INTRODUCTION:

The heat transfer in a viscous incompressible fluid neglecting the buoyancy force has been studied by several authors notably Jain and Bansal (14), Kapoor and Malik (15), Gopalan and Sundaram (10), Hwang and Hong (12), Balaram (3) and Hahn (11). But it was shown by Gill and Casal (9) that viscosity variations and temperature differences on the horizontal forced convection flow between two parallel infinite plates can induce such flows as might significantly increase or decrease the tendency towards instability. Many authors have shown that the fluids with low Prandtl number are to a good deal more sensitive to gravitational field effects than the fluids with high Prandtl number. This is due to the fact that the low Prandtl number fluids are characterised by thicker thermal boundary layers. All fluids with low Prandtl number are electrically conducting and hence the flow pattern is affected by an external magnetic field. This property has been used profitably in magnetohydrodynamic channel flows. Liquid metals and highly ionised air are electrically conducting and have low Prandtl number (say Pr~1/40 for mercury at ordinary temperature and Pr~1/135 for liquid sodium at 200°C). Hence the extent to which the thermal buoyancy force influences a forced flow is a topic of interest. Keeping this in view several authors viz., Soundalgekar (37), Acharya and Padhy (1), Schlichting (33),
Singh, Sharma and Misra (36), Nanda and Sharma (24), Kelleher and Yang (17), Eshghy et al (8), Muhuri and Maiti (23), Singh (35), Varma and Singh (42), Gupta (10a), Pop (25), Kafousias et al (16), Vajravelu (39, 40) and Schneider (32) have discussed the fully developed combined free and forced convection flows of a viscous fluid.

The study of buoyancy driven convection flows through porous media has been stimulated by its applications in several geophysical and engineering problems. The two main configurations in which the heat transfer driven flow in a porous medium generally considered are (1) porous layers heated from below (2) porous layers heated from the side. This convection heat transfer potential flow through porous medium is rapidly growing as an independent branch in Fluid Mechanics and Heat Transfer. Interest in understanding the convective transportation in porous material is increasing owing to the development of geothermal energy technology, high performance insulation for building and cold storage, drying technology and many other areas.

Porous materials such as sand, crushed rock in the underground are saturated with water which under the
influence of local pressure gradients migrate and transport energy through the material. The rapid depletion of the unrenewable reserves of fossil fuel and their increasing costs with attendant environmental pollution have provided the inpetus for the development of the supplementary energy sources. Meteoric water, percolating down to great depths in a permeable formation is heated directly or indirectly by the intruded magma and is then driven buoyantly upwards to the top of the aquifer where it can be tapped through drill holes.

A few other technological applications of a natural convection in a porous medium are cooling of nuclear fuel in shipping flasks and water filled storage bays, insulation of high temperature gas cooled reactor vessels burrying of drums containing heat generating chemicals in the earth, thermal energy storage tanks, regenerative heat exchangers containing porous materials, petroleum reservoirs and chemical catalytic reactions. Combarnous and Bories (7), Cheng (5,5a) and Combarnous (6) have recently provided extensive reviews of state of the art on free convection in fluid staturated porous medium.
The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of magnetohydrodynamic heat transfer. This MHD heat transfer has gained significance owing to recent advancement of space technology. The MHD heat transfer can be divided into two sections. One contains problems in which the heating is an incidental by product of the electromagnetic fields as in MHD generators and pumps etc and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer (22). With the fuel crises deepning allover the world, there is great concern to utilise the enormous power beneath the earth’s crust in the geothermal region (1). Liquid in the geothermal region is an electrically conducting liquid because of high temperature. Hence the study of interaction of the geomagnetic field with the fluid in the geothermal region is of great interest, thus leading to interest in the study of Magnetohydrodynamic convection flows through porous medium.

