CHAPTER II

MIXED CONVECTION FLOW THROUGH A

POROUS MEDIUM IN A CYLINDER OF NON-UNIFORM GAP
1. FORMULATION OF THE PROBLEM

We consider the steady axisymmetric flow of a viscous, incompressible fluid through a porous medium in a vertical pipe of variable cross-section maintained at a constant temperature in the presence of a constant internal heat source. The Boussinesq approximation is used so that the density variations will be retained only in the buoyancy force. The viscous dissipation is neglected in comparison to the heat flow by conduction and convection. The kinematic viscosity \( \nu \), the thermal conductivity \( \lambda \), the coefficient of volume expansion \( \beta \), and the specific heat at constant pressure \( C_p \) are assumed to be constants. The cylindrical polar system \((r,z)\) is chosen with \(z\) axis along the axis of the pipe. The boundary of the pipe is assumed to be \( r = a f (\delta z / a) \). A linear density temperature variation is assumed with \( \rho_c \) and \( T_c \) being the density and the temperature of the fluid in the equilibrium state. The flow is maintained by a constant volume flow rate for which a characteristic velocity \( q \) is defined as

\[
q = 2/a^2 \int_0^{af} w r dr
\]

(1.1)
The governing equations of motion for a steady axisymmetric flow of a viscous incompressible fluid through a porous medium are

\[ P_e (u u_Y + w u_z) = -p_y + \mu \left( u_{yy} + u_r - \frac{u_z}{r} - \frac{u}{K} + u_{zz} \right) \quad (1.2) \]

\[ P_e (u w_Y + w w_z) = -p_z + \mu \left( w_{yy} + \frac{w_r}{r} + w_{zz} - \frac{w}{r} \right) - fg \quad (1.3) \]

The equation of continuity is

\[ u_Y + \frac{u}{r} + w_z = 0 \quad (1.4) \]

The equation of energy is

\[ P_e C_p (u T_y + w T_z) = \lambda T \left( T_{yy} + \frac{T_r}{r} + T_{zz} \right) + Q \quad (1.5) \]

The equation of state is

\[ P = P_e \left[ 1 - \beta_p (T - T_e) \right] \quad (1.6) \]

suffices \( r, z \) denote differentiation w.r.t. the respective variable. We write \( P = P_e + P_d \) where \( P_e \) is the hydrostatic pressure and \( P_d \) is the hydrodynamic pressure. In the equilibrium state

\[ -p_z - P_e g = 0 \]
and also in this state the temperature gradient balances the heat flux generated by the constant internal heat source \( Q \).

The boundary conditions relevant to the problem are

\[
\begin{align*}
\quad u(0, z) &= 0, \quad w(af, z) = 0, \\
(\omega_Y)_{y=0} &= 0, \quad (T - T_e) = T_w.
\end{align*}
\]

We introduce the following nondimensional variables

\[
\begin{align*}
\theta &= \frac{T - T_e}{\Delta T_e}, \\
\theta &= \frac{T - T_e}{\Delta T_e},
\end{align*}
\]

where

\[
\Delta T_e = \frac{Q a^2}{2 \lambda T}.
\]

Substituting these non-dimensional variables in equation (1.2) and (1.3) the non-dimensional equations governing the flow reduces to (on dropping the asterisks)

\[
\begin{align*}
R(u_w w_Y + w w_z) &= -R P_y + (u_{yy} + u_{rr} - \frac{u}{r} + u_{zz} - \sigma^2 u), \\
R(u w w_Y + w w w_z) &= -R P_z + (w_{yy} + w_{rr} + w_{zz} - \sigma^2 w) - \frac{\sigma}{r} \theta, \\
R(\theta w_{Y}) &= \theta_{yy} + \frac{\theta_y}{r} + \theta_{zz} + 1,
\end{align*}
\]

(1.8a)
where
\[
\sigma = \frac{a^2}{k}, \text{porous parameter}
\]
\[
R = \frac{q a}{k}, \text{Reynolds number}
\]
\[
P = \frac{\mu C_p}{\lambda_T}, \text{Prandtl number}
\]
\[
G = 2 \gamma \beta \Delta T e a^3 / \gamma^4, \text{Grashoff number}
\]

