CHAPTER I

INTRODUCTION
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1. INTRODUCTION

In the modern applied research reliability and its applications are identified as part and parcel of the industrial management. This gained momentum due to the keen interest shown by the researchers to develop optimal reliability systems. In recent times optimisation procedures in reliability are attracting a large interest from the system designers and financial managers.

In an ever increasing complexity of the system design, the system designers often encounter with the problem of ensuring that the system performs its intended function to the expectations of its minimum requirements for which it is being designed. With the development of science and technology although one observes sophistication in the systems, yet the complexity involved, results in the increase in costs. Therefore it becomes essential to prevent from providing expensive redundancies in already reliable sub-systems.

A high degree of reliability is a prime requirement in many practical systems. For e.g., nuclear systems, aero-space systems, defence combat systems, computer net works, communication systems and so on. This high degree of reliability
is usually achieved through use of redundancy. Through the introduction of more number of units into the system, the mean time to failure (MTTF) of the system may increase. But there are other conflicting criteria of installation costs, space and repairs. Therefore optimal space, optimal costs and optimal repairs provide ample of scope for further research in optimisation procedures.

The objective of the investigations carried out in this thesis is to develop the space optimal reliability systems through technological progress.

Before demonstrating the results in the next chapter, some basics needed for the further work and a review of optimisation techniques in reliability theory are presented in the following sections.

2. Basics

The concept of reliability is often useful while assessing the performance of any system and is also described as the science that studies the laws governing the failures in technical equipments and methods of prediction.
Since functions of different systems vary over a wide range, it may not always be possible to give a general quantitative definition of system reliability. However, for a particular type of system, one may define the system reliability in a somewhat arbitrary manner. A more precise definition of the system can be given only after the description of the mathematical model. The most commonly accepted definition of reliability is as follows:

The reliability of a system (or a unit or a product) is the probability that the system performs its intended function adequately for a given period of time under the stated operating conditions or environment.

The other important concepts in reliability theory are (i) failure time distribution (ii) hazard rate and (iii) mean time to failure (MTTF) of a system. We now present these concepts in brief.

(i) Failure time distribution

A failure is defined as the partial or total loss or change of the properties of components or system so that their function is seriously affected or stopped. The failure modes are generally of two kinds:
(i) Initial failures or infant mortality (which occurs at early stages) and (ii) Random failures or catastrophic failures (which occurs during operation of the system).

Mathematically:

Consider a system whose life time or the time to failure is denoted by $X$, assumed to be a non-negative continuous random variable.

The probability function denoted by $U(t)$, defined as:

$$U(t) = \Pr(X \leq t), \quad t > 0,$$

is called the failure time distribution.

The Reliability function $R(t)$ defined as:

$$R(t) = 1 - U(t) = \Pr(X > t), \quad t > 0.$$  \hfill (1)

The Reliability function is also termed as the "survivor function", since it is the probability that the component survived upto time "$t$".

The hazard rate or instantaneous failure rate, denoted by $h(t)$, is defined as:

$$h(t) = \frac{u(t)}{1 - U(t)} \quad \text{(3)}$$

(Where $u(t) = U'(t)$)
It is observed that $R(t)$ and $h(t)$ are uniquely connected by the relation.

$$- \int_{t}^{\infty} h(x) \, dx$$

$$R(t) = e^{-R(t)} \quad . \quad (4)$$

It can also be written as:

$$- \int_{t}^{\infty} h(x) \, dx$$

$$u(t) = h(t) e^{-R(t)} \quad . \quad (5)$$

The mean time to failure (MTTF) of the system, denoted by $\mu$, is given by:

$$\mu = \int_{0}^{\infty} t \, u(t) \, dt \quad (6)$$

since $u(t) = -R'(t)$, then

$$\mu = - \int_{0}^{\infty} t \, dR(t)$$

$$\mu = -t \left[ R(t) \right]_{0}^{\infty} + \int_{0}^{\infty} R(t) \, dt$$

(by integration by parts)

$$\mu = \int_{0}^{\infty} R(t) \, dt$$

We now present the Reliability systems that are needed for the further work in the dissertation.
In practice the systems are designed with several components based on different structures. According to the structures we may broadly classify those systems as follows:

(i) Series systems,
(ii) Parallel systems,
(iii) Parallel - series systems,
(iv) Series - parallel systems and
(v) k-out-of-N systems.

(i) **Series Systems**

Consider a system which consists of \( N \) components: \( U_1, U_2, \ldots, U_N \), assumed to function or fail independently of each other. The system is called a series system if operation of the system implies operation of each of the components.

