The task of measuring the value of an asset or a firm's stock is most interesting and challenging as the process of valuation is vexingly difficult. The value of a particular asset or stock of a firm is assumed to be the fair price that an investor would be willing to pay. The determination of this fair price depends on different factors like - the expected return arising out of the proposed asset, the burden that is to be borne by the investor to earn such an amount of expected return (i.e. risk) and the period during which the investor has to wait for this return or yield. Thus, the task of such a valuation involves forecasts for all of these determinants but probably the most difficult part of the job is to measure the risk of a particular asset as well as its incorporation into a valuation model.

Thus, a body of positive microeconomic theory dealing with risk is most essential for the prediction of the behaviour of share prices (or for that matter, the prices of the capital assets) and the capital market in general. Different attempts have been made to tackle the problem of determining the attractiveness of a particular asset and thus a number of valuation methods have been developed. While the older valuation techniques were concerned with the evaluation of individual assets in isolation, the modern view of the
investment is oriented more toward the assembly of judicious combinations of individual assets or securities to form a portfolio and all the investments are evaluated in a portfolio sense. This modern view considers the risk of an investment in terms of its effect on the investor's portfolio. The inclusion of an individual asset is characterised by a trade-off of its expected return against its contribution to the portfolio risk. With this basic premise, the Modern Portfolio Theory (MPT) ultimately develops into general equilibrium models of the determination of the price of capital assets under conditions of uncertainty. The two major directions of these developments are -(i) the 'mean-variance models' following the pioneering work of Harry Markowitz and (ii) the 'stock-preference models' originally developed by Arrow and Debru.¹


These two approaches are actually the generalisation of the work of Inving Fisher of the theory of interest to a world of uncertainty. Of these two approaches, perhaps, the State preference approach is more 'general', and provides an excellent framework for investigating theoretical issues. But unfortunately, it is very difficult to provide any empirical content on it. And hence, it is not considered in the present study.

The research on Investment Management and on the behaviour of the capital market have gained tremendous momentum with the development of the Portfolio Selection Model by Harry Markowitz. Following his work, the model which first dealt with the task of constructing a market equilibrium theory of asset prices under condition of risk was developed by W.F. Sharpe. Several other models have also been developed by


3 W.F. Sharpe: "Capital Asset Prices, A Theory of Market Equilibrium under Conditions of Risk". Journal of Finance, 19, Sept., 1964, 425-42. Before developing this model, the author presented a simplified version of the Markowitz model which is known as 'Diagonal' or 'Market' model. This will also be discussed here in brief. A brief review of the other alternative equilibrium models will also be presented here.
many authors like John Lintner, F. Fama, Mossin and others. These models are popularly known as Capital Asset Pricing Models which in comparison with other adaptation models of the MPT" can give us a better feel for the CAPM's value as a method of divining the future". And of them, the Sharpe model is, perhaps, the most accepted one. This chapter however, has been designed to consider the MPT and the CAPM on the basis of the Markowitz Model of Portfolio Selection and the Capital Market Equilibrium Theory developed by Sharpe. The main thrust will surely be on the notion of risk that has been used in these two models.

Both the models argue that a portion of the total risk of an individual asset can be eliminated by its inclusion in a portfolio in a judicious manner. The remaining part of the risk i.e. the variability of its return is related to the general market changes which is measured by the average covariances in the Markowitz model. The same, on the other hand, has been measured in the Sharpe's model by the slope of the Security market line, denoted as beta (β) coefficient which in this study will be used as the relevant measure of risk.

The 'Mean-variance Portfolio Selection Model' or the Markowitz model and the Diagonal model developed by Sharpe will be analysed in the first section of this Chapter. Section II will be devoted to provide an in-depth study of the equilibrium model of the capital market developed by Sharpe. A brief review of the other alternative models will be presented in Section III, while Section IV will contain the concluding remarks on the basis of the analysis presented in the previous sections.

Section - I

A. Markowitz's Mean-variance Model

The pioneering work of Harry Markowitz is the first attempt to deal with risk in a portfolio sense. The model rests on the following assumptions:

1. The investors are guided by the objective of the maximisation of the utility of their terminal wealth;

2. They have the homogeneous expectations about the risk and return;

3. They have identical time horizons;

4. All the informations are readily available to each of them;

5. The investors are basically risk averters i.e., they
prefer low risk to high; and

6. All investment decisions are made on the basis of Portfolio return and Portfolio risk.

According to this model, the investors will choose investments that provide the highest return for a given level of risk or those that offer least risk for a given return. The model defines the riskiness of a portfolio in terms of variance of portfolio returns\(^5\), and the return of a portfolio is simply the sum of the returns from the stocks which are included in the portfolio. The model develops the concept of an 'efficient frontier' that has been defined as 'a set of alternative efficient portfolios of assets'. These portfolios are efficient in the sense that each of them provides maximum expected return at its level of risk. Of these alternative portfolios, the investor's attitude towards risk which

\(^5\) Variance can be an appropriate measure of risk subject to the conditions as under:

1. The utility function of the investor has two properties (i) the first derivative be positive and (ii) the second derivative be negative (if, the risk-averse utility function for wealth) and

2. The distributions of returns of the individual securities are stable with a finite variance (i.e., normal distribution).
is represented by his utility function acts as the ultimate guiding factor to select the most suitable portfolio for him.

Although the Mean-variance model uses 'variance' as the measure of portfolio risk, there are some other measures also. Those alternative measures such as the mean-absolute deviation, semi-standard deviation, the range, the interquartile range and other central moments of the distribution can also be used in place of variance. The empirical evidence collected by Fama has shown that the variance is highly related with other popular dispersion measures at the portfolio level\(^6\). In the modern portfolio theory, however, variance and the standard deviation are used for measuring the portfolio risk. On the other hand, the risk of an individual security is measured by its average covariance with other securities in the portfolio - where covariance is simply the co-relation co-efficient multiplied by the product of the standard deviations of the expected returns of the two securities.

The portfolio variance is composed of two elements - (i) the mean of variances of the individual securities and (ii) the mean of the covariance for individual security with every other security.

Assuming that equal amount is invested in each security of the portfolio, it can be mathematically expressed in the following manner:

\[ \sigma^2(\tilde{R}_p) = \frac{1}{N} \sigma^2(\tilde{R}_i) + \frac{N-1}{N} \sigma(\tilde{R}_i, \tilde{R}_j) \]  \hspace{1cm} (1.1)

where, \( \sigma^2(\tilde{R}_p) \) = variance of the portfolio returns;

\( \sigma^2(\tilde{R}_i) \) = mean of the individual security's variance in the portfolio;

\( \sigma(\tilde{R}_i, \tilde{R}_j) \) = mean of the covariance for individual security with every other security,

\[ \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma(\tilde{R}_i, \tilde{R}_j)} / N \]

and \( N \) = number of securities in the portfolio.

As \( N \) increases the first term of the equation 1.1 tends to zero and the second term converges to the average covariance among the securities in the portfolio. Hence, for a well diversified portfolio (where \( N \) is sufficiently large), the portfolio risk is given by the average covariance only because in that case the first term of the equation i.e., the mean of the individual security's variance becomes zero.

The above proposition leads to the conclusion that by efficient diversification an investor can reduce the portfolio
risk. For the reduction of the portfolio risk an investor should put together assets whose returns are not highly correlated - or in other words, by increasing the number of assets with uncorrelated returns in his portfolio. The portion of the total risk that can be diversified away is called the non-market related risk. To put it in another way, the basic market variability or the effect of general economic activity, on the returns of all stocks can not be eliminated. An investor, however, can almost completely eliminate the remaining variability of returns that can be termed as the firm specific or industry specific risk by investing his funds in an efficient portfolio.

The application of the Markowitz model requires the estimation of the future returns, variances of returns of each of the investments, the portfolio variance and the correlation between the returns of each pair of investments, that comprise the portfolio. The main problem in using this models is the need for enormous parameter estimation especially when \( N \) becomes large. For example, for a portfolio composed of four securities, the investor is required to estimate the values of 10 parameters only. When \( N \) becomes 10, the estimation of 55 parameters are required to be made. Accordingly for a portfolio with 1000 securities included in it, the required parameter estimation goes up to 500500 which is almost impossible for an individual if no computer facilities are available to him. Now-a-days, however, the readily available
computer programme packages have made the task quite easy.