Eliminating the pressure from (1.8) and (1.8a) and introducing the non-dimensional stream function \( \psi \), we obtain
\[
R \left[ \gamma \psi_z (D^2 \psi)_z - \gamma \psi_{zz} (D^2 \psi)_z + 2 \psi_{zz} D^2 \psi \right] = \gamma^2 D^2 \psi + \gamma^3 G \theta_y - \sigma^2 \gamma^4 D^2 \psi
\]
Also the equation of energy reduces to
\[
R P \left[ \frac{1}{\gamma} \psi_{zz} \theta_z - \frac{1}{\gamma} \psi_{zz} \theta_y \right] = \theta \psi + \theta_y + \theta_{zz} + 1
\]
where
\[
D^2 = \frac{\partial^2}{\partial z^2} - \frac{1}{\gamma} \frac{\partial}{\partial y} + \frac{\partial^2}{\partial z^2}
\]
These coupled equations (1.10) and (1.11) are to be solved subject to boundary conditions
\[
\psi(f, z) = \frac{1}{z}, \quad (\psi_y)_{y=0} = 0, \quad \psi(0, z) = 0
\]
\[
(\psi_{zz} - \gamma \psi_y) = 0, \quad (\theta)_{y=0} = \theta_w, \quad (\theta_y)_{z=0} = 0
\]
The value of on the boundary assures the constant volumetric flow in consistence with the hypothesis.

**Analysis of the flow**

We introduce the transformation

\[ \tilde{z} = \delta z \]

\[ \eta = \gamma f(\tilde{z}) \]

(1.13)

We assume \( \frac{3}{\delta z} \sim O(\delta) \) such that \( \frac{3}{\delta z} \sim O(1) \). For small values of \( \delta \), the flow develops slowly along the axial direction with gradient of \( o(\delta) \).

Making use of the asymptotic expansion

\[ \psi = \psi_0 + \delta \psi_1 + \delta^2 \psi_2 + \cdots \]

\[ \theta = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 + \cdots \]

(1.14)

and substituting (1.13) and (1.14) in (1.10) and (1.11) and on separation of like powers of \( \delta \), the equations corresponding to the zeroth order are

\[ \Theta_0,\eta \eta' + \frac{1}{\eta} \theta_0,\eta + f^2 = 0 \]

(1.15)

\[ F_1^4 \psi_0 + \eta \frac{G}{K} \theta_0,\eta f^4 - \sigma^2 f^2 F_1^2 \psi_0 = 0 \]

(1.16)
The corresponding conditions are

\[ \psi_0 (1, \bar{z}) = \frac{1}{2}, \quad (\psi_0, \eta) \eta = 0 \]

\[ (\eta \psi_0, \eta \psi_0 - \psi_0, \eta) \eta = 0 \]

\[ \psi_0 (0, \bar{z}) = 0, \quad \theta_0 (1, \bar{z}) = \theta_0, \quad (\theta_0, \eta) \eta = 0 \]

The equations corresponding to the first order are

\[ \frac{R}{\eta} \left[ \psi_0, \eta \left( \frac{1}{F^2} \psi_0 \right)_z - \frac{2}{F^2} \psi_0, \eta \left( \frac{1}{F^2} \psi_0 \right)_z \right. \left. - \psi_0, \bar{z} \left( \frac{1}{F^2} \psi_0 \right)_\eta \right. \]

\[ + \frac{2}{\eta} \psi_0, \bar{z} \left( \frac{1}{F^2} \psi_0 \right)_z - \eta \frac{G}{R} \theta_0, \eta \psi_0^4 = F^4 \psi_0^4 - \sigma^2 F^2 \frac{2}{F} \psi_0 \]

\[ \eta \theta_0, \eta + \theta_0, \eta = R P \left[ \theta_0, \bar{z} \psi_0, \eta - \theta_0, \eta \psi_0, \bar{z} \right] \]  

\[ (1.19) \]