In all the above investigatiose the boundaries are assumed to be flat. But there are many physical situations in which the surface of the solid boundaries are
wavy in nature. For example, the surface formed by cleavage of mica contains irregularities of the order of 20°A in size, and the irregularities of the surface of an ideally smooth quartz crystal can be up to 100°A in height (18). Flow over a wavy wall has attracted the attention of several authors in the recent times owing to its applications in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching of an ablative surface and film vapourisation in combustion chambers. In view of these applications, several authors (2,3a,13,21) have made some investigations of the fluid flow over a wavy wall. Ackert (2) has treated compressible, inviscid fluid flows over wavy walls. Lighthill (21) has extended the work of Ackert (2) by including the effects of viscosity in both the mean and disturbed flows. Inger (13) analysing compressible flow over wavy walls, has extended the analysis of Lighthill (21) by including the effects of heat transfer. In all these investigations, the mean flow is assumed to vary linearly within the disturbance sub-layer. Benjamin (3a) has carried out an analysis for incompressible flow over flexible walls and pointed out that the mean flow may not be linear in the viscous sub-layer. Keeping this in view, Lekeoudis et al (19) have discussed the compressible viscous flow past wavy
walls using linear analysis without restricting the mean flow in the disturbance layer. Sankar and Sinha (31) have studied the Raleigh problem over a wavy wall for low and high Reynolds numbers. A transformation of the coordinate system was used which reduces the wavy wall to a flat wall in the transformed system. They have shown that at low Reynolds numbers the flow field is dominated by viscous effects and the waviness of the wall quickly ceases to be of the importance as the fluid is dragged along by the wall. At high Reynolds number, the effects of viscosity are confined to a thin boundary layer adjacent to the wall and the well known potential solution emerges in time. In their study, Lessen and Gangwani (20) have examined the effects of small amplitude wall waviness on the stability of the laminar boundary layer. Vajravelu and Nayfeh (41) have investigated the influence of the wall waviness on friction and pressure drop of the generalised Couette flow. They have considered an incompressible viscous fluid confined in a horizontal wavy channel with the upper plate move with a uniform velocity under a constant pressure gradient in the axial direction. The equations governing the flow and heat transfer have been solved analytically as well as numerically by a variable step
Runge-Kutta-Fehlberg integration scheme of Scott and Watts (34) subject to the relevant boundary conditions. It is found that for small and moderate Reynolds number the analytical solutions for the perturbed part is in good agreement with the numerical solution. However at high Reynolds number the analytical solution differs very much from the numerical solution. The amplitude of skin-friction at the upper plate decreases with increased Reynolds number. The effect of wave number on amplitude is to increase it considerably and increase the phase. The effects of Reynolds number, the pressure gradient parameter and the undulation wavy number on friction and pressure drop are found to be quite significant. Rao and Sastri (29) have analysed the laminar natural convection flow and heat transfer of a viscous incompressible fluid confined between two long vertical wavy walls by taking the fluid properties constant or variable. Approximate solutions of the governing equations have been obtained by Galerkin’s method employing orthogonal polynomials which has proved to give results valid for all values of the wave number of the wavy walls and for all values of the Grashoff number. The flow and heat transfer characteristics have been evaluated numerically. It is found that the effect of viscous dissipation is to
increase the values of the amplitudes considerably and to enhance the effect of variable fluid properties. The phases of the perturbed shear-stress or perturbed Nusselt number at either of the walls are considerably smaller in magnitude than their amplitudes and that they increase for small values of wavelength but decrease asymptotically to zero for its large values. The fluid pressure at either wall lags behind (exceeds) that at the center of the channel whenever the pressure drops are positive (negative) in nature. Rao et al (27) have studied the heat transfer aspect of an incompressible viscous conducting fluid in a horizontal porous channel bounded by wavy wall and a flat wall in the presence of a constant heat source. Using the long wave approximations, the equations governing the flow and heat transfer have been solved. It is observed that for all values of the governing parameters the maximum of total axial velocity occurs very nearly the boundary, the minimum value being nearly wavy wall. For an increase in the wave length the growth in the magnitude of axial velocity is almost proportional to the variation in wave length. The total axial velocity retards in general except in a narrow layer abetting the boundary, due to an increase in the suction parameter. The effect of waviness of the boundary is to
induce reversal transverse velocity whose peak values are attained nearly on the wavy wall. The waviness of the boundary reduces the total temperature in the entire flow field. Also the magnitude of the skin-friction at the wavy wall is very much large in comparison to its magnitude which implies that the waviness of the boundary reduces the skin-friction on it to a very large extent. Vajravelu et al (40) have analysed the free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall in the presence of a constant heat source by using the long-wave approximations. The equations governing the flow and heat transfer characteristics have been solved analytically. It is found that whether the Prandtl number is small or large, the effect of the frequency parameter on the first order axial velocity is to increase it significantly in the presence of heat sources and to diminish it considerably in either the absence of heat sources or the presence of heat sinks. This situation gets reversed in the other half of the channel. The skin-friction at the wavy wall is an increasing function of Grashoff number, the Prandtl number, frequency parameter and the heat source or sink parameter. This problem has been extended to hydromagnetics by Rao et al (26). In the
presence of the heat sink. The first order axial velocity which increases positively in the vicinity of wavy wall suddenly gets reversed in the middle region. The reversed flow with negative velocity persists almost upto the flat wall, though very close to it the fluid moves with small positive velocity. The secondary velocity is always directed towards the wavy wall and its magnitude increases with increase in either wall waviness parameter or Grashoff number in the entire channel. In the presence of heat sources the magnitude of skin-friction at the wavy wall is greater than that at the flat wall and it decreases with increase in \( G \) or \( \lambda \). But in either the absence of heat sources or the presence of heat sinks the magnitude of the skin-friction at the wavy wall is less than that at the flat wall.

Sarojamma (30) has analysed the convective heat transfer of an incompressible, viscous fluid through a horizontal wavy channel. It is found that the appearance of reversed flow regions with respect to the primary velocity solely depends on the variation in the entry temperature at the boundaries for any moderate value of the Grashoff number, \( G \). The primary velocity and secondary velocity increase in
their magnitude due to the reduction in the wave length of
the boundary. The shear stresses at the boundaries decrease
in magnitude with an increase in the magnitude of $G$.

Murthy and Krishna (38) have analysed the
unsteady magnetohydrodynamic free convection flow of an
electrically conducting viscous fluid in a vertical channel
bounded by wavy walls which are maintained at different
temperatures. The flow is basically asymmetric and in view of
the nonuniformity of the boundary surface the flow can not be
unidirectional. The perturbation analysis has been carried
out with the assumption that the flow varies slowly along the
axial direction of the channel. The velocity, the
temperature, the shear stress and the rate of heat transfer
have been been evaluated analytically and their behaviour for
variations in the governing parameters has been numerically
discussed using computational methods.

In this dissertation an attempt has been made
to study the combined free and forced convection effects on
the flow of an electrically conducting viscous incompressible
fluid through a porous medium in a horizontal channel bounded
by two wavy walls at $y=f(\delta x/L)$ under the influence of an
axial magnetic field. The wavy walls are maintained at non-uniform temperature. The perturbation analysis has been carried out with the assumption that the flow varies slowly along the axial direction of the channel. The velocity, temperature distribution, the shear stress and the rate of heat transfer have been evaluated analytically and their behaviour for variations in different governing parameters has been discussed using the numerical results.

2. FORMULATION:

We consider the steady flow of an electrically conducting, viscous incompressible fluid through a porous medium confined in a horizontal channel bounded by two wavy walls which are maintained at a nonuniform wall temperature in the presence of a constant heat source. The Boussinesque approximation is used so that the density variations will be considered only in the buoyancy force. The viscous dissipation is neglected in comparison to the transport of heat by conduction and convection. Also the kinematic viscosity $\nu$, the thermal conductivity $k$ are treated as constants. We chose the cartesian coordinate system $O(x,y)$ with $y$-axis in the vertical direction the walls of the
channel are at \( y = \pm f(\delta x/L) \). The flow is maintained by a constant volume flow rate for which a characteristic velocity \( u_c \) is defined as

\[
u_c = \frac{1}{L} \int_{-L}^{L} u \, dy \quad (2.1)
\]

The applied axial magnetic field \( B_0 \) is uniform, no electric field is applied and hence (21a) there is no induced electric field for the constraints given.