B. Diagonal Model

The intuitive logic of the Markowitz model has attracted a number of people as a result of which several simplified versions have been developed. Of them, the 'Diagonal model', which is popularly known as the 'Market model', developed by W.F. Sharpe is perhaps the best known among those applications of the Markowitz model. It is the logical extension of the modern portfolio theory both intuitively and mathematically.

The problem relating to the enormous amount of parameter estimation of the Markowitz model has been greatly simplified by Sharpe in this model with the help of an index of the market returns. This index represents the weighted average of the returns of all the assets available in the market. For example, in order to estimate the portfolio risk with 20 securities included in it, the Markowitz model requires the estimation of 210 parameters while the 'Diagonal model' can perform the same job with only 40 parameter estimates.

The model specifies the stochastic process generating security returns as under:

\[ \tilde{R}_i = \alpha_i + \beta_i \tilde{R}_m + \tilde{\varepsilon}_i \]  

where, \( \tilde{R}_i \) represents the return on security \( i \), \( \tilde{R}_m \) stands for the return on all assets available in the market i.e., market return; \( \alpha_i \) and \( \beta_i \) are the intercept and slope respectively.

\[ \tilde{\beta}_i = \frac{\sigma ( \tilde{R}_i, \tilde{R}_m )}{\sigma^2 ( \tilde{R}_m )} \]

\( E(\tilde{\varepsilon}_i) = 0 \) for all \( i \neq 0 \),

\( \sigma ( \tilde{R}_m, \tilde{\varepsilon}_i ) = 0 \) for all \( i \neq 0 \),

\( \sigma ( \tilde{\varepsilon}_i, \tilde{\varepsilon}_j ) = 0 \) for all \( i \neq j \).

and the tilde is used to denote the random variable.

Now on the basis of an assumption that equal amount is invested in each security the portfolio variance can be written in the following manner:

\[ \frac{2}{N} \sigma^2 ( \tilde{R}_p ) = \left( \frac{1}{N} \sum_{i=1}^{N} \beta_i \right)^2 \sigma^2 ( \tilde{R}_m ) + \left( \frac{1}{N} \sum_{i=1}^{N} \sigma^2 ( \tilde{\varepsilon}_i ) \right) \]

If the \( \beta_p \) is given by \( \frac{1}{N} \sum_{i=1}^{N} \beta_i \) then the above equation (1.3) can be rewritten as:

\[ \sigma^2 ( \tilde{R}_p ) = \beta_p^2 \sigma^2 ( \tilde{R}_m ) + \frac{1}{N} \sigma^2 ( \tilde{\varepsilon}_i ) \]  

Now, let it be assumed that the portfolio is composed of only one security, then the equation 1.4 comes down to:

\[ \sigma^2 ( \tilde{R}_i ) = \beta_i^2 \sigma^2 ( \tilde{R}_m ) + \sigma^2 ( \tilde{\varepsilon}_i ) \]  

15.
The above expression shows that the security variance is composed of two elements which is analogous to the Markowitz model (equation 1.1). If \( N \) becomes sufficiently large, the last term of the equation (1.4) comes down to zero and the portfolio variance given by \( \sigma^2 (\bar{R}_p) \) can be measured by the first term \( \sum \beta_p^2 \sigma^2 (\bar{R}_m) \) only. Furthermore, the return on the market portfolio \( (\bar{R}_m) \) is equal for all securities at a particular point of time, and thus the market variance can never be the source of any difference between the variances of two different portfolios and hence, can be eliminated. For a well diversified portfolio, however, \( \beta_p \) is the sole measure of its riskiness and similarly, for an individual security the relevant risk which it contributes to the portfolio variance is represented by \( \beta_i \).

It may be recalled that in the Markowitz model the two elements of the individual security's riskiness have been referred to as market risk and the individualistic or non-market risk. The 'Diagonal model', equivalently, defines them as systematic risk and the unsystematic risk respectively. According to this model the market risk can be called as the systematic risk and it can never be eliminated as it represents the effect of general economic changes on the security return. On the other hand, the individualistic trend of a security can be completely diversified away by means of assembling it with other unrelated securities to form an efficient portfolio and this portion of the security risk is
called diversifiable risk or industry and firm specific risk. The empirical evidence of B. King shows that the industry effect on the security risk account for about 10 per cent of the realised return of the security\(^8\). Thus it is evident that the unsystematic risk is attributable to the specific features of the industry as well as of the firm itself.

Probably the most important feature of the 'Diagonal model' is that it brings about a dramatic change in the efficient frontier concept of the Markowitz model. Markowitz views the efficient frontier as a set of alternative portfolios of different securities. On the other hand, Sharpe describes the market portfolio as the ultimate limit of the efficient diversification\(^9\). As the market portfolio is the most efficient, the unsystematic risk of a security is completely diversified away. And thus, the variance of the market portfolio represents the systematic risk only. Sharpe has shown


\(^9\) The 'Market Portfolio' has been defined as the complete universe of all available risky assets which is almost impossible to calculate precisely. Thus, a proxy for it may be used to represent \(R_m\) in equation (1.2). The stock price index is often used as the proxy for \(R_m\). Any other index, however, can be used for this purpose.
that beta (\(\beta\)) for the market portfolio will be equal to unity. This implies average riskiness of the portfolio, because every one per cent change in the market rate of return results in an equivalent change in the portfolio return (as the portfolio itself is the market portfolio). On the basis of the magnitude of beta, Sharpe defines a security as defensive or aggressive. Any security with \(\beta < 1\) is called defensive, as it responds at a lower rate to the changes in the market as a whole. On the other hand, aggressive stocks are those the return of which changes at a higher rate due to any change in the economy and in that case \(\beta\) will be greater than unity. According to Sharpe, larger the value of \(\beta\), the more aggressive the security\(^{10}\).

The possibility of a negative beta is negligible but can not be completely ruled out.

According to Sharpe, the intercept of the equation (1.2) is the average return on a security which is independent of the market return (i.e., specific component of return). It may assume any value (either positive, or negative or even

Finally, the Markowitz's definition of the riskiness of a portfolio in terms of variance, is appropriate only when the return distribution of the individual securities is stable with a finite variance (i.e., normal distribution). While the market model can be applied in general where the return distributions of a security are characterised by a family of distributions of which normal distribution is a special care. The empirical evidence by Fame show that $\beta$ can be used as an appropriate measure of risk in those cases also where the return distributions of a security have finite expected values but infinite variances and covariances.

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11 According to Sharpe, "There is some interest rate ($r_t$) at which money can be lent with virtual assurance that both principal and interest will be returned; at the least, money can be buried in the ground ($r_t=0$). Such an alternative could be included as one possible security ($A_i = 1+r_t$, $B_i=0$, $Q_i=0$) but this would necessitate some needless computation. In order to minimise computing time, lending at some pure interest rate is taken into account explicitly in the diagonal code", - W.F. Sharpe: A Simplified Model For Portfolio Analysis". - Management Science, 9, (January, 1963), p.285.

Section - II

Capital Market Theory

In the previous section, the Modern Portfolio Theory and the 'Diagonal' or 'Market Model' have been analysed. Those models can be considered as the basis for the development of the theory of equilibrium capital market. One of the pre-requisites for the analysis of the risk-return equilibrium relationship of the capital market is the introduction of the notion of an 'Efficient Market'.¹³ A number of equilibrium pricing models have been developed of which the CAPM is a sub-set. The model was originally developed by W. F. Sharpe and subsequently extended by Lintner, Mossin and Fama.¹⁴


The present section has been devoted to review the risk-return equilibrium relationship of the Capital Market on the basis of the CAPM. As the main purpose of this venture is to search out the appropriate measure of risk of an asset, an in-depth analysis of the CAPM is needed. A bird's eye view of some of the alternative models will also be presented here.