The corresponding conditions are

\[ \psi_i (1, \bar{z}) = 0, \quad (\psi_i, \eta) \eta = 0, \quad \psi_i (0, \bar{z}) = 0 \]

\[ (\eta \psi_i, \eta - \psi_i, \eta) \eta = 0, \quad \theta_i (1, \bar{z}) = 0, \quad (\theta_i, \eta) \eta = 0 \]

\[ (1.20) \]

where

\[ F^2_i = \frac{\delta^2}{\delta \eta^2} - \frac{1}{\eta} \frac{\delta}{\delta \eta} \]
SOLUTION OF THE PROBLEM

Solving the coupled equations (1.15) and (1.16) subject to the corresponding conditions (1.17), we get the expressions for zeroth order

\[ \Theta_0 = f^2 (1 - \eta^2)/4 + \theta_0 \]  

(1.21)

and

\[ \psi_0 = H \eta \mu \frac{I_1(\eta)}{I_3(\eta)} + \frac{M}{H} \left[ \frac{\eta^2}{\mu} (I_1(\eta) k_2(\eta) - I_2(\eta) k_1(\eta)) + \frac{9}{4 \mu \eta^2} \right] - \frac{H_0}{2h} \psi_0 \]  

(1.22)

where

\[ \eta = \eta h \]

\[ I_n(\eta) = \text{Modified Bessel function} \]

\[ K_n(\eta) = \text{Modified Bessel function} \]

\( n = 0, 1, 2, \ldots \ldots \)

\[ \theta_0 = \text{Constant wall temperature} \]

Solving the coupled equations (1.18) and (1.19) subject to the corresponding conditions (1.20), the solution for \( \psi_1 \) is

\[ \psi_1 = A \eta I_1(\eta) + \eta I_1(\eta) \int f(\eta, \xi) k_1(\eta) d\eta - \eta k_1(\eta) \int f(\eta, \xi) I_1(\eta) d\eta \]

where

\[ f(\eta, \xi) = \frac{R}{\eta} \left[ \psi_{0, \xi} \left( \psi_0 - \psi \right) \right] - \frac{2f^4}{\psi_0} \psi_{0, \xi} \left( \frac{\psi_0^2}{\psi_0} \right) - \psi_{0, \xi} \left( \frac{\psi_0^2}{\psi_0} \right) + \frac{2}{\eta} \psi_{0, \xi} \left( \frac{\psi_0^2}{\psi_0} \right) - \frac{\eta G}{k} \theta_0, \psi_1 \]
and the solution for \( \theta_1 \) is

\[
\theta_1 = M_1 \left[ \frac{H_0}{K} I_0 + \frac{M_1}{K^2} \left( I_1 (\eta_1) + B_{15} (\eta_1) + \frac{4}{h} \eta_1 \right) - \frac{H_4}{4K^2} \eta_1^2 + \frac{T_{ef}}{K} \left( \eta_1^2 I_1 (\eta_1) - \frac{\pi}{16} \eta_1^4 \right) \right] + T_3
\]

where

\begin{align*}
B_{14} (\eta_1) &= I_1 (\eta_1) K_2 (\eta_1) - I_2 (\eta_1) K_1 (\eta_1) \\
B_{15} (\eta_1) &= I_3 (\eta_1) K_4 (\eta_1) - I_3 (\eta_1) K_1 (\eta_1) \\

\text{Near the axis of the cylinder (} \eta \to 0\text{), } \Psi_1 \text{ reduces to}
\end{align*}

\[
\Psi_1 = H_{35} \eta^2 I_1 (\eta_1) - \frac{1}{h} H_{21} \eta^3 + \frac{1}{h} H_{23} B_4 (\eta_1) + \frac{1}{h^5} H_{24} B_5 (\eta_1) + \frac{1}{h^6} H_{26} B_6 (\eta_1) + \frac{1}{h^6} H_{36} \eta^2
\]