The equations governing the flow are

\[
\rho_e (\vec{v} \times \vec{q}) = -\nabla (p + \rho_e \dot{q}^2/2) + \mu \, \nabla \cdot \vec{q} + \rho_e (\vec{J} \times \vec{B}) - \rho_e \dot{q} (\mu/k_1) \vec{q} \quad (2.2)
\]

\[
\nabla \cdot \vec{q} = 0 \quad (2.3)
\]

\[
\rho_e c_p (\bar{v} \cdot \nabla) T = \lambda \nabla^2 T + Q \quad (2.4)
\]

\[
\rho = \rho_e (1 - \beta (T - T_e)) \quad (2.5)
\]

The Maxwell's equations are

\[
\nabla \cdot \vec{B} = 0 \quad (2.6)
\]

\[
\nabla \times \vec{B} = 4\pi \vec{J} \quad (2.7)
\]

From the Ohm's law

\[
\vec{J} = \sigma \mu_e (\vec{q} \times \vec{B}) \quad (2.8)
\]
Where $\rho_e$ is the density of the fluid in the equilibrium state, $\bar{q}$ is the velocity, $\bar{\xi}$ is the vorticity, $p$ is the pressure, $T$ is the temperature in the flow region, $\rho$ is the density of the fluid, $Q$ is the strength of the heat source, $\mu$ is the coefficient of viscosity, $c_p$ is the specific heat at constant pressure, $\lambda$ is the coefficient of thermal conductivity, $\beta$ is the coefficient of volume expansion, $B$ is the magnetic induction vector, $J$ is the current density, $\sigma$ is the electrical conductivity of the fluid and $\mu_e$ is the magnetic permeability and $k_1$ is the permeability parameter.

In the equilibrium state:

$$-\frac{\partial p_e}{\partial x} - \rho_e \bar{g} = 0$$

$p = p_e + p_0$ where $p_0$ is the hydrodynamic pressure.

Introducing the non-dimensional variables

$$\bar{q}^* = \bar{q}/u_c ; \quad p^* = p/\rho_e u_c^2 ; \quad (x^*, y^*) = (x, y)/L$$

$$B^* = B/B_0 ; \quad J^* = J/\sigma u_c B_0 ; \quad \phi^* = (T - T_e)/\Delta T_e$$

under the equilibrium state $\Delta T_e = T_e(L) - T_e(-L) = \frac{Q L^2}{\lambda}$

(2.9)
Using (2.9) the equations (2.2)-(2.8) reduce to (on dropping the asterisks)

\[ R(\overline{r} \overline{x} \overline{q}) = \nabla (p + q^2/2) + \overline{q} + M (Jx\overline{B}) + (G/R) \theta - \sigma_i^2 \overline{q} \]  
\[ \nabla \cdot \overline{q} = 0 \]  
\[ P_e (q, \overline{q}) \theta = \nabla^2 \theta + 1 \]  
\[ \nabla \cdot \overline{B} = 0 \]  
\[ \nabla \times \overline{B} = R_m \overline{J} \]  
\[ \overline{J} = \overline{q} \times \overline{B} \]  

where

- \( R = u_c L / \nu \), the Reynolds number.
- \( R_m = \sigma \mu_e u_c \), the magnetic Reynolds number.
- \( M = L B_e (\sigma / \rho v)^{1/2} \), the Hartmann number.
- \( G = \beta g (\Delta T_e) L^2 / \nu^2 \), the Grashoff number.
- \( P_e = \mu_e u_c c_p L / \nu \lambda \), the Peclet number.
- \( \sigma_i^2 = L_i^2 / k \), the porous parameter.

In view of continuity equation (2.3) and the magnetic induction equation (2.6) velocity and magnetic field may chosen to be \( \overline{q} = (u, v) \); \( \overline{B} = (f, g) \).
where \((u,v)\) are the velocity components and \((f,g)\) are the components of the magnetic field along the \((x,y)\) directions respectively.

The boundary conditions relevant to the problem are

\[
\begin{align*}
    u &= 0, \quad v = 0 \quad \text{on} \quad y = \pm f(\delta x/L) \\
    T - T_e &= \gamma(\delta x/L)
\end{align*}
\]

Equations (2.10)-(2.15) constitute a system of six equations for the six unknowns \(u, v, f, g, J_z\) and \(p\). These may be reduced to two equations in terms of the stoke's stream function \(\psi(x,y)\) and the magnetic stream function \(\phi(x,y)\) given by in view of (2.11) and (2.13)

\[
\begin{align*}
    u &= -\psi_y \quad ; \quad v = \psi_x \quad (2.16) \\
    f &= -\phi_y \quad ; \quad g = \phi_x \quad (2.17)
\end{align*}
\]

(The subscripts \(x\) and \(y\) denote the respective partial derivatives). Combining (2.7) and (2.8) to eliminate \(J\), we find

\[
\nabla^2 \phi = R_m(\psi_x \phi_y - \psi_y \phi_x) \quad (2.18)
\]

where the operator \(\nabla^2\) is defined by

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]
Eliminating $J$ between (2.2) and (2.7) and taking the curl of the former to eliminate the pressure, we have

$$R[\psi_x (v^4_\psi)_y - \psi_y (v^4_\psi)_x] = v^4 (v^4_\psi) + \frac{M^2}{Rm} (\phi_x (v^4_\phi)_y - \phi_y (v^4_\phi)_x)$$

$$+ \left( \frac{G}{R} \right) \theta_x - \sigma^4_1 (v^4_\psi)$$

(2.19)

and the energy equation is

$$P_e (\theta_y \phi_x - \theta_x \phi_y) = \theta_{yy} + \theta_x x + 1$$

(2.20)

The current density can be found once $\psi$ and $\phi$ are known, for from equation (2.15) $J_x = (\psi_x \phi_y - \psi_y \phi_x)$

These coupled equations (2.18)-(2.20) to be solved subject to non-dimensional boundary conditions for $\psi$ and $\theta$

$$\psi(x,1) - \psi(x,-1) = 1$$

(2.21)

$$\psi_{yy} = 0 \quad & \quad \theta_y = 0 \quad \text{on } y = 0$$

(2.22)

$$\psi_x = \psi_y = 0 \quad \text{at } y = \pm (\delta x)$$

(2.23)

$$\theta = \gamma (\delta x)$$

Condition (2.21) assures the constant volumetric flow in consistence with the hypothesis (2.1), condition (2.22) corresponds to the axial symmetry of the
flow and (2.23) corresponds to no-slip condition and the condition related to the prescribed nonuniform boundary temperature \( \gamma(\bar{x}) \), which is assumed to be twice differentiable.