A. Capital Asset Pricing Model

To construct the CAPM, following three additional assumptions are used (along with those that underlie the MPT and the Diagonal Model):

i. All investors can borrow or lend at a risk-free rate. It refers to a return about which the investors are certain with respect to its magnitude and timing. But other risks associated with an investment such as the loss of purchasing power remain present at this investment (risk-free asset).

ii. For the sake of simplicity it is assumed that there are no taxes and transaction costs. The quantity of total assets is fixed and all of them are infinitely divisible and marketable.

iii. The desirability of a particular asset is determined on the basis of different parameters such as mean (expected return), variance (risk) and covariance between the returns of the two assets. The investors possess homogeneous expectations about them.
Analysing the investor behaviour and on the basis of the correlations between the expected returns of each pair of investment plans available in the market, the model specifies the investment opportunity curve such that - (i) all investment plans should lie along this curve and (ii) any investment beyond this curve is impossible. And in order to satisfy his objective an investor should select that efficient plan lying on a particular point on the investment opportunity curve where the indifference curve (that represents the investors' preference) becomes tangent to it.

With the introduction of the risk-free rate, the notion of the opportunity curve is approximated to a straight line representing the efficient portfolios which are composed of an investment (or disinvestment) in the risk-free asset and that in the most efficient portfolio lying at the point where the straight line is tangent to the opportunity curve. This straight line (which is called the capital market line) portrays the equilibrium risk-return relationship of the efficient portfolios.

The author defines the efficient plan as "A plan is said to be efficient if (and only if) there is no alternative with either (1) the same $E_R$ and a lower $\sigma_R$, (2) the same $\sigma_R$ and a higher $E_R$ or, (3) a higher $E_R$ and a lower $\sigma_R$" - W.F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", Journal of Finance, 19, (September, 1964); p.429.
Depending on the demand for the assets that comprise the most efficient portfolio above, the process of price revision goes on continuously so that a number of new combinations of risky assets crop up (those which were previously inefficient). And this process continues until all assets are included in at least one such combination and a set of prices is reached. Thus, the capital market equilibrium condition requires that each asset should be included in one of the efficient combinations of risky assets and hence, there will be a perfect correlation between them as they lie along a linear border of the investment opportunity curve.

The capital market line represents a linear relationship between the expected return and risk in equilibrium and can be mathematically shown as under\textsuperscript{16}:

\[
E(\bar{R}_M) = P + r_e \delta (\bar{R}_M)
\]

\textsuperscript{16} For any efficient portfolio, the equation of the CML can be written as \(- E_p = P + r_e \delta p\). The characteristics of the CML are that: (a) the intercept is equal to the risk-free rate and this risk-free rate can be called as the price of time or the price of immediate consumption; (b) the slope represents the trade-off between the expected return and risk. As any increase in the expected return is accompanied by a corresponding increase in risk and vice versa, the slope representing such a relationship indicates the amount of expected return that is to be foregone for the reduction of risk. In other words, the slope of the CML determines the price of risk.
where $E(\tilde{R}_M) = \text{the expected return on the market portfolio.}$

$P = \text{the pure interest rate or the intercept of the linear equation (or the CML)}$

$r_e = \text{the slope of the CML}$

and $\sigma (\tilde{R}_M) = \text{the standard deviation of the return on the market portfolio.}$

Thus, the slope of the CML (or the price of risk reduction $r_e$) can be written as: $r_e = \frac{E(\tilde{R}_M) - P}{\sigma (\tilde{R}_M)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 1.7.$

While the CML represents the risk-return equilibrium relationship for the efficient portfolio the same for the individual security has been described by the Security Market Line (SML) in CAPM. The equation of the SML can be written as:

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17 The CML portrays the equilibrium risk-return relationship for the efficient portfolios only and thus, it does not hold for the individual security (or the inefficient portfolios). On the other hand, a study of the relationship between the individual security and the efficient portfolio will be useful for this purpose. Another important point of disagreement between the two equilibrium relationships stems from the diversity in the assumptions of the appropriate measures of risk for individual security and the portfolio. While the S.D. or variance is used to measure the portfolio risk, the same for an individual security is measured by the covariance of its return with that of the portfolio (or the market portfolio in equilibrium). To describe the risk-return equilibrium relationship of an individual security the author has derived the security market line.
\[ E(\tilde{R}_i) = P + r_s \cdot C(\tilde{R}_i, \tilde{R}_M) \]  

where the slope of the SML is given by \( r_s \).

From the equation (1.8) the slope of the SML can be shown as:

\[ r_s = \frac{\sqrt{E(\tilde{R}_i)} - P}{\text{Cov}(\tilde{R}_i, \tilde{R}_M)} \]  

The risk-return equilibrium condition requires, as before, that all securities should lie along the security market line. A study on the relationship between the returns of a security and that of a portfolio reveals the following feature:

\[ \sqrt{E(\tilde{R}_i)} - P = \frac{E(\tilde{R}_M) - P}{\sigma(\tilde{R}_M)^2} \cdot C_{iM} \]  

where the left hand side of the equation represents the risk-premium for bearing excess risk with regard to an investment in security \( i \) and \( C_{iM} \) represents the Cov \( (\tilde{R}_i, \tilde{R}_M) \).

From the equation (1.10) it is evident that reward for bearing risk \( (E_i - P) \) is a function of the covariance between the rate of return of the security and that of the market portfolio.

Equivalently, the equation (1.10) can be written as:

\[ E(\tilde{R}_i) = P + \beta_i \cdot \sqrt{E(\tilde{R}_M)} - P \]  

---

18 The mathematical manipulation is given in the Appendix at the end of this chapter.
or, $\sqrt{E(\bar{R}_i)} - P = \beta_i \sqrt{E(\bar{R}_M)} - P \quad 1.12$

or, $E(\bar{R}_i) = P (1 - \beta_i) + \beta_i \sqrt{E(\bar{R}_M)} \quad 1.13$

where $\beta_i = \text{Cov.} (\bar{R}_i, \bar{R}_M) / \sigma^2 (\bar{R}_M)$

$\bar{R}_i$ and $\bar{R}_M$ denote the returns on the security $i$ and that of the market portfolio respectively.

It is now evident from the above expressions that the expected return on an individual security is equal to the risk free rate plus a premium for bearing risk proportional to its beta ($\beta_i$)$^{19}$. The relationship is linear and greater the value of beta ($\beta$) higher the value of the expected return for a security.

**Beta (or the Volatility as a Measure of Risk)**

As the market portfolio refers to the complete universe of risky investments available in the market, the covariance between the returns of the market portfolio and that of an individual asset will reveal the effect of an asset's dependence on the overall level of economic activity.

$^{19}$ As the pure interest rate ($P$) and the risk premium on the market portfolio $\sqrt{E(\bar{R}_M)} - P$ are same for an securities, the expected return on security $i$ depends mainly on the beta ($\beta_i$).
To examine the relationship, a regression analysis can be made. If a set of observations relating to the returns of security \(i\) and that of the market portfolio are plotted in a '\(R_i - R_M\)' plane and a line of best fit is drawn (which is called the regression line or the security's characteristic line), the slope of this line represents the responsiveness of \(R_i\) with respect to \(R_M\) (common market factor).

The equation of the characteristic line is

\[
\tilde{R}_i^* = a_i + b_i R_M
\]

and as the line passes through a point representing both of \(E_{R_i}\) and \(E_{R_M}\), then -

\[
E_{R_i} = a_i + b_i E R_M
\]

where \(a_i\) and \(b_i\) are the intercept and the slope respectively and \(\tilde{R}_i^*\) stands for the predicted value of \(R_i\).

Now the equation (1.14) and (1.15) can be used directly to show that -

\[
\text{Cov.} ( R_i, R_M ) = b_i \sigma^2_R M
\]

or, \(b_i = \frac{\text{Cov.} ( R_i, R_M )}{\sigma^2_R M}\)
which is the equation of the slope of the characteristic line\textsuperscript{20}.