where

\begin{align*}
B_1 (\eta_1) &= I_1 (\eta_1) K_2 (\eta_1) + I_3 (\eta_1) K_1 (\eta_1) \\
B_2 (\eta_1) &= I_1 (\eta_1) K_3 (\eta_1) - I_2 (\eta_1) K_1 (\eta_1) \\
B_3 (\eta_1) &= I_1 (\eta_1) K_4 (\eta_1) + I_2 (\eta_1) K_1 (\eta_1) \\
B_4 (\eta_1) &= -h^3 \eta_1^2 - 2h^3 \eta_1^4 B_2 (\eta_1) \\
B_5 (\eta_1) &= -h^4 \eta_1^5 + 15 \left( h^4 \eta_1^3 + 6 \eta_1 I_1 (\eta_1) K_1 (\eta_1) - 3 \eta_1^3 \right) \\
B_6 (\eta_1) &= -h^5 \eta_1^6 + 48h^5 \eta_1^5 B_2 (\eta_1) - 4h^5 \eta_1^6 B_4 (\eta_1) + 8 \eta_1^4 \\
B_7 (\eta_1) &= -h^7 \eta_1^7 + 5h^7 \eta_1^7 B_1 (\eta_1) + 105 \eta_1 (20 B_1 (\eta_1) + 5 \eta_1 I_1 (\eta_1)) + 2 \eta_1^3 B_3 (\eta_1) + \eta_1^3 B_4 (\eta_1) \\
B_8 (\eta_1) &= -h^8 \eta_1^8 - 192 \eta_1^7 B_1 (\eta_1) + 12 \eta_1^3 B_2 (\eta_1) + \eta_1^5 B_4 (\eta_1) + 2 \eta_1^3 \right)
\end{align*}
The general expression for $\psi_1$ near the boundary ($\eta > 0$) reduces to:

$$\psi_1 = \frac{H_{125}}{\lambda} I_i(\eta) + M_s \left[ \beta_1(\eta) + \beta_{10}(\xi) + \beta_{11}(\xi) + \beta_{12}(\xi) + \beta_{13}(\xi) \right]$$

where

$$B_{ij}(\eta) = \frac{H_{109}}{\lambda} I_i(\eta) \gamma^3 I_i(\eta) - \frac{H_{121}}{\lambda} K_i(\eta) \gamma^4 I_i(\eta) -$$

$$+ \frac{H_{107}}{\lambda} I_i(\eta) + \frac{H_{108}}{\lambda} I_i(\eta) \gamma^1 I_i(\eta) \exp(-2\eta)$$

The local heat transfer coefficient (Nusselt number) on boundary of the pipe is calculated using the formula

$$N = \frac{1}{f} \left( \Theta_\infty - \Theta_\infty \right) \left( \Theta_\eta \right)$$

where

$$\Theta_{\eta} = 2 \int \Theta \eta \, d\eta$$

and the corresponding expression is

$$N = \left( 0.5f^2 + \delta T_9 \right) / \left( f^2/8 + \delta T_9 \right)$$

(The constant coefficients $A, H, ..., S, ..., T, ..., M, ...$ are explained in detail in Appendix)
DISCUSSION

The axial and radial velocity profiles are drawn for different values of the governing parameters $G$, $\beta$ and $\sigma$ and are shown in figures 1 to 9. Fig.1 indicates the behaviour of axial velocity ($w$) for different $G$ keeping $R$, $\beta$ and $\sigma$ fixed. For numerical purpose the profiles of the non-uniform cylinder has been chosen as $r = f(z) = 1 + \beta \exp(-z^2)$ in the non-dimensional form. $\beta > 0$ corresponds to the dilation of the pipe while $\beta < 0$ corresponds to the constriction of the pipe. It is interesting to note that for $G$ negative, the axial velocity is positive except near the boundaries for sufficiently large $G$, where as for $G$ positive reversed flow appear near the axis for large $G \approx 2 \times 10^4$. However we notice that such a reversed flow does not appear anywhere in the flow field as long as $G \approx 10^3$ and the amplitude of the wall $\beta$ is either small or the coefficient of porosity is greater than or equal to 2. For sufficiently large $G$ ($\beta > 0$) the axial velocity attains the maximum near the axis in the mid region (at a distance $y \approx 0.2$). Also we observe that the magnitude of the axial velocity increases with an increase in $G$, although for small $G \approx 10^3$ the maximum of axial velocity is attained on the axis. The occurrence of the
back flow very much depends on the Grashoff number, the porous parameter and the non-uniformity of the pipe. This is evident from the fact that this reversed flow appears near the axis for positive $G$, while it appears near the boundary for $G$ negative. We also find that the occurrence of such back flow for small value of porous parameter eventhough $G$ is of order $10^3$ irrespective of the amplitude of dilation or constriction. The axial velocity profiles for different amplitudes of the dilation or constriction for fixed $R, G$ and $\sigma$ of pipe have been plotted in fig. 2. It is interesting to note that reverse flow does not appear at a constriction of the pipe, however small the constriction may be. Such back flow occur either in the vicinity of the axis or the boundary for amplitude of the dilation $\beta \approx 0.1$. For amplitudes $\beta \approx 0.3$ the reverse flow appears near the axis where as for smaller amplitudes it appears near the boundary. The maximum of $w$, however occurs on the axis.