Diagramatically, a series system with \( U_1, U_2, \ldots, U_N \) units can be represented as follows:

```
   __________    __________    __________
 o  ------->   |     |   |     |   |     |
   |     |   |     |   |     |
   | 1   |   | 2   |   |  \ldots |   | N   |
 o     o   o     o   o     o   o
```

Fig.1: Series system
The reliability function, \( R(t) \) for this system is given by:

\[
R(t) = \prod_{j=1}^{N} R_j(t)
\]  

(8)

where \( R_j(t) \), \( j=1,2,\ldots,N \) respectively represents the reliability functions of the components:

\[
U_j(t), \quad j=1,2,\ldots,N.
\]

When all the components in the system are identical then

\[
R(t) = (R(t))^N.
\]

(9)

i.e., where \( R(t)^J_R(t) \forall j, j=1,2,\ldots,N \)

Also \( U(t) = 1-R(t) \) and \( u_j(t) = 1-R_j(t) \), so that

\[
U(t) = 1-(\prod_{j=1}^{N} (1-u_j(t))).
\]

(10)

(ii) Parallel systems:

Suppose that the system consists of \( N \) units: \( U_1, U_2, \ldots, U_N \) assumed to function or fail independently of each other. The system is called parallel system if and only if the successful functioning of any one of the units leads to the failure free operation of the system.
A parallel system with \( U_1, U_2, \ldots, U_N \) components can be represented as follows:

![Diagram of parallel system](image)

**Fig. 2: Parallel system**

In parallel system

\[
U(t) = \sum_{j=1}^{N} u_j(t) \quad (11)
\]

and

\[
R(t) = \prod_{j=1}^{N} (1-R_j(t)) \quad (12)
\]

For the identical unit system

\[
R(t) = \left[1 - (1-R(t))\right]^N, \text{ where } R_j(t) = R(t) \quad \forall j, \quad j = 1, 2, \ldots, N.
\]
Again, when all components of the system have the same failure time distribution $U(t)$, and the MTTF denoted by $\mu_N$, is given by:

$$\mu_N = \int_0^{\infty} \left[ 1 - U(t)^N \right] dt.$$  \hspace{1cm} (13)

(iii) Parallel-series systems:

A system is called a parallel-series system of order $(m,N)$ if the system consisting of $m$ identical series, each of order $N$ is connected in parallel. A typical schematic representation is given in Fig.3.

![Parallel-series system of order $(m,N)$]

Fig.3: Parallel-series system of order $(m,N)$
If the components are independent, then the system reliability function is given by:

\[ R(t) = \prod_{i=1}^{N} (1 - (1 - R_{i}(t)))^m. \]  
(14)

And the system MTTF is given by

\[ \mu_{m}^{N} = \int_{0}^{\infty} \left( 1 - F(t) \right)^{m} dt. \]  
(15)

(iv) **Series-parallel system:**

A system consisting of \( Nm \) components arranged in \( m \) series of parallel systems each comprising of \( N \) units is called a series-parallel system of order \((m,N)\).

The reliability function of the system is given by

\[ R(t) = \prod_{j=1}^{N} \left( 1 - (1 - R_{j}(t)) \right)^{m}. \]  
(16)

Suppose that all the units have the same failure time distribution \( U(t) \), then the system's MTTF, denoted by \( \mu_{m}^{N} \), is given by

\[ \mu_{m}^{N} = \int_{0}^{\infty} \left( 1 - F(t) \right)^{m} dt. \]  
(17)
Diagramatic representation of the system is given in Fig.4.

\begin{center}
\begin{tabular}{ccc}
1 & 1 & 1 \\
2 & 2 & 2 \\
\vdots & \vdots & \vdots \\
1 & 2 & N \\
\end{tabular}
\end{center}

\textbf{Fig.4. Series-parallel system of (a, N)}

(v) \textbf{k-out-of-N systems}

Suppose that a system consisting of N components assumed to function or fail independently of each other. A system is called a k-out-of-N system, if it operates successfully if and only if at least k (1 \leq k \leq N) of the N components function.

Let us consider that all the units have the same failure time distribution function \(U(t)\), then the system reliability \(R(t)\), is given by:

\[ R(t) = \sum_{j=k}^{N} \binom{N}{j} (1-U(t))^j (U(t))^{N-j}, \quad (18) \]

and the system MTTF is given by:

\[ \mu_N(k) = \int_0^\infty R(t) \, dt. \quad (19) \]
3. OPTIMISATION TECHNIQUES IN RELIABILITY - A REVIEW

We now illustrate in some details about notable contributions made by the researchers in this field, and also which are particularly relevant for the present investigation. Reliability theory has its origin since Nineteen thirties. The earliest work in optimisation methods was due to Lotka (18). He developed a model in which the system is subjected to random failures and suggested a replacement policy. Later, Cambell (8) for the Lotka's model provided a preventive maintenance policy.

Morrison and David (20) considered the distribution of the random operating life of an identical unit series system with a set of spare components for the situation when failed units are in turn replaced by the spares and also provided the reliability of such systems.

canfield (9) employed an optimisation procedure based on an iterative procedure which provides an optimal preventive maintenance policy.

Barlow and Proschan (3) studied planned replacement policy for a unit based on replacement and exchange costs. They suggested three types of replacement policies:

(1) strictly periodic replacement, (2) Random periodic replacement and (3) sequentially determined replacement.
Drink water and Hastings (11) studied a repair limit replacement policy. Diveroli (10) suggested the optimal replacement policies for equipment subjected to failures with randomly distributed repair costs.

Gaver (13) considered reliability systems operating in alternating random environments exposed to two different types but with known and constant hazard rates. He studied explicit expression for the system's mean time before failure.

