
3 \ddot{a} + 3 \left( \frac{\dot{a}}{a} \right)^2 = -\chi \rho - \frac{3}{4} \beta^2
\]

(9)
Taking divergence of (2) with $T_{ij}$ given by (5), the Bianchi identity gives

$$\lambda \dot{\rho} + \frac{3}{2} \beta \dot{\beta} + \left[ \lambda (\rho + p) + \frac{3}{2} \beta^2 \right] \left( 3 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) = 0$$

(10)

The dots denote time-derivatives.

**5.3 SOLUTIONS OF FIELD EQUATIONS**

The field equations (7)-(9) are a system of three equations with five unknown parameters $a, b, \rho, p, \beta$. Therefore to obtain exact solutions of the field equations we need two more equations. The fifth dimension should die away as the physical four dimensions evolve. Hence a choice has to be adopted keeping this fact in mind. We assume the following relation between $a(t)$ and $b(t)$:

$$b(t) = k_1 a^n (t)$$

(11)

where $k_1 (> 0)$ and $n$ are constants. We take as the second equation, the equation of state of the chaplygin gas (1)

On using (11), (8) and (9) yield

$$\frac{\dot{a}}{a} + (n + 2) \frac{\dot{a}^2}{a^2} = 0$$

(12)

The solution of (12) is
\[ a(t) = \left( c_1 t + c_2 \right)^{\frac{1}{n+3}} \]  \hspace{1cm} (13)

where \( c_1 \) and \( c_2 \) are constants of integration and we take \( n > -3 \).

The metric coefficient \( b(t) \) is obtained on using (11) and (13)

\[ b(t) = k_1 \left( c_1 t + c_2 \right)^{\frac{n}{n+3}} \]  \hspace{1cm} (14)

The conservation law for energy-momentum tensor \( T_{ij} = 0 \) leads to

\[ \dot{\rho} + (\rho + p) \left( 3 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) = 0 \]  \hspace{1cm} (15)

Using (15) in (10) we have a relation between \( \beta(t) \) and the metric coefficients \( a(t) \) and \( b(t) \)

\[ \beta \ddot{\beta} + \beta^2 \left( 3 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) = 0 \]  \hspace{1cm} (16)

Integrating equation (16) [using (13) and (14)] we obtain

\[ \beta(t) = \frac{c_4}{(c_1 t + c_2)} \]  \hspace{1cm} (17)

where \( c_4 \) is a positive constant.

Equation (15), on using (11), (13) and (14), yields the following expression for \( \rho \)

\[ \rho = \left[ A + \frac{c_4}{(c_1 t + c_2)^2} \right]^{\frac{1}{2}} \]  \hspace{1cm} (18)
where $c_4$ is a positive constant.

From equations (1) and (18), we obtain the expression for pressure

$$ p = -\frac{A}{\left[ A + \frac{c_4}{(c_1t + c_2)^2} \right]^2} \quad (19) $$

We define, $H$, the Hubble parameter and $q$, the deceleration parameter in terms of four dimensions as they only are relevant to cosmological observations (Chatterjee et al.[25]).

The Hubble parameter is given by

$$ H = \frac{\dot{a}}{a} = \frac{c_1}{(n+3)(c_1t + c_2)} \quad (20) $$

and the deceleration parameter is given by

$$ q = -\frac{a\ddot{a}}{\dot{a}^2} = n + 2 \text{, (a constant)} \quad (21) $$

5.4 PHYSICAL CHARACTER OF THE MODEL
For a physical model, \( a(t) \) should increase and \( b(t) \) should die out. From (13) we see that for \(-3 < n < -2.5\), \( a(t) \) and \( \dot{a}(t) \) increase with the increase of the cosmic time as shown in Fig.1 and Fig.2 respectively. For \(-3 < n < -2.5\), \( \dot{a}(t) > 0 \) and \( \ddot{a}(t) > 0 \) but \( q < 0 \) which implies that the universe accelerates. From (20) we observe that for \(-3 < n < -2.5\), the Hubble parameter \( H \) decreases as the cosmic time \( t \) increases as shown in Fig.3. Using (14), we observe that for \(-3 < n < -2.5\), the scale factor \( b(t) \) decreases as the cosmic time increases as is shown in Fig. 4.

Further from (17) and (18) we see that the displacement vector, \( \beta(t) \) and the energy density, \( \rho(t) \), decrease with the increase of the cosmic time, \( t \). Fig.5 and Fig.6 show the variation of \( \beta(t) \) and \( \rho(t) \) with \( t \).

5.5 CONCLUSIONS

For \(-3 < n < -2.5\), equation (13) implies that \( \dot{a} > 0 \) and \( \ddot{a} > 0 \). That is the universe accelerates. From equation (11) one can see that for \(-3 < n < -2.5\), the fifth dimensional metric coefficient \( b(t) \) decreases while \( a(t) \) increases with the increase of time. Thus the extra dimension becomes insignificant as the time proceeds after creation and we are left with the real four-dimensional world. A type of dark energy, the so-called pure chaplygin gas model which obeys an equation of state \( p = -\frac{A}{\rho} \) (1), where \( A \) is a positive constant and \( p \) and \( \rho \) respectively the pressure and density of the
fluid, is taken for study as it possesses negative pressure which is responsible for accelerating the universe. It is observed that the pressure decreases with the increase of time.

In this paper, we have observed that our universe, after the big bang, rapidly expands with the chaplygin gas and the Lyra field quickly dying out as the displacement field $\beta(t)$ decreases with the increases of time. The radiation or matter is yet to show up. Therefore our model represents the early era of the universe after birth.

![Graph showing the variation of scale factor $a(t)$ against cosmic time $t$ for $n = -2.6$, $c_1 = 3$, $c_2 = 0$.](image)

**Fig. 1**

Fig. 1 shows the variation of scale factor $a(t)$ against cosmic time $t$ for $n = -2.6$, $c_1 = 3$, $c_2 = 0$. 
Fig. 2 shows the variation of scale factor $\dot{a}(t)$ against time $t$ for $n = -2.6, \ c_1 = 3, \ c_2 = 0$.

Fig. 3
Fig. 3 shows the variation of Hubble parameter $H$ against cosmic time $t$ for $c_1 = 3$, $c_2 = 0$, $n = -2.6$.

Fig. 4

Fig. 4 shows the variation of $b(t)$ with cosmic time $t$ for $n = -2.6, c_1 = 3, c_2 = 0, k_1 = 1$.
Fig. 5 shows the variation of $\beta(t)$ with cosmic time $t$ for $c_1 = 3, c_2 = 0, c_3 = 4$
Fig. 6

Fig. 6 shows the variation of $\rho(t)$ with cosmic time for

$$n = -2.6, c_1 = 3, c_2 = 0, c_4 = 5, A = 5.$$
5.6 REFERENCES


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