Now, the equation of the security market line can be expressed in terms of volatility as under (It may be recalled that in equation (1.8) the relationship has been shown in terms of covariance):

\[ E(R_i) = p + \sqrt{E(R_M)} - \sqrt{E(R_M)} b_i \quad \ldots \quad 1.16. \]

or,

\[ \sqrt{E(R_i)} - \sqrt{E(R_M)} = \sqrt{E(R_M)} - \sqrt{E(R_M)} b_i \quad \ldots \quad 1.17. \]

\textsuperscript{20} The predicted covariance between the return from the security \( i \) and that of the market portfolio can be written as

\[ \text{Cov}(R_i, R_M)^* = \bar{\beta} \text{Pr}(R_M) (R_i - E(R_i)) (R_M - E(R_M)) \]

putting the values for \( R_i^* \) and \( E(R_i) \) from equation (1.14) and (1.15), we get -

\[ \tilde{C}_{iM}^* = \bar{\beta} \text{Pr}(R_M) \sqrt{\sum (a_i + b_i R_M) - (a_i + b_i E(R_M)) (R_M - E(R_M))} \]

where \( \tilde{C}_{iM}^* = \text{Cov}(R_i, R_M)^* \)

Now, \( \tilde{C}_{iM}^* = b_i \bar{\beta} \text{Pr}(R_M) \sqrt{R_M - E(R_M)} \sqrt{R_M - E(R_M)} \]

\[ = b_i \bar{\beta} \text{Pr}(R_M) \sqrt{R_M - E(R_M)} \sqrt{R_M - E(R_M)} \]

\[ = b_i \bar{\beta} (R_M)^2. \]

If the line portrays the relationship between \( R_i \) and \( R_M \) properly then \( C_{iM}^* \) should equal to \( C_{iM} \), i.e., predicted covariance will be equal to the actual covariance.
Hence, the risk premium for security \( i \overline{E}(\tilde{R}_i) - P \) is proportional to its risk measured by beta (\( b_i \)) and thus, for an individual security beta is the appropriate measure of risk.

A study on the relationship between the returns of an individual security and that of the market portfolio reveals that the risk can be decomposed into two distinct parts: systematic and unsystematic.\(^2\)

The effect of the swings in the market as a whole on the individual security represented by beta (\( \beta_i \)) is the systematic risk. And as the efficient diversification results in the complete elimination of the unsystematic portion of the total risk, systematic risk or beta can be regarded as the appropriate measure of risk for an individual asset.

\(^2\) The characteristic line is drawn in such a way that the algebraic sum of deviations in the individual observations is equal to zero. The scatter of the \( R_i \) observation around their mean \( \overline{E}(R_i) \) is the evidence of its total risk - \( \sigma R_i \). But part of the scatter is due to an underlying relationship with the return on the market portfolio. This relationship is measured by the slope of the characteristic line (\( b_i \)) and this part of the total risk is called the systematic or undiversifiable risk. The other part which is uncorrelated with the market rate of return represents the individualistic factor of the security and is called the unsystematic or diversifiable risk. The model asserts that by efficient diversification this portion of the total risk can be completely eliminated.
The last few paragraphs have been used to provide a brief account of the MPT and the Capital Asset Pricing Model. In both the theories the twin aspects of an investment plan i.e., return and risk measured by its mean and variance respectively, have been aptly used to describe a useful technique for the evaluation of a project. On the basis of some logical economic objectives, these theories describe the effect of investor's behaviour on the prices of assets or securities rather than providing a narration of the capital market in general. This normative (rather than descriptive), quality of these models is the main cause for its being criticised by the practitioners on the one hand and for its acceptance by the academicians on the other. Both the models rest on some restrictive and unrealistic assumptions which have been described by the practitioners as "ivory tower" distortions of reality. Of them probably, the assumption relating to the market efficiency has led the management practitioners to bark against the models. An attempt will, however, be made to analyse the assumptions that underlie both the MPT and the CAPM briefly in the next few paragraphs.

The assumptions relating to the market efficiency will be taken up first. The other conditions that underlie the CAPM will also be considered here.

1. Both the MPT and the CAPM assume that the investors are guided by the objective of the maximisation of the utility
of their terminal wealth (not the wealth or return itself). This assumption is required to describe the differences in the individual preferences. The investor's utility functions depend on their attitude towards risk. For example, for a risk averse investor the utility function can be described as the diminishing marginal utility for wealth. On the other hand, the same for a risk lover/taker is characterised by an increasingly positive marginal utility for wealth. But for the risk-neutral investor the utility is neither increasing nor diminishing, it is constant. In that case, it depends only upon the available combinations of risk and return.

The main problem associated with this assumption is that the objective of the maximisation of utility of the terminal wealth does not consider the distinctive features of the dividends and the capital gains. Though, in practice, they are not equally attractive to all types of investors (i.e., short-time speculators, or the 'buy and hold' type investors) the models do not recognise them. If such differences were considered the use of covariance to measure the risk would become inadequate and this would make the model more complex.

2. The models assume that the investment (portfolio) decisions are made on the basis of risk and return measured respectively by the variance and the mean of the returns. Beta is also used to describe the portfolio risk. The use of portfolio variance as a measure of portfolio risk limits the
scope of application of the models. They can be used accurately in cases where the return distributions are normal. The CAPM is unable to provide adequate description of an investor behaviour if the returns are skewed. Some authors opine that semi-variance can be an alternative for the variance. But, the use of semi-variance (in place of variance) for measuring risk will produce different results from any decision made by using variance as a risk measure.

3. The notion of single efficient frontier requires that investors possess homogeneous expectations about the risk and return of an investment. This assumption, indeed, is an oversimplification of the real world situation. Because, investors will seldom agree on the future prospect of an investment plan. For example, some investors may prefer dividends, while others may be interested in the magnitude and timing of the capital gains only. In addition to that, investors may use different definitions for the market factor.

Such a problem of investor behaviour (non-homogeneous expectations) has been adequately considered in the 'State

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Preference Models' which are originally due to Arrow and Debru\textsuperscript{23}.

Another alternative explanation of investors' behaviour has been developed by Miller (1977)\textsuperscript{24}. But all these models provide the description of the capital market activity in a complex manner.

4. The assumption of identical time horizon for all investors is quite unrealistic. Because, the capital market is composed of different types of investors. The characteristics of the assets are also different. In such a situation, the assumption of a common time horizon can not provide an appropriate description of the investor behaviour and then, a single-period model can not be a good fit. Instead, a continuous model may do the job more accurately\textsuperscript{25}.


Another serious limitation of the theories in question is that they do not make any distinction between the short term fluctuations and the long term cycles in so far as the measurement of risk is concerned. Empirical evidences are sufficient to rule out the validity of this assumption. Fama and MacBeth have shown that the single period model can be approximated to a multi-period investment behaviour subject to the fulfilment of some other conditions.

5. Another important assumption for both the MPT and the CAPM (as well as for the market efficiency) is that all informations are readily and freely available to the investors. This assumption is, of course, not realistic. Because, in case of small firms the informations are not readily available in the market.

From the above analysis however, it is revealed that efficient

26 R.F. Vandell, D. Harrington and S. Levkoff, "Cyclical Timing : More Return for Less Risk", Darden School Working Paper 78-12 (Charlottesville, va : Darden Graduate School of Business Administration, 1978) cited in D.R. Harrington: Modern Portfolio Theory and Capital Asset Pricing Model (1983, Prentice Hall, Inc., Cliffs); p.27. They have shown that 'risk' changes over time. Such a change would bring about corresponding changes in the assessment of risk free rate, market premium etc. consequently, the single period model can no longer be considered efficient.

market hypothesis rests on some restrictive and unrealistic assumptions. The efficient market dogma describes the prices of assets or stocks as the 'best estimate' of the future prospects for the assets.

In addition to the above mentioned assumptions, following assumptions underlie the Capital Asset Pricing Model:

1. The model assumes that all investors can borrow or lend at a pure interest which has no covariance with the market rate of return i.e. a risk-free rate. Such an assumption allows the model to construct the capital market line which represents the efficient portfolios in equilibrium. This assumption is also not accepted in the academic community as well as by the management practitioners without criticism. This is undoubtedly true that there are some securities (such as govt. bonds etc.) the returns (not in real terms) on which are certain. But this is no guarantee to compensate for the uncertainty arising out of price level changes. Several authors have suggested different modified versions of the CAPM to solve this problem.

Another point of criticism is that all investors cannot borrow or lend at the same rate of interest. This, obviously, does not reflect the real world properly. In spite of that, the CAPM, with this basic assumption, is considered as the most simple and thus, acceptable proposition for the description of the risk-return equilibrium condition in the capital
market.

2. Another assumption that underlies the CAPM is the absence of transaction costs or taxes. This assumption is surely, a clear distortion of the real world situation. In this case also, the model is unable to make any distinction between the dividend and the capital gains. Several adaptation models of the CAPM have been evolved.