For large $G \approx 10^4$ and $\sigma \approx 3$, the profiles of $w$ reveal wavy character in a dilated case with $w$ increasing from its zero value on the boundary to certain value in the region $0 < \eta < 0.7$ and further decrease to lower value in the mid region $\eta \approx 0.5$. Later it increases to its maximum on the axis in the remaining mid half of the region. For small values of $\beta \approx 0.5$ however $w$ increases in the mid half
abetic the boundary and later decreases to a lower value at
the vicinity of the axis \( \eta \approx 0.2 \) before increasing
rapidly to reach maximum on the axis. A similar behaviour
may be observed in a constricted case for \( \beta \approx -0.05 \). For
sufficiently large \( \beta \), the velocity \( w \) decreases from its
maximum on the axis to zero on the axis. The decay is found
to be rapid in the vicinity of the axis and the boundary.

Fig 3 reveals that no reverse flow appear for small value \( G \approx 10^3 \) and \( \beta \approx 0.05 \) and \( w \) decreases from its maximum to zero
on the axis for all \( \sigma \gg 1 \), keeping \( \beta \) small for large \( G \approx 10^4 \). Fig 4 shows that \( W \) is completely negative for \( \sigma \approx 1 \)
with magnitude \( w \) gradually decreasing from its maximum on the
axis to zero. For \( \sigma \gg 2 \) the profile exhibits wavy nature
with sudden decay near the axis in the range \( 0 < \eta < 0.2 \) and
a gradual increase towards \( \eta \approx 0.7 \) before attaining zero on
the boundary, although at \( \sigma \approx 2 \) reverse flow is found near the
axis while no such flow appears for higher \( \sigma \). A comparison of
fig 3 & 5 shows that the larger the amplitude of dilation
lesser the magnitude of \( w \) at all points in the flow field for
all \( \sigma \). Also for \( \sigma \approx 1 \) we find the development of back flow
near the axis in consistent to the case of small amplitude
dilation. Again comparing Fig 4 and 6 we observe that for
large dilation (fig 6) and for small \( \sigma \approx 1 \) the reversal
flow does not appear in the mid region \(0.2 < \eta < 0.7\). Also \(w\) attains its maximum at \(\eta = 0.2\) in case of \(\sigma = 3\). The flow is wavy with magnitude of \(w\) increasing and decreasing at every quarter of the radial distance. For higher values of \(\sigma\), the reversed flow gets confined to the vicinity of the axis and the wavy nature continues to exist. The point of maximum \(w\) which appears at \(\eta = 0.2\) for \(\sigma = 2\) pushes to the axis.

The non-uniformity in the boundary geometry gives rise to secondary radial flow (Fig 7, 8). In view of the axisymmetry the radial velocity \(u\) vanishes on the axis and hence increases in its magnitude from zero to a maximum value on the mid layer and gradually reduces to zero on the axis. Thus the radial velocity profiles are parabolic type in general for variation in \(G, \sigma\). In general \(u\) increases with \(G\) and growth of \(u\) with \(G\) is very rapid in a dilated case in comparison to constricted case (Fig. 7, 8). From fig. 9 we observe that \(u\) increases with an increase in \(\sigma\) and for a fixed \(\sigma\) we find a steep rise towards the mid layer either from the axis or the boundary.