Electric currents within the fluid induces a magnetic field exterior to the channel as well as within. This external field, \( \mathbf{B} = (f, g) \) is given by \( f = -\phi_y, \ g = \phi_x \). Since \( R_m = 0 \) in the exterior region equation (2.14) requires

\[ \nabla^2 \phi = 0 \quad (2.24) \]

Both the potential and the field itself must be continuous at the wall (29a). Hence we may write the matching conditions

\begin{align*}
\phi &= \hat{\phi} \\
\phi_y &= \hat{\phi}_y \\
\text{at } y &= \pm f(\delta x) \\
\end{align*}  
(2.25)

Because of the symmetry the axial component of the magnetic field must vanish at the centreline, i.e.

\[ \phi_x = 0 \quad \text{at } y = 0 \quad (2.26) \]
Also the uniform applied field far from the channel (|y|>1) must be retrieved, i.e.

\[ \phi_y = -1 \text{ and at large } (|y| >> 1) \lim_{x \to \pm \infty} \phi_y = -1 \]  

(2.27)

It follows from equation (2.24) that \( \hat{\phi} \) and \( \hat{\phi}_y \) satisfy the Laplace equation and it is known that a harmonic function can attain its extrema only on the boundary. Hence \( \hat{\phi}_y \) can attain its maximum or minimum only on the channel walls \( y = \pm 1 \), \( \hat{\phi}_y \) should be either a constant or a function of \( x \). In case, it is a constant in view of the continuity requirement it should be \(-1\) in which case the induced magnetic field in the exterior region is absent, leading to a contradiction. Hence it can attain only its minimum. Thus \( \hat{\phi}_y = 0 \) on \( y = \pm 1 \).

However by matching conditions (2.25)

\[ \phi_y = \hat{\phi}_y \text{ on } y = \pm 1 \]  

(2.28)

3. ANALYSIS OF THE FLOW:

Following the analysis of Mc Michael and Deutsch (21a) we introduce the transformation

\[ \bar{x} = \delta x \]
We assume $\frac{\partial}{\partial x} \approx 0(\delta)$ such that $\frac{\partial}{\partial x} \approx 0(1)$ for small values of $\delta$, the flow develops slowly along the axial direction of the channel with this transformation (2.18) to (2.20) reduce to

$$F^2 \phi = \delta R_m (\psi_x \phi_y - \psi_y \phi_x)$$  \hspace{1cm} (3.1)

$$\delta R \left[ \left( \psi_x \left( F^2 \psi \right)_y - \psi_y \left( F^2 \psi \right)_x \right) \right] = F^4 \psi - (\delta M^2 / R_m) (\phi_x \left( F^2 \phi \right)_y - \phi_y \left( F^2 \phi \right)_x) + \frac{(G \phi_x - \sigma^2 F^4 \psi)}{R} \hspace{1cm} (3.2)$$

$$\delta \text{Pe} \left( \delta \psi_x \phi_y - \delta \phi_x \psi_y \right) = \delta \psi_{yy} + \delta \psi_{xx} + 1 \hspace{1cm} (3.3)$$

$$J = \delta (\psi_x \phi_y - \psi_y \phi_x) \hspace{1cm} (3.4)$$

Where

$$F^2 = (\delta^2 \partial^2 / \partial X^2 + \partial^2 / \partial Y^2)$$

Taking $R \sim 0(1)$ in the limit $\delta \rightarrow 0$, the viscous term remain and the inertial term vanish in the equation (3.2). From (3.1), we find that $F^2 \phi \sim 0(\delta)$ for $R_m \sim 0(1)$ in which case the magnetic term in (3.2) is of order $(\delta M^2)$. Thus even choosing $M$ as large as $(\delta^{-1/3})$ at the zeroth order (equation corresponding to $\delta \rightarrow 0$), neither the inertial terms nor the magnetic terms are present and their effect appear in equations of higher order. The viscous effects alone appear
at the zeroth order simplifying the analysis to a large extent. Using the regular perturbation method, we make use of the asymptotic expansion

$$\psi = \psi_0 + \delta \psi_1 + \delta^2 \psi_2 +$$

$$\theta = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 +$$

$$\phi = \phi_0 + \delta \phi_1 + \delta^2 \phi_2 +$$

Rewriting the Equations (3.1), (3.2) and (3.3) in terms of $$\eta = y/f(x)$$ and substituting (3.5) on separating the like power of $$\delta$$, the equations corresponding to the zeroth order are

$$\bar{\psi}_0, \eta \bar{\eta} = -\beta^i \bar{\psi} \eta \bar{\eta} = 0$$  \hspace{1cm} (3.6)

$$\bar{\theta}_0, \eta \bar{\eta} = f^i$$  \hspace{1cm} (3.7)

$$\bar{\phi}_0, \eta \bar{\eta} = 0$$  \hspace{1cm} (3.8)

$$\bar{\phi}_0, \eta \bar{\eta} = 0$$  \hspace{1cm} (3.9)

where

$$\sigma^i f^i = \beta^i$$
The corresponding conditions on \( \psi_o, \theta_o \) and \( \phi_o \) are

\[
\psi_o(x, +1) - \psi_o(x, -1) = 1 \quad (3.10)
\]

\[
\psi_o, x = \psi_o, \eta = 0 \quad \text{on } \eta = \pm 1
\]

\[
\theta_o = \gamma(x) \quad (3.11)
\]

\[
\psi_o, \eta \psi_o = 0, \quad \theta_o, \eta = 0, \quad \phi_o, x = 0 \quad \text{on } \eta = 0 \quad (3.12)
\]

At far infinity

\[
\phi_o, x = -1 \quad \text{on } \eta = \pm 1 \quad (3.13)
\]

\[
\phi_o, \eta = \phi_o, \eta = 0
\]

The equations to the first order are

\[
\theta_1, \eta \eta - \frac{\gamma'}{\gamma^2} \psi_1, \eta \eta = \text{Re} \left[ \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \right] + (G/R) \theta_o, x \quad (3.14)
\]

\[
\psi_1, \eta \psi_1, \eta \psi_1, \eta \psi_1, \eta \psi_1, \eta = \text{Re} \left[ \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \right] \quad (3.15)
\]

\[
\phi_1, \eta \phi_1, \eta \phi_1, \eta \phi_1, \eta \phi_1, \eta = \text{Re} \left[ \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \psi_o, \eta \right] \quad (3.16)
\]