3. The formulation of the model (for equilibrium in the Capital Market) requires that all assets are divisible and marketable and the number of assets is fixed. This is also an unrealistic assumption. Because the shares of all firms especially the small firms are not actively traded in the capital market and thus they appear to be less attractive or rather dangerous to the investment managers. The model does not consider the non-marketable assets such as human resources at all. This is also a serious limitation of the model. An adaptation model of the CAPM has, however, been developed by Myres which considers the non-marketable assets. But this is more complex and very difficult to give any empirical content on it.

From the above analysis it is evident that all the assumptions that underlie the mean variance security valuation

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28 Some of these adapted models will be mentioned in the subsequent section.
Theorem are purely restrictive and unrealistic. Inspite of that these models can not be rejected simply on the basis of the fact that all these assumptions are violated in the real world. Besides, one thing should be remembered that the CAPM is an expectational model. It describes the beliefs of the investors only and these expectations or beliefs are assumed to determine the security prices.

The acceptance of the CAPM is however conditional on its empirical validity. To end the analysis of the present section it is, therefore, necessary to present a brief review of the studies relating to the empirical evidences on the model. The increased interest of the academicians in the problems of the capital market during the last few decades, has resulted in a large number of empirical studies which are sufficient to provide evidences relating to the validity of the model.

As it is not possible to mention all of such a large number of empirical studies, it becomes necessary to limit the citations for the present purpose. A brief review of some of the important studies covering the different aspects of the Capital Asset Pricing Model will be presented here. All the tests available till date can be grouped into two broad categories: firstly, on the problem of misspecification of the model and secondly, on its inadequacy with respect to the inclusion of all possible factors for the purpose of describing the investor behaviour. Most of these studies have been conducted
on the U.S. stock market.

The assumption of the market efficiency is considered to be the most important taproot in developing CAPM. It is, therefore, essential to mention the test results on the merit of this concept. Fama has conducted a weak-form test of the market efficiency which involves the testing on the dependence of the successive share price changes overtime through regression analysis\(^2\). A semi-strong test has been presented by Pinches which is less direct\(^3\). The empirical results show that it (market efficiency) is, however, an 'average' kind of law that may be true for all securities at all times.

Test of the Capital Asset Pricing Model

Most of the empirical tests relating to the empirical validity of the model have been designed either to consider the forecasting ability of it or to examine if the model could be able to provide more accurate predictions regarding the parameters used in this model, by using some other added factors with the original form.

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Forecasting Ability Tests

It may be recalled that the model specifies the risk-return equilibrium relationship of the capital market. The model uses the historical records to predict the future (ex-ante-estimates). Thus, if the model is correct, it should explain the historical relationship accurately and by using this model one should be able to predict the ex-ante values of $R_f$, $R_m$ and beta.

The earlier findings of Douglas (1969) and Lintner show that the intercept of the model is not equal to the risk-free rate\(^{31}\).

Another important empirical study which is considered to be the foundation stone for the other subsequent studies, has been presented by Miller & Scholes (1972)\(^{32}\). This study also


provides similar results to that of Lintner. These studies are considered to be inefficient because they have used informations relating to the single securities only.

More clearer view of the risk-return relationship can be obtained by grouping the securities in portfolios, as this procedure allows one to eliminate the possibility of any 'noise' created by the large number of random components of the realised returns.

Such a grouping procedure has been used in the studies conducted by Jacob, Black-Jensen-Scholes, Blume and Friend, and Fama and MacBeth while Jacob uses the cross-sectional technique, Black, et. al. apply both time-series and cross-sectional tests on the portfolio returns. For the time-series

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and cross-sectional tests on the portfolio returns. For the
time-series test B-J-S use the following equation:

\[ r_p = \alpha_p + \beta_p r_m + \epsilon_p. \]

Their test is concerned with the estimation of the value of
the intercept (portfolio) given by \( \alpha_p \). It is argued that
if the simple CAPM is valid, the value of the \( \alpha_p \) will be
equal to zero as they have regressed the excess returns of
the portfolio on the excess market returns over time. Unfor-
tunately, they have found that the value of \( \alpha_p \) is different
from zero. Another point of difference between the CAPM and
their tests is that high beta portfolios give low returns
and vice versa which represents a completely opposite rela-
tionship from that is expected by the model.

In a subsequent study Fama and MacBeth have obtained similar
results to that of B-J-S. Another study conducted by Fama has
experienced almost the same thing as before except in one
year during which the market has been described well by the
model\(^{34}\).

Blume and Friend have also concluded their study with the
same view as others. They have used an expanded cross-sec-
tional regression of the following form:

\[ \tilde{r}_p = \delta_0 + \delta_1 \hat{\beta}_p + \delta_2 (\hat{\beta}_p) + \epsilon_p. \]

\(^{34}\) Eugene F. Fama, Foundation of Finance, (New York: Basic
In another test the authors (Blume & Friend) have shown that, in practice, the investors do seldom hold the diversified portfolios. To them, the traditional measures of risk (i.e., standard deviation or variance) appear more comfortable than beta. This study includes other assets such as real estate and human capital also in addition to the securities.

Cooley, Roenfeldt, and Modani provide an interesting observation that instead of beta, Skewness and Kurtosis provide distinct and useful information about the risk. While Levy has found that the expected relationship is strongly supported for bull markets. Thus, he suggests that beta estimates should be made separately for the two different situations in the market. Levy's result has been contradicted by another


study conducted by Johnson and Baesel\textsuperscript{38}.

The model assumes that the return distributions have finite mean and finite variance i.e., the assumption of normal distribution. This is, however, contradicted in a number of empirical studies. Arditti provides evidence that Skewnen may be an important factor in explaining return behaviour\textsuperscript{39}.

Another more encouraging result has been presented by Kraus and Litzenberger\textsuperscript{40}. Their study reveals that (it is not Skewnen) the systematic and non-diversifiable skewness is important.

Simkowitz and Beedles have examined the effect of diversification on Portfolio Skewness and their study derives a reverse relationship between diversification and portfolio


\textsuperscript{39} Arditti, Fred D., "Risk and Required Return on Equity", Journal of Finance, (March, 1967); pp.19-36.

They suggest that for determining portfolio strategy, a measure that relates skewness to variance may be very useful.

There are empirical evidences which are in contrast with the above findings. One such important study has been conducted by Fama. He has found that the model's assumption regarding the return distribution is appropriate.

The study of Miller & Scholes concludes that the real or imagined skewing of returns results in a non-linear risk-return relationship or an inaccurate prediction of the regression coefficients.

The empirical works involve ex-ante prediction by using ex-post data. Thus, in turn, requires that the risk-parameter beta be stationary overtime. Unless this assumption of beta stationerity is satisfied, the CAPM will no longer be compatible with the market model. Several studies have found that the stock beta does not remain stable over contiguous

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sample periods. On the other hand, two studies—one conducted by Altman, Jacquillat and Levasseur and the other by Sharpe and Cooper have justified the assumption of beta stability over time at the individual shares as well as portfolio level.

The researchers have argued that the model remains silent about the index to be used for portfolio selection. And this has, however, created controversy among the academicians as

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to which index can be appropriately used in this model.

In this context, the works of Roll and Friend, Westerfield and Granito can be mentioned\textsuperscript{46}. Both of these studies have found that the use of different indexes (such as bond index, stock index, joint index and the use of national or global index) produce different results.

Another point of controversy among the researchers is concerned with the use of a single market index. Empirical evidences show that the use of multiple index may provide better prediction of the different parameters used in this model. King's study concludes that the use of an industry-specific index may add value to the regression and a similar result has been obtained in a study conducted by Cohen and Pogue\textsuperscript{47}.

Probably, the most important contribution towards the controversy over the use of single or multiple index has been


presented by Farrell\textsuperscript{48}. His study has proved that the reduction of risk at a faster rate can be facilitated by the use of a multiple index to form portfolios and these portfolios require fewer number of securities than the other portfolios of the same risk class constructed on the basis of a single index.