Fig. 10 and 11 exhibits the behaviour of the temperature field and it is found that \(\theta\) rises rapidly in the radial direction from its prescribed value on the boundary to a maximum attained on the axis of the pipe. Also
we observe that the temperature in a dilated pipe at any radial distance is comparatively larger to that corresponding value in constricted pipe. Fig 11 indicates the behaviour of the perturbation in temperature with reference to the variation in $G$. This perturbed temperature is of order $10^{-4}$ in either of the cases $G>0$ or $G<0$. For a fixed $\sigma$, $R$, and $\beta$, the actual values of $\theta_1$ evidently show that the perturbation in case of positive $G$ is to reduce the total temperature in the entire field while the perturbation in case of negative $G$ raises the total temperature slightly. Supposing the boundary is at a lower temperature level than in the mid region in the equilibrium state ($G>0$) the total temperature decreases with an increase in $G$. This is because the corresponding perturbed temperature (Fig 11) contributes to its reduction at all radial points, and its magnitude increasing with an increase in $G$. However, when the equilibrium temperature is greater than the corresponding axial temperature, ($G<0$), contributes to growth in the total temperature, the magnitude of which increases with an increase in $G$ ($G<0$). The decay in the case of $G>0$ or the rise in the case of $G<0$ of the total temperature reaches its peak at a radial distance $\eta \approx 0.2$ near the axis.
The average Nusselt number is calculated for different $G$ and is Tabulated in Table. 1,2. In general the Nusselt number is negative and both in the dilated or constricted cases its magnitude decreases with an increase in positive $G$ while it increases with an increase in $G (< 0)$. Also the magnitude in constricted case is larger than the magnitude in dilated case for all $G (< > 0)$. This confirms that the rate of heat transportation is much efficient in a constricted pipe in comparison to a dilated pipe.
# THE AVERAGE NUSSELT NUMBER

## TABLE-1

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Variation with $G$ when $\beta=0.5$, $\sigma=2$, $R=50$, $\varepsilon=\Pi/4$.

\begin{align*}
G. & \quad a,d \quad b,e \quad c,f \\
& \quad \pm 2 \times 10^3 \quad \pm 2 \times 10^4 \quad \pm 3 \times 10^4
\end{align*}

FIG. 1
Axial velocity $W$ with $\beta$ when

$G=30000$, $R=50$, $\sigma=3$, $\phi=\pi/4$

$\beta = 0.05$  $0.1$  $0.3$  $0.5$

FIG. 2a
Axial velocity $W$ with $\beta$ when $G=30000, \sigma = 3, R = 50, \pi = 3\pi/4$.

$\beta$ -0.05 -0.1 -0.3 -0.5

FIG. 2b
Axial velocity $W$ with $\sigma$ when $G=2000$, $\beta=0.05$, $\epsilon=\eta/\epsilon$, $R=50$.

$\sigma$:  
1  2  3

FIG. 3
Axial velocity $W$ with $\sigma$ when $\theta = 20000, \beta = 0.5, z = r/4, R = 50$. 

FIG. 4
Axial velocity $W$ with $\sigma$ when $G=2000$, $\rho=0.5$, $R=50\pi/4$. 

FIG. 5
Axial velocity $W$ with $\sigma$ when $G=20000$, $B=0.5$, $z=1/4$, $R=50$. 

FIG. 6
Radial velocity $U$ with $G$ when $R = 50, B = 0.5, \sigma_m = 2, \pi = 1/4$

- $a, d$
- $b, e$
- $c, f$

$G \pm 2 \times 10^3 \pm 2 \times 10^4 \pm 3 \times 10^4$

FIG. 7
Radial velocity $U$ with $G$ when $R = 50$, $\sigma = 2$, $\beta = -0.5$, $z = \frac{w}{4}$.

$G = 2 \times 10^3$  
$/ 2 \times 10^4$  
$/ 3 \times 10^4$

FIG. 8
Radial velocity $U$ with $\sigma$ when $G=2000$, $\beta = 0.5$, $\xi = TV/4$

$\sigma$ \quad a \quad b \quad c
\quad 1 \quad 2 \quad 3

FIG. 9
(a) $\beta = 0.05$  (b) $\beta = -0.05$, $x = \pi/4$

Temperature ($\theta$) profile

FIG. 10
Temperature ($\Theta_1$) profile with $G$

$B = 0.05$, $\sigma = 3$, $R = 50$, $\pi = \pi/4$.

$\eta = 0$, $e = \pm 2 \times 10^4$

$\pm 3 \times 10^4$  $\pm 4 \times 10^4$

FIG: 11