The respective conditions are

\[
\psi_1(x, +1) - \psi_1(x, -1) = 1 \quad (3.17)
\]

\[
\psi_1, x = \psi_1, \eta = 0 \quad \text{on } \eta = \pm 1 \quad (3.18)
\]

\[
\theta_1 = 0
\]

\[
\psi_1, \eta \psi_1 = 0, \quad \theta_1, \eta = 0, \quad \phi_1, x = 0 \quad \text{on } \eta = 0 \quad (3.19)
\]
At far infinity

\[ \phi_1, x = 0 (3.20) \]
\[ \phi_1, \eta = \hat{\phi}_1, \eta = 0 \quad \text{on} \ \eta = \pm 1 \]

The equations to the second order are

\[ \psi_1, \eta = \mathcal{F}_1 (\psi_1, \eta) = 0 \quad \text{on} \ \eta = \pm 1 \]

where, \( \hat{M} = \delta M \)

The corresponding conditions are

\[ \psi_1(x, +1) - \psi_1(x, -1) = 1 \quad (3.24) \]
\[ \psi_1, \bar{x} = \psi_1, \eta = 0 \quad \text{on} \ \eta = \pm 1 \]
\[ \theta_1 = 0 \quad (3.25) \]
\[ \psi_1, \eta = 0, \theta_1, \eta = 0, \phi_1, \bar{x} = 0 \quad \text{on} \ \eta = 0 \quad (3.26) \]
At far infinity

\[ \phi_i, \bar{\chi} = 0 \]  \hspace{1cm} (3.27)

\[ \phi_i, \eta = \hat{\phi}_i, \eta = 0 \quad \text{on} \ \eta = \pm 1 \]

4. **SOLUTION OF THE PROBLEM**

The zeroth order solution for the stream function and temperature satisfying the corresponding conditions (3.10), (3.11), (3.12) and (3.13) are

\[ \psi_0 = -a_1 \eta + a_1 \text{Sh} \beta_1 \eta \]

\[ \theta_0 = -(f^2 \eta^2 / 2) + (2 \gamma + f^2 / 2) \]

\[ \phi_0 = \hat{\phi}_0 = -\eta \]

The solutions for \( \psi_i, \beta_i \) and \( \phi_i \) using conditions (3.17), (3.18), (3.19) and (3.20) corresponding to the first order perturbations are

\[ \psi_i = a_{30} \eta + a_{4 \eta} \text{Ch} (\beta_1 \eta) + a_{5} \text{Sh} (\beta_1 \eta) + H(\eta) \]

where

\[ H(\eta) = a_{31} \eta^2 \text{Sh} (\beta_1 \eta) - a_{32} \eta \text{Ch} (\beta_1 \eta) + a_{33} \text{Sh} (2\beta_1 \eta) + a_{34} \eta^3 \]

\[ + a_{35} \text{Ch} (\beta_1 \eta) - a_{36} \eta \text{Ch} (2\beta_1 \eta) - a_{37} \text{Ch} (3\beta_1 \eta) + a_{38} \eta^2 \]

\[ \beta_i = f \mathcal{P} e [a_{4} \eta^4 + a_4 \text{Ch} (\beta_1 \eta) + a_5 \eta \text{Sh} (\beta_1 \eta) - a_6 \eta^4 \text{Ch} (\beta_1 \eta) + (a_7 \eta^4 / 12)] - a_{10} \]

\[ \phi_i = \hat{\phi}_i = 0 \]
Similarly the solutions for $\psi_1$, $\theta_1$, and $\phi_1$ using (3.24), (3.25), (3.26) and (3.27) corresponding to the second order perturbations are

$$\psi_1 = e_{143} \eta + e_{142} \text{Ch}(\beta_1 \eta) + e_{141} \text{Sh}(\beta_1 \eta) + H_1 \eta$$

where

$$H_1(\eta) = \text{Ch}(\beta_1 \eta) [e_{151} \eta^4 + e_{152} \eta^5 + e_{153} \eta^6 + e_{154} \eta + e_{155}] + \text{Sh}(\beta_1 \eta) [e_{156} \eta^4 + e_{157} \eta^5 + e_{158} \eta^6 + e_{159} \eta^7 + e_{160} \eta^8 + e_{161} \eta^9 + e_{162} \eta^{10}]$$

$$+ \text{Ch}(2\beta_1 \eta) [e_{153} \eta^4 + e_{154} \eta^5 + e_{155} \eta^6 + e_{156} \eta^7 + e_{157} \eta^8 + e_{158} \eta^9 + e_{159} \eta^{10}]$$

$$+ \text{Ch}(3\beta_1 \eta) [e_{155} \eta^4 + e_{156} \eta^5 + e_{157} \eta^6 + e_{158} \eta^7 + e_{159} \eta^8 + e_{160} \eta^9 + e_{161} \eta^{10}]$$

$$+ \text{Sh}(2\beta_1 \eta) [e_{151} \eta^2 + e_{152} \eta^3 + e_{153} \eta^4 + e_{154} \eta^5 + e_{155} \eta^6 + e_{156} \eta^7 + e_{157} \eta^8 + e_{158} \eta^9 + e_{159} \eta^{10}]$$

$$+ \text{Sh}(3\beta_1 \eta) [e_{157} \eta^2 + e_{158} \eta^3 + e_{159} \eta^4 + e_{160} \eta^5 + e_{161} \eta^6 + e_{162} \eta^7 + e_{163} \eta^8 + e_{164} \eta^9 + e_{165} \eta^{10}]$$

$$+ \text{Sh}(4\beta_1 \eta) [e_{159} \eta^2 + e_{160} \eta^3 + e_{161} \eta^4 + e_{162} \eta^5 + e_{163} \eta^6 + e_{164} \eta^7 + e_{165} \eta^8 + e_{166} \eta^9 + e_{167} \eta^{10}]$$

$$+ \text{Sh}(5\beta_1 \eta) [e_{161} \eta^2 + e_{162} \eta^3 + e_{163} \eta^4 + e_{164} \eta^5 + e_{165} \eta^6 + e_{166} \eta^7 + e_{167} \eta^8 + e_{168} \eta^9 + e_{169} \eta^{10}]$$