Natarajan (26) considered a single unit system with (N-1) spares and single repair facility and derived the distribution of time to system failure (TSF) period by using the general process probabilities got by solving difference differential equation. He further using renewal theoretic arguments obtained expected time to system failure, expected down-time (SDT), mean recurrence time to the system down state, long run availability of the system, expected number of failures in a given interval of time (0,t) and interval reliability.

Nakagawa and Osaki (25) studied a single unit system and studied optimum repair limit replacement policies by introducing a policy, where the repair of the failed units starts immediately and if its repair is not completed within a specified time it is replaced by a new unit.
Elton and Gruber (12) considered the optimality of equal life policy for the equipment subjected to technological progress. Later Berg (5) considered an age replacement policy in which a unit is replaced on failure or when it reaches a predetermined age and developed optimal critical age for a stochastically failing equipments.

Rade (28) obtained explicit results for mean time before failure for a N-Identical unit parallel systems subjected to shocks generated by a random enviornment. Later Nakagawa (21) obtained a replacement policy for Rade's model.

Bergman (6) obtained optimal replacement policies based on measurements of wear, accumalated damage or accumulated stress. He also suggested an optimal rule for some generalisations and special cases. Menipaz (19) based on variable maintenance cost and a positive discount factor suggested the problem of optimal checking policy for a single stochastic system. Later Keigo Yomada (16) studied some typical stopping problems for group processess and reported in testing results.

Nguyen and Murthy (27) obtained optimal preventive maintenance policies for a repairable systems based on the assumption that its failure rate increases with the increasing number of failures. Ansell etal (2) based on three alternative
cost criteria obtained the relationships between the optimal replacement policies. They also compared the optimal sequential and fixed-age replacement policies through an illustration.

Nakagawa (22) studied a N-identical unit parallel system and obtained a procedure by minimising the cost function, which leads to optimal number of elements \( N \). He also developed an optimal replacement time \( T \) and the jointly optimal pair \( (N, T) \).

Yoo and Sung (40) considered an age replacement policy for a system comprised of a single piece of equipment and an indicator, and the stage of the equipment is monitored through the indicator, with the provision that both the equipment and the indicator can fail.

Nakagawa (23) studied the K-out-of-n system based on the assumption that the failure rate of each unit is constant and obtained a method of suggesting the most economical optimal number of units, \( n \) and the optimal replacement time for the system failure.

Block et al (7) outlined the recent developments concerning the problem of determining the expected cost per unit time for various refinements of age replacement, block replacement and periodic replacement with minimal repair at failure. They also attempted to use a set of unified notation which in many cases differs.
Venugopal and Rami Reddy (32) provided an optimal repair stage for a single unit reliability system by introducing a new approach for optimality criteria in terms of absolute differences in per unit costs of replacement and repair. Later these results were extended in (33) for N-unit parallel system. Further, adopting the same modelling set up and incorporating salvage costs they obtained (34) optimal replacement policy for the general reliability model.

Venugopal et al (39) studied N-identical unit-parallel system by introducing the maintenance cost and repair cost into the modelling set up and suggested an optimal number of units N for fixed repair stage (n). Later Venugopal, Rami Reddy and Meenakshi Bai (38) developed an optimal replacement in terms of repair stage (n) for the above system and obtained an algorithm to obtain the optimal system (N,n).

Venugopal, Krishna Reddy and Rami Reddy (37) considered N-identical unit parallel system and developed an optimal replacement-policy based on comprehensive cost considerations (comprising of acquisition cost, per unit time repair cost and salvage cost).

Venugopal et al (36) considered a single unit reliability system and suggested an optimal replacement policy incorporating the crucial factor of technological progress into the modelling set up.
Venugopal et al. (35) considered a parallel-series reliability system with a repair facility and developed cost-space optimal \((N, m, n)\) systems.

Venugopal and Krishna Reddy (31) considered a series-parallel system with fixed repair facility and developed an optimal procedure leading to the most economical number of parallel units \((N)\), based on the optimal criterion of minimal per unit costs (including repair costs).

Nakagawa (24) considered a replacement policy which maximizes mean time to failure (MTTF) of a system with \(N\) spare unit. He developed the optimum replacement time of system with \(K\) spares \((k=1, 2, \ldots, N)\) successively from MTTF with \((K-1)\) spares by induction.

Srinivas Iyer (30) considered series and parallel systems and developed the increase in reliability of a coherent system due to the addition of a redundant element to a component is expressed as a function of the Birnbaum reliability importance of that component. Further he derived an analog of the pivotal decomposition formula for the increase in reliability and upper bounds for the increase in the mean time to failure (MTTF) of the system.
Andreas Kossow and Wolfgang Preuss (1) considered the $k$-out-of-$n$:F system with $n$ components and presented a topological formula for the exact system reliability of linear and circular consecutive $k$-out-of-$n$:F networks.

Other notable works in this direction are excellently reported in Barlow and Proschan (4), Jorgenson et al. (15), Gnedenko et al. (14) and Lewis (17).

We now present in the next chapter our results leading to space optimal systems with technological progress.


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