The above discussion reveals that the model is not sufficient to describe the past performance or the future expectation very well and the risk description according to this model, by beta, is also insufficient. Though several researchers in a number of studies, have pleaded for the validity of the model, the limitations cited above with respect to both of its theoretical and empirical inefficiency should not be oversighted. But, in spite of all such limitations, the simple CAPM has been favoured by the academicians for several reasons as under:

i. To start with, the model is simple. As an expectational model it should not be assessed so rigorously which is normally done in case of a descriptive model.

ii. There is no doubt that the model is an abstract of the reality. Still, in case of the individual securities the use of ex-post data can provide useful explanation. For portfolio, the model is able to provide even better results.

iii. The use of any other complex model of CAPM or some other models can not guarantee to provide better estimation of the parameters and moreover, these models are very difficult to use for practical purposes.

Other Studies

Most of the studies cited in the foregoing discussion are related to U.S. Stock markets. The studies relating to the capital markets of U.K., Japan, France or Europe as a whole are also available. But, empirical evidence on the Indian

context, is, however, too limited to mention here. Probably, Gupta (1981) is the first man to apply the model in the Indian stock market conditions. Other studies relate to the empirical investigation on the equity share price behaviour following random walk hypothesis and the valuation of shares etc. An empirical study, however, on the CAPM has been presented for the first time by S.K. Choudhuri which covers various aspects of the model. The another provides a useful technique relating to the estimation of different parameters of the model using the Indian data. He has also considered the problem of beta stability and has used the Bayesian adjustment technique for beta forecast.

The authors' findings (using time series estimates) primarily, are in general agreement with the simple CAPM: the intercepts are in all cases insignificantly different from zero using the risk-premium form of the model. He has also noticed


51 S.K. Choudhuri, "Some Evidence on the CAPM with Indian Share Data," Journal of Indian Accounting Association - Research Foundation, 1990. According to Prof. Choudhuri two other valuable papers are expected to be published very shortly. These two studies are concerned with the examination of Markowitz model in the Indian context and an empirical study on the Random walk hypothesis.
that there is ample scope for the investors to diversify and thus, they may control their portfolio risk by systematically selecting the securities on the basis of their beta values. His findings, however, reveal that for a sizeable proportion of the sample used in this study, the beta values do not remain stationary over time (period). He has observed that the beta instability depends on the length of the estimation period—i.e., an inverse relationship between beta and the length of the estimation period. He has, therefore, concluded that the assumption of beta stability for individual shares and as well as portfolios is not reasonable. He has conducted a cross sectional test or the model. This test, however, produces results which are not in agreement with the CAPM. Both the scope and the intercept are different from the value predicted by the model.

Section - III

Alternative Models

From the foregoing discussion it is evident that the basic single period asset pricing model has created abundant controversy among the academicians. The major points of criticism against it are:

i. The model rests on some restrictive and unrealistic assumptions and thus unable to reflect the real world.
ii. The model does not consider all relevant factors while deriving the equilibrium risk-return relationship of the capital market.

iii. 'Beta' is not sufficient to measure the risk; some other measures may be more appropriate to capture the return distributions in so far as the real world situation is concerned.

But the empirical evidences remain inconclusive as to obscure the model. 'Beta' is still used as an important economic determinant of the equilibrium pricing.

Several adaptation models have been developed by many authors to give it a shape of best-fit to the real world situation. The models, however, maintain the linearity of the risk-return relationship. An attempt will be made to provide a brief review of some of the popular adaptation models in the following paragraphs:

1. Non-marketable assets and equilibrium pricing

The simple CAPM assumes that all assets are marketable which is violated in reality. Investors may hold claim on the labour income (human capital) which is not marketable. Again the physical assets such as real estate involve large amount of transaction costs. Mayers provides an adaptation model which involves these non-marketable assets though the
implications of the basic CAPM remain the same. The author finds that the use of an incomplete market return as the proxy for the market factor is the source of trouble in the simple CAPM.

2. Zero-beta portfolio and equilibrium

The assumption of the existence of a risk-free rate has been questioned by many authors for several reasons. To avoid this problem, some authors have developed adaptation models which do not involve the risk-free asset. Of them, probably, the model which is believed to be the most popular, has been developed by Black, Jensen and Scholes. The authors have used the short-selling in place of risk-free asset and derived a linear risk-return equilibrium relationship as under:

\[ E(\tilde{R}_1) = (1 - \beta_1) E(\tilde{R}_Z) + \beta_1 E(\tilde{R}_M). \]

Where \( E(\tilde{R}_Z) \) represents the expected return on the 'beta factor' (zero-beta portfolio). This 'beta factor', according to


B-J-S represents a portfolio which has no relation (uncorrelated) with the market portfolio. According to them, there will be an equilibrium between the short-selling and lending of securities. Though, it is not clear which asset is used as a proxy for 'zero-beta portfolio'. This model provides a higher intercept and comparatively less slope than those predicted by the simple CAPM. This model is, however, widely used in practice.

3. After-tax version of CAPM

The simple CAPM is based on the assumption that there are no taxes. This is, however, a clear distortion of the reality. The fact is that the investors pay taxes and the investor's reaction to the stock price changes depends on the tax-bracket with which he or she belongs. This, in turn, influences the stock-price behaviour and thus tax is an important factor that may influence the risk-return relationship. Brennan has developed an after-tax version of CAPM\textsuperscript{54}. According to basic CAPM, the investors are indifferent between receiving income in the form of capital gains or dividends and it is also assumed that all investors hold the same portfolio or risky assets. But, in practice, they are

not the same in so far as the differential tax rate is concerned. To be more specific, the capital gains are, in general, taxed at a lower rate than dividends. Consequently, the efficient frontier faced by each investor will be different. The authors are, however, of the opinion that a general equilibrium relationship should still exist since in the aggregate markets must clear. The extended version of CAPM, on the basis of the differential taxes on income and capital gains can be written as under:

\[ E(R_i) = R_F + \beta_i (E(R_M) - R_F) - \tau (\delta_M - R_F) + \tau (\delta_i - R_F). \]

Where \( \delta_M \) = the dividend yield (dividends divided by price) of the market portfolio;

\( \delta_i \) = the dividend yield for stock \( i \);

and \( \tau \) = a tax factor which is a complex function of investors' tax rates and wealth.

The after-tax version of CAPM predicts that before-tax expected returns are the increasing function of dividend yield, if the beta remains constant. This proposition has been further investigated by Litzenberger & Ramaswamy (1979)\(^\text{55}\).

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In a subsequent work Litzenberger and Ramaswamy find that tax-induced clienteles form and the incremental expected return per unit of dividend yield is a decreasing function of dividend yield provided the short-sales are restricted\(^{56}\). In other words, in the absence of any restrictions on short-sales, the expected return per unit of dividend yield is constant across securities and independent of dividend yield under the conditions of equilibrium. In an earlier work also, Black and Scholes did not find any significant effect of dividend on the prices of assets\(^{57}\).

4. Inflation and CAPM

Some authors are of the opinion that inflation is an important factor influencing the risk-return relationship. Bigger concludes that the composition of a portfolio may be affected by the price level changes as both of the two assets—risky and riskless—are influenced by it\(^{58}\). He presents the

\(^{56}\) Litzenberger and Ramaswamy, "Dividends, Short-selling Restrictions, Tax Induced Investor Clienteles and Market Equilibrium", Journal of Finance, 35 (May, 1980); pp.469-482.


following model to account for the inflation:

\[ R_j = \text{real } R_f + \text{inflation} + \beta_j (R_m - R_f). \]

Another model has been developed by Hagerman and Kim as under\(^{59}\):

\[ E(\tilde{R}_j) = E(\tilde{R}_f) + \frac{\text{Cov.}(R_j, R_m - R_f)}{\text{Cov.}(R_m, R_m - R_f)} \sqrt{E(\tilde{R}_m) - E(\tilde{R}_j)} \]

where the beta is given by:

\[ \frac{\text{Cov.}(R_j, R_m - R_f)}{\text{Cov.}(R_m, R_m - R_f)}. \]

They have argued that there is a relationship among the market rate of return, stock return and the inflation.