$$+ \text{Sh}(6\beta_1 \eta) [e_{163} \eta^2 + e_{164} \eta^3 + e_{165} \eta^4 + e_{166} \eta^5 + e_{167} \eta^6 + e_{168} \eta^7 + e_{169} \eta^8 + e_{170} \eta^9 + e_{171} \eta^{10}]$$

$$+ \text{Sh}(7\beta_1 \eta) [e_{165} \eta^2 + e_{166} \eta^3 + e_{167} \eta^4 + e_{168} \eta^5 + e_{169} \eta^6 + e_{170} \eta^7 + e_{171} \eta^8 + e_{172} \eta^9 + e_{173} \eta^{10}]$$

$$+ \text{Sh}(8\beta_1 \eta) [e_{167} \eta^2 + e_{168} \eta^3 + e_{169} \eta^4 + e_{170} \eta^5 + e_{171} \eta^6 + e_{172} \eta^7 + e_{173} \eta^8 + e_{174} \eta^9 + e_{175} \eta^{10}]$$

$$+ \text{Sh}(9\beta_1 \eta) [e_{169} \eta^2 + e_{170} \eta^3 + e_{171} \eta^4 + e_{172} \eta^5 + e_{173} \eta^6 + e_{174} \eta^7 + e_{175} \eta^8 + e_{176} \eta^9 + e_{177} \eta^{10}]$$

$$+ \text{Sh}(10\beta_1 \eta) [e_{171} \eta^2 + e_{172} \eta^3 + e_{173} \eta^4 + e_{174} \eta^5 + e_{175} \eta^6 + e_{176} \eta^7 + e_{177} \eta^8 + e_{178} \eta^9 + e_{179} \eta^{10}]$$
\[ \Theta_1 = \text{Sh}(\beta_1 \psi)[b_{\delta_6} + b_{\delta_1} \psi + b_{\delta_8} \psi^2 - b_{\delta_3} \psi^3] \\
- b_{\delta_9} \psi^4 - b_{\delta_0} \psi^6 \text{+ Ch}(\beta_1 \psi)[b_{\delta_7} + b_{\delta_9} \psi] \\
+ m_1 \psi^2 - b_{\delta_6} \psi^3 - b_{\delta_8} \psi^4 + b_{\delta_1} \psi^5 \text{+ Ch}(2\beta_1 \psi)[b_{\delta_4} \\
+ b_{\delta_7} \psi - b_{\delta_6} \psi^5 + b_{\delta_4} \psi^6 + b_{\delta_1} \psi^7] \text{+ Sh}(2\beta_1 \psi)[b_{\delta_5} \\
+ b_{\delta_3} \psi^3 - b_{\delta_6} \psi^4] \text{+ Sh}(2\beta_1 \psi)[b_{\delta_5} \\
- b_{\delta_9} \psi^7 - b_{\delta_9} \psi^6 \\
- b_{\delta_0} \psi^5 + b_{\delta_0} \psi^4 + b_{\delta_0} \psi^2 - b_{\delta_0} \psi - b_{\delta_0} \psi \\
\] \\
\Phi_2 = h_{1,0}^5 \psi^2 - \text{Ch}(\beta_1 \psi)[h_{1,0} \psi - h_{1,0} \psi \\
- h_{1,1} \psi^2 + h_{1,1} \psi^3] \text{+ Sh}(\beta_1 \psi)[h_{1,0} \psi + h_{1,0} \psi \\
+ h_{1,2} \psi^2 - h_{1,2} \psi^3] \text{- Ch}(2\beta_1 \psi)[h_{1,0} - h_{1,0} \psi \\
+ h_{1,1} \psi^4] \text{- Sh}(2\beta_1 \psi)[h_{1,4} + h_{1,4} \psi + h_{1,4} \psi \\
+ h_{1,2} \psi^4 - h_{1,2} \psi^3 + (h_{\delta_4}/2) \\
\hat{\Phi}_2 = m_2 - m_0 \text{Sh}(\beta_1) - m_4 \text{Ch}(\beta_1) + m_5 \text{Sh}(2\beta_1) + m_6 \text{Ch}(2\beta_1) \]
5. SHEAR STRESS AND NUSSELT NUMBER:

The Shear Stress ($\tau$) on the channel walls is given by

$$\tau' = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

which in the non-dimensional form reduces to

$$\tau = \tau' / (\mu u_c/L) = u_y + v_x = \delta^2 \psi_{1x} - \psi_{yy}$$

Therefore

$$\tau = \frac{-(1-f^{'1})}{2F^*(1+f^{'1})} \left( \psi_{1x} + \delta \psi_1 - \delta^2 \psi_{1x} \right)$$

$$(\tau)_{\eta=1} = \frac{-(1-f^{'1})}{2F^*(1+f^{'1})} \left[ a_1 \beta_1^2 Sh(\beta_1) + \delta (m_1 Ch(\beta_1) + m_6 Sh(\beta_1)) + m_3 Ch(2\beta_1) + m_{10} Sh(2\beta_1) - m_4 \right] + \mathcal{O}(\delta^2)$$

$$(\tau)_{\eta=-1} = \frac{-(1-f^{'1})}{2F^*(1+f^{'1})} \left[ -a_1 \beta_1^2 Sh(\beta_1) + \delta (m_1 Ch(\beta_1) - m_6 Sh(\beta_1)) - m_5 Ch(2\beta_1) - m_{10} Sh(2\beta_1) - m_4 \right] + \mathcal{O}(\delta^2)$$
The local heat transfer coefficient (Nusselt number) on the channel walls has been calculated using the formula

\[
\text{Nu} = \frac{1}{(8m_{\infty} W)} \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=1}
\]

and corresponding expression is

\[
\text{Nu} = \frac{-f^l + \delta (m_{1,0} + m_{1,1} \text{Ch}(\beta_i) + m_{1,2} \text{Sh}(\beta_i))}{2f_1 (\gamma^2 + 2f/3) + \delta (m_{1,0} + m_{1,1} \text{Sh}(\beta_i) + m_{1,2} \text{Ch}(\beta_i))}
\]

6. DISCUSSION OF THE NUMERICAL RESULTS

The aim of our analysis in this problem is to discuss the behaviour of the temperature induced buoyancy flow through a porous medium bounded by a horizontal nonuniform channel. The flow phenomenon is investigated for different sets of parameters \(G, R, M, \sigma, \alpha\) and \(\beta\). The variation in \(\alpha\) and \(\beta\) would throw light on the effects of wall temperature and the surface geometry on the flow field. The configuration being horizontal the buoyancy is in transverse direction to the primary flux and hence the flow field is basically asymmetric. For computational purpose we assume
Variation of primary velocity ($u$) with $G$ when $R = 50$, $\alpha = 2$, $P = -0.5$, $Pe = 35$, $M = 50$, $X = n/4$.
the boundaries to be \( y = \pm f(x) = \pm (1 + \beta \exp(-x)) \), \( \beta \) positive corresponds to the dilated channel and \( \beta \) negative corresponds to the constricted channel. However the use of the transformation \( \eta = y / f(x) \) transforms the boundaries \( y = \pm f(x) \) to \( \eta = \pm 1 \). Also the temperature field on the boundaries is prescribed as \( \gamma(x) = \alpha \sin(\bar{x}) \). The nonuniformity in the geometry gives rise to a secondary flow in the transverse direction. The computation of the individual velocity components enable us to investigate the effect of each governing parameter acting on the flow and its relative influence on the the primary and secondary flows.

We observe in general, for all variations in parameters no reversal flow appears in the flow field. Figs.(1a,1b) show that, for positive \( G \) the axial velocity \( u \) exhibits the same behaviour either in the dilated or constricted channel where as for \( G \) negative the behaviour of \( u \) in a dilated channel differs from that of a constricted channel. When \( G \) is positive, the other parameters being fixed, we find that \(|u|\) increases with \( G \) in the left region \((-1 < y < 0)\) and decreases in the right fluid region \((0 < y < 1)\).
Fig. 2
Profiles for $u$ with $\sigma$
$G = 3 \times 10^3, \sigma = 2, Pe = 35, H = 50, X = \pi/4, R = 50$
\[\begin{array}{cccccc}
\alpha & b & c & d & e & f \\
2 & 5 & 7 & 2 & 5 & 7 \\
\beta & 0.5 & 0.5 & 0.5 & -0.5 & -0.5 & -0.5
\end{array}\]

Fig. 3
Profiles for $u$ with $\alpha$
$G = 3 \times 10^3, H = 50, \sigma = 2, X = \pi/4, Pe = 35, R = 50$
\[\begin{array}{cccccc}
\alpha & b & c & d & e & f \\
2 & 5 & 10 & 2 & 5 & 10 \\
\beta & 0.5 & 0.5 & 0.5 & -0.5 & -0.5 & -0.5
\end{array}\]
Fig. 4

$u$ with $R$

$G = 3 \times 10^3$, $\sigma_1 = 2$, $\alpha = 2$, $X = \pi/4$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
<td>200</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>35</td>
<td>70</td>
<td>140</td>
<td>35</td>
<td>70</td>
<td>140</td>
</tr>
<tr>
<td>$\beta = 0.5$</td>
<td>$0.5$</td>
<td>$0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
</tr>
</tbody>
</table>

Fig. 5

$u$ with $\tilde{H}$

$G = 3 \times 10^3$, $\sigma_1 = 2$, $\alpha = 2$, $X = \pi/4$, $Pe = 35$, $R = 50$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
<td>200</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>$\beta = 0.5$</td>
<td>$0.5$</td>
<td>$0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
</tr>
</tbody>
</table>

$h = 3 \times 10^3$, $\sigma_1 = 2$, $\alpha = 2$, $X = \pi/4$, $Pe = 35$, $R = 50$
$u$ with $\beta$

$G = 3 \times 10^3$, $M=50$, $\sigma_1 = 2$, $\alpha = 2$, $X = \pi/4$, $Pe=35$, $R=50$

$\beta = 1.5 \ 0.5 \ 0.1 \ -1.5 \ -0.5 \ -0.1$
But for negative G in a dilated channel, |uj| decreases in the left region and increases in the right side while in a constricted channel |uj| increases with G in the whole flow field. Fig. 2 shows that u increases with \( \sigma_1 \) in both the channels although in the dilated case we observe for values \( \sigma_1 > 5 \) it increases rapidly. The behaviour of u with \( \alpha \) once again from dilated to a constructed case (Fig. 3). In a dilated case |uj| increases with \( \alpha \) in the left region and decreases in the right region whereas in a constricted case |uj| increases with \( \alpha \) everywhere in the flow field. For an increase in \( R \), |uj| decreases in the left side and increases in the right region in either of the channel Fig. (4). Fig. (5) indicates the influence of magnetic parameter M on |uj| and we observe that for all G and \( \beta \), an increase in M increases |uj|. Also we find that |uj| decreases with an increase in the amplitude of a dilation while an increase of in the constriction increases |uj| (Fig. (6)).

The behaviour of the secondary flow v has been plotted in Fig. 7-12. In general, v is always directed towards the midplane of the channel on either sides and its behaviour for positive G is in contrast to that for negative
Fig. 7

Variation of secondary velocity \( (v) \) with G

\( H=50, \quad \sigma_i=2, \quad \alpha = 2, \quad X=\pi/4, \quad Pe=35, \quad R=60 \)

\( a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \quad i \quad j \quad k \)

\( G = 10^4 \quad 3\times10^4 \quad 5\times10^4 \quad -10^4 \quad -3\times10^4 \quad -6\times10^4 \quad 10^4 \quad 3\times10^4 \quad 5\times10^4 \quad -10^4 \quad -3\times10^4 \)

\( \beta = 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad -0.5 \quad -0.5 \quad -0.5 \quad -0.5 \quad -0.5 \)
Fig. 6