Friend, Landskroner and Losq have studied the effect of uncertain inflation on the Market Price of Risk (MPR)\(^{60}\). Their study concludes that:

i. If the market rate of return and the rate of inflation is positively correlated, the traditional asset pricing model (CAPM) undertakes the Market Price of Risk (MPR). On the


other hand, if they show a negative correlation between them, the theory provides an overestimation of the MTR.

ii. If the investors' are risk neutral, the expected rate of return on a risky asset is equal to the risk-free return plus the covariance between the return on the risky asset and the rate of inflation.

iii. The expected rate of return on a rising security has been shown as under:

\[ E(R_j) = R_f + \text{Cov.}(R_j, R_p) + \frac{E(R_m - R_f) - \text{Cov.}(R_m, R_p)}{\kappa^\gamma 6_m^2 - \text{Cov.}(R_m, R_p) - \text{Cov.}(R_j, R_m) - \text{Cov.}(R_m, R_p)} \]

where \( R_p \) represents the expected inflation and \( \kappa \) is used to denote the proportion of the total capital of an investor employed for the acquisition of risky assets.

5. General Capital Asset Pricing Model

H. Levy has developed a new version of the simple CAPM which he calls as the General Capital Asset Pricing Model\(^{61}\). The author assumes that instead of holding all of the available risk assets the investor holds a given number of securities

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in his portfolio. The simple CAPM relies that the investors, independent of their preferences, hold the same proportion of risky assets in their portfolio as they are available in the market. On the other hand, the author assumes that the investors' portfolio differ with respect to the proportion of risky assets in each portfolio and different portfolio may use different types of risky assets.

Under conditions of an imperfect market, the author has derived an equilibrium relationship between the risk and return of a security \( i \) as under:

\[
M_i = r + \beta k_i (M_k - r)
\]

where \( M_i \) the expected return on the security \( i \);

\( r \) = risk-free rate of interest;

\( M_k \) = the expected return of the portfolio selected by the investor \( K \);

\[
\text{Cov.}(R_i, R_k)
\]

and \( \beta k_i \) = \( \frac{\text{Cov.}(R_i, R_k)}{k^2} \) i.e., the systematic risk of the security \( i \) in the portfolio of investor \( K \). According to him, the systematic risk is not sufficient to determine the equilibrium price of risk. Instead, it is the \( \beta_{ik} \) which appropriately portrays the risk of the \( i \)th security. The difference between these two parameters is that \( \beta \) is obtained by \( (\text{Cov.}(R_i, R_m) / \sigma_m^2) \) dividing the covariance of returns between the \( i \)th security and the market portfolio by the market variance (where all risky securities are included in the market portfolio). On the other hand, to derive \( \beta_{ik} \), the return
from the K th portfolio (which is held by the investor 'K' only and which does not include all risky assets), is used in place of the market portfolios' return \((R_m)\) for calculating the variance (the denominator) and the covariance factor and is given by - 

\[
\beta_{ik} = \frac{\text{Cov.}(R_i, R_k)}{\sigma_k^2}.
\]

He remarks, "........ even when investor hold stocks of three or four companies, we still obtain the same result: the ith security variance is much more important in price determination than one would expect from the analysis of the traditional CAPM"\(^{62}\).

Thus, the security variance, according to this model, plays a crucial role in the measurement of risk of each stock which is quite different from the equilibrium results of the simple CAPM.

However, he finally concludes as - "For securities which are widely held (i.e., AT & T) we expect that beta will provide a better explanation for price behaviour, while for most securities, which are not held by many investors we would expect that the variance \(\sigma_i^2\) would provide a better explanation for price behaviour"\(^{63}\).


The above finding is similar to what has been derived in an earlier work by Tobin and Brainard. According to them, "Suppose that transaction costs limit the number of assets a typical investor can hold in his portfolio. The "undiversifiable" risk of a particular asset to him then depends on its co-variation not with the entire market but with his own portfolio. Obviously as asset will be a higher proportion of the portfolios in which it is held than in the aggregate market portfolio. Hence, its own variance will be more important." 64

Another adapted version of CAPM has been formulated by J. Mayshar. He has incorporated the transaction cost (both fixed and volume related transaction costs) to the basic CAPM. He explains the expected return on the security \( i \) with the help of the following form:

\[
E(R_i) = R_f + t + (\alpha_i \beta_i - \delta_i \sigma_i^2) \sqrt{E(R_m)} - R_f
\]

Where \( \delta_i = \) a measure of the relative concentration of holding of the \( i \)th asset; the higher the concentration the greater the value of \( \delta_i \) such that

\[
0 \leq \delta_i \leq 1
\]

\[ \alpha_i = 1 - \beta_i \] and

\[ t = \text{transaction cost.} \]

6. **International Asset Pricing Model**

The International Asset Pricing Model has been developed first by Solnik\(^6\). It has, further, been extended and tested by Grauer, Litzenburger and Stehle; Senbet and Others\(^6\). The model considers the exchange risk and its influence on both the price of goods and the assets' return.

7. **Arbitrage Pricing Theory**

Another alternative theory to the CAPM is the Arbitrage Pricing Theory which is originally due to S.A. Ross\(^6\). This


theory defines the systematic risk as the covariability of the assets return with several economic factors such as - interest structure, inflation, industrial production etc. though the model does not specify or identify all possible factors that should be considered for measuring the non-diversifiable risk of an investment. The model describes the compensation for bearing risk as the summation of several risk premia as opposed to only one risk premia in the simple CAPM.

The theory assumes that (i) capital market is perfectly competitive, (ii) more wealth is preferred to less wealth by the investors, and (iii) the process of the generation of an assets' return can be given by a K-factor model.

The theory requires that the economy has no arbitrage opportunities. Several empirical tests have been made on this theory, but the evidences are not sufficient to establish its superiority (and simplicity also) over the CAPM.

Section - IV

Conclusion

In the previous sections, it has been tried to provide a brief outline of the evolution of the mean variance security valuation theorem from the development of the portfolio selection technique by H. Markowitz to the simple CAPM. Some
other adapted models of the CAPM and a short review of the APT have also been considered in the context of the measurement of security risk.

The critical estimates of the assumptions that underlie the CAPM and the empirical test results on it are sufficient to question the models' ability to describe the real world situation and consequently, its empirical validity. Probably, one better alternative to the simple CAPM is the 'State Preference Model' which is considered to be much better, more descriptive and more specific but unfortunately much more difficult to give an empirical content on it. As to the other adapted models of the simple CAPM, one common feeling among the academicians and the practitioners as well, is that those versions are quite complex in nature and thus very difficult to use them for practical purposes. On the other hand, the increased ability and lower cost per unit of computer time, at present, have offered the opportunity to adopt Markowitz's technique for portfolio selection. Other two promising constructs of the capital asset pricing are, namely, the APT and the option pricing theory. These two theories are,


however, not so popular among the academicians and the practitioners for several reasons.

In spite of its various limitations, the simple CAPM is widely used in the corporate world.

In order to provide an idea about the risk complexion of different securities, 'beta' statistics are widely quoted by the firms in U.S.A. According to Crowell, though the theoretical content of the simple CAPM can be criticised in various ways, the beta can still be used for different purposes as under:

i. In order to control the portfolio risk by making adjustments of the security betas of a portfolio.

ii. As the portfolio performance is greatly influenced by the conditions of the overall-market, the market timing should be given greater emphasis than stock selection.

iii. For the purpose of performance measurement by way of distinguishing between market timing and stock selection skills.

iv. It can be used to draw the attention of the analysts to those stocks with most residual risk; and

v. It can be used for the purpose of risk versus return considerations.

Morrison and Crowell opine that 'beta' is a part of the whole story of the total risk complexion of an asset or security. It represents only one among the large number of factors which altogether make the future stream of income from an asset or security uncertain. The question of the effect of other factors on the return of a security or investment still remains inconclusive. But, the effect of changes in the economy as a whole on the security prices has been recognized unanimously by the academicians as well as by the practitioners as the most important factor to determine the future returns of an investment.

Finally, the analysis can be concluded in the words of Bernstein as under:

Is beta Dead? The question is ridiculous. As long individual securities continue to move up and down in sympathy or as a

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72 Peter L. Bernstein, "Dead or Alive and Well", Journal of Portfolio Management, 7 (Winter, 1981); p.4.
part of broad market movements, portfolio manager must be concerned about the sensitivity of their portfolios to the general market, the stability of that relationship, and the accuracy of the measurement tools that they employ .......... 
..................................................................................
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..................................................................................
Are the tools that we use to measure the sensitivity of individual securities to the general market, their sensitivity to one another alive and well? Ah! There's the rub!"

However, it may be expected that in the near future the quantitative techniques will be sufficiently improved to cope with the problem.
Let an investor invests a part of his fund (denoted by $\alpha$) in the security $i$ and the remainder amount (denoted as $1-\alpha$) in the market portfolio (of which security $i$ is a component part) to form 'Z'.

The expected return on the new portfolio $Z$ can be given by -

$$E(R_Z) = \alpha E(R_i) + (1-\alpha) E(R_M)$$ .. (1).

Similarly, the S.D. of the portfolio returns can be written as -

$$\sigma(R_Z) = \sqrt{\alpha^2 \sigma(R_i)^2 + (1-\alpha)^2 \sigma(R_M)^2 + 2\alpha(1-\alpha) \rho_{iM} \sigma(R_i) \sigma(R_M)}$$ .. (2).

Let a curve be drawn to connect the security $i$ with the market portfolio $M$ and be called the $iM$ curve.

The risk-return equilibrium condition requires that the $iM$ curve should be tangent at point $M$ on the capital market line. It may be recalled that the slope of the capital market line ($r_e$) is given by -

$$r_e = \frac{\frac{\sum E(R_M) - P}{\sigma(R_M)}}$$ .. (3).

Let the slope of the $iM$ curve be denoted as $S_M$, and mathematically be given by $dE(R_Z)/d \sigma(R_Z)$.

The equation is a function of $\alpha$. $E(R_Z) = f(\alpha)$.

Now, $\frac{dE(R_Z)}{d(\alpha)} = 1 \cdot \alpha^{1-1} E(R_i) + E(R_M) - \alpha E(R_M) = E(R_i) - E(R_M)$.
Similarly the equation (2) is a function of function.

Let \( U = \alpha^2 \sigma(R_i)^2 + (1 - \alpha)^2 \sigma(R_M)^2 + 2r_i M \alpha (1 - \alpha) \sigma(R_i) \sigma(R_M) \), and \( \sigma = f(u) \).

\[ \frac{d\sigma}{du} = \frac{d\sigma}{d\alpha} \times \frac{du}{d\alpha} \]

Now \[ \frac{d\sigma}{du} = \frac{1}{2} \sqrt{\sigma^2 \sigma(R_i)^2 + (1 - \alpha)^2 \sigma(R_M)^2 + 2r_i M \alpha (1 - \alpha) \sigma(R_i) \sigma(R_M)} \]

\[ = \frac{1}{2} \sqrt{\sigma^2 \sigma(R_i)^2 + (1 - \alpha)^2 \sigma(R_M)^2 + 2r_i M \alpha (1 - \alpha) \sigma(R_i) \sigma(R_M)} - \frac{1}{2} \sqrt{5}(i) \]

Similarly,

\[ \frac{dU}{d\alpha} = 2 \alpha - 1 \sigma(R_i)^2 + 2(1 - \alpha)^2 \sigma(R_M)^2 + 2r_i M (1 - 2\alpha) \sigma(R_i) \sigma(R_M) \]

Now putting the values for \( \frac{d\sigma}{du} \) and \( \frac{du}{d\alpha} \) to derive \( \frac{d\sigma}{d\alpha} \), we get,

\[ \frac{d\sigma}{d\alpha} = \frac{1}{2} \sqrt{\sigma^2 \sigma(R_i)^2 + (1 - \alpha)^2 \sigma(R_M)^2 + 2r_i M \alpha (1 - \alpha) \sigma(R_i) \sigma(R_M)} - \frac{1}{2} \]

\[ = \frac{1}{2} \sqrt{\sigma(R_M)^2 - \sigma(R_i)^2 - \sigma(R_M) \sigma(R_i) \alpha M} \]

At the tangency \( \alpha = 0 \),

\[ \frac{d\sigma}{d\alpha} = \frac{1}{2} \sqrt{\sigma(R_M)^2} - \frac{1}{2} \sqrt{6 + 2(1 - 0)(-1) \sigma(R_M)^2 + 2r_i M \sigma(R_i) \sigma(R_M)(1 - 0)} \]

\[ = \frac{1}{2} \sqrt{\sigma(R_M)^2} - \frac{1}{2} \sqrt{2 \sigma(R_M)^2 + 2r_i M (1 - 2\alpha) \sigma(R_i) \sigma(R_M)} \]

\[ = - \sigma(R_M) \sigma(R_i) \sigma(R_M) \sigma(R_i) \alpha M \]

\[ = - \sqrt{\sigma(R_M)^2 - \sigma(R_i)^2 \sigma(R_M) \sigma(R_i) \alpha M} \]

\[ = - \sqrt{\sigma(R_M)^2 - \sigma(R_i)^2 \sigma(R_M) \sigma(R_i) \alpha M} \]
Thus $dE/d\delta$ can be written as -

$$\frac{dE}{d\delta} = \frac{\sqrt{E(R_M) - E(R_1)}}{\sqrt{\sigma(R_M) - \sigma(R_1) \cdot \text{riM}}}$$

The equation (8) represents the slope of $iM$ curve $S_M$. Thus

$$S_M = \frac{\sqrt{E(R_M) - E(R_1)}}{\sqrt{\sigma(R_M) - \sigma(R_1) \cdot \text{riM}}}$$

The equilibrium condition requires that the slope of the $iM$ curve will be equal to the slope of the capital market line.

Symbolically $r_e = S_M$

Putting values for $r_e$ and $S_M$, we get -

$$\frac{\sqrt{E(R_M) - E(R_1)}}{\sigma(R_M)} = \frac{\sqrt{E(R_M) - E(R_1)}}{\sqrt{\sigma(R_M) - \sigma(R_1) \cdot \text{riM}}}$$

or Alternatively,

$$\frac{\sqrt{\sigma(R_M) - \sigma(R_1) \cdot \text{riM}}}{\sqrt{E(R_M) - E(R_1)}} = \frac{\sigma(R_M)}{\sqrt{E(R_M) - E(R_1)}}$$

or

$$1 - \frac{\sigma(R_1)}{\sigma(R_M)} \cdot \text{riM} \right) = \frac{1}{\sqrt{E(R_M) - E(R_1)}}$$

or

$$1 - \frac{\sigma(R_1)}{\sigma(R_M)} \cdot \text{riM} \right) = \frac{\sqrt{E(R_M) - E(R_1)}}{\sqrt{E(R_M) - E(R_1)}}$$

or

$$1 - \frac{\sigma(R_1)}{\sigma(R_M)} \cdot \text{riM} \right) = \frac{\sigma(R_1)}{\sigma(R_M)}$$
This is the equation of a straight line and as \( \frac{\sigma(R_i)}{\sigma(R_M)} \) is the slope of the regression line \( b_{iM} \), the above equation can be written as under:

\[
b_{iM} = -\frac{p}{\overline{E(R)} - \overline{P} - \overline{E(R_i)}} + \frac{1}{\overline{E(R)} - \overline{P} - \overline{E(R_i)}} \overline{E(R_i)}
\]

The equation (9) can be alternatively shown as:

\[
\frac{\sigma(R_i)}{\sigma(R_M)} = \frac{\overline{E(R_i)} - \overline{P} - \overline{E(R_i)}}{\overline{E(R)} - \overline{P} - \overline{E(R_i)}}
\]

or \( \overline{E(R_i)} - \overline{P} = \frac{\sigma(R_i)}{\sigma(R_M)} \overline{E(R)} - \overline{P} - \overline{E(R_i)} \)

\[
= \frac{C_{iM} \overline{E(R)} - \overline{P} - \overline{E(R_i)}}{\sigma(R_M)} \overline{E(R)} - \overline{P} - \overline{E(R_i)}
\]

As \( \sigma R_i = C_{iM} \), i.e., the covariance between the returns of the security \( i \) and that of the market portfolio.

Now the equation (10) can be alternatively given as:

\[
\overline{E(R_i)} - \overline{P} = \frac{\sigma(R_i)}{\sigma(R_M)} \overline{E(R)} - \overline{P} - \overline{E(R_i)} C_{iM}
\]

Equation (11) represents the security market line where the slope is given by \( \frac{\overline{E(R)} - \overline{P} - \overline{E(R_i)}}{\sigma(R_M)} \) and \( P \) is the intercept.