$v$ with $g$

$G = 3 \times 10^5$, $\alpha = 2$, $X = \pi/4$, $N = 50$, $Pe = 35$, $R = 50$

$a$ $b$ $c$ $d$ $e$

$\sigma = 2$ $5$ $7$ $2$ $7$

$\beta = 0.5$ $0.5$ $0.5$ $-0.5$ $-0.5$
Fig. 9

$\nu$ with $\alpha$

$G = 3 \times 10^3, \sigma_1 = 2, X = \frac{\pi}{4}, H_1 = 50, Pe = 36, R = 50$

$\alpha_1 = 2 \ 5 \ 7 \ 10 \ 2 \ 5 \ 7 \ 10$

$\beta = 0.6 \ 0.6 \ 0.5 \ 0.5 \ -0.5 \ -0.5 \ -0.5 \ -0.5$
Fig. 10

\[ v \text{ with } R \]

\[ G = 3 \times 10^0, \sigma_i = 2, \alpha = 2, X = \pi / 4, \hat{M} = 50 \]

\[ \begin{array}{cccccc}
  a & b & c & d & e & f \\
  R & 50 & 100 & 200 & 50 & 100 & 200 \\
  Pe & 35 & 70 & 140 & 35 & 70 & 140 \\
  \beta & 0.5 & 0.5 & 0.5 & -0.5 & -0.5 & -0.5 \\
\end{array} \]
Fig. 11a

$v$ with $\hat{M}$

$G = 3 \times 10^3$, $\sigma_t = 2$, $\alpha = 2$, $X = \pi/4$, $Pe = 35$, $R = 50$

$\hat{M} = 50$  100  150  50  100  150

$\beta = 0.5$  0.5  0.5  -0.5  -0.5  -0.5

Fig. 11b

$v$ with $\hat{M}$

$G = -3 \times 10^3$, $\sigma_t = 2$, $\alpha = 2$, $X = \pi/4$, $Pe = 35$, $R = 50$

$\hat{M} = 50$  100  150  50  100  150

$\beta = 0.5$  0.5  0.5  -0.5  -0.5  -0.5
Fig. 12a

\[ v \text{ with } \beta \]
\[ G = 3 \times 10^3, \sigma_1 = 2, \alpha = 2, X = \pi / 4, Pe = 35, R = 50, \hat{M} = 50 \]

\[ \beta = 1.5 \ 0.5 \ 0.3 \ 0.1 \]

Fig. 12b

\[ v \text{ with } \beta \]
\[ G = 3 \times 10^3, \sigma_1 = 2, \alpha = 2, X = \pi / 4, Pe = 35, R = 50, \hat{M} = 50 \]

\[ \beta = -1.5 \ -0.5 \ -0.3 \ -0.1 \]
G. From fig.7 we find that in a dilated channel $|v|$ decreases in the left region and increases in the right side for an increase in positive $G$. However, reversal behaviour is observed for negative $G$. In a constricted case for $G>0$ its magnitude increases with $G(>0)$ in the left side, where as for $G(<0)$ we find this behaviour in the right region. In the rest of the regions growth in $|v|$ is found for $|G| \approx 3 \times 10^4$ and at higher $G$ it starts decresing (fig.7). In a dilated channel growth of this secondary velocity with the porous parameter $\sigma_1$ may be observed from (fig.8) while in a constricted case $|v|$ decreases with $\sigma_1$. An increase in $\alpha$ decreases $|v|$ in the left region and increases in the right side (fig.9). With respect to variation in $R$ a reversal behaviour may be observed with $|v|$ increasing in the right region and decreasing in the left side (fig.10). The influence of a magnetic field on the secondary velocity may be observed from figs.(11a,11b) and we find a growth in $|v|$ with an increase in the parameter $\hat{M}$. The secondary velocity however decays rapidly with an increase in the amplitude of either dilated or constricted channel figs.(12a,12b). The rate of decay in a dilated channel is rapid compared to its decay in a constricted case.
Fig. 13

Variation of temperature ($\theta$) with $G$

$\sigma_1 = 2, \alpha = 2, \beta = 0.5, X = \pi/4, Pe = 35, R = 50, \hat{M} = 50$

$G = 10^3, 10^4, 2 \times 10^4, 5 \times 10^4, -10^3, -10^4, -2 \times 10^4, -5 \times 10^4$
Fig. 14

8 with G

$\sigma_1 = 2, \alpha = 2, \beta = -0.5, X = \pi/4, P_e = 35, R = 50, \hat{M} = 50$

a b c d e f

$G = 10^3 \quad 10^4 \quad 2 \times 10^4 \quad -10^3 \quad -10^4 \quad -2 \times 10^4$
Fig. 15a

\[ \theta \text{ with } \sigma \]

- \( \sigma = 2, 5, 7, 10 \)
- \( G = 3 \times 10^{9}, \alpha = 2, \beta = -0.5, X = \pi/4, Pe = 35, R = 50, H = 50 \)

Fig. 15b

\[ \theta \text{ with } \sigma \]

- \( \sigma = 2, 5, 7, 10 \)
- \( G = 3 \times 10^{9}, \alpha = 2, \beta = -0.5, X = \pi/4, Pe = 35, R = 50, H = 50 \)
Fig. 16a

θ with α

G = 3 \times 10^3, \sigma_1 = 2, \beta = 0.5, X = \pi/4, Pe = 35, R = 50, \hat{M} = 50

a, b, c

α = 2 5 7

Fig. 16b

θ with α

G = 3 \times 10^3, \sigma_1 = 2, \beta = -0.5, X = \pi/4, Pe = 35, R = 50, \hat{M} = 50

a, b, c

α = 2 5 7
Fig. 17a

\[ G = 3 \times 10^3, \quad \sigma_1 = 2, \quad \alpha = 2, \quad \beta = 0.5, \quad X = \pi/4, \quad \hat{M} = 50 \]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Pe</td>
<td>35</td>
<td>70</td>
<td>140</td>
</tr>
</tbody>
</table>

Fig. 17b

\[ G = 3 \times 10^3, \quad \sigma_1 = 2, \quad \alpha = 2, \quad \beta = -0.5, \quad X = \pi/4, \quad \hat{M} = 50 \]

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Pe</td>
<td>35</td>
<td>70</td>
<td>140</td>
</tr>
</tbody>
</table>
Fig. 18

$\theta$ with $\hat{H}$

$\sigma_1 = 2$, $\alpha = 2$, $X = \pi/4$, $Pe = 35$, $R = 50$

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{H}$</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$G$</td>
<td>$3 \times 10^0$</td>
<td>$3 \times 10^0$</td>
<td>$3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td>$3 \times 10^3$</td>
<td>$3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
<td>$-3 \times 10^3$</td>
</tr>
</tbody>
</table>
Fig. 19

\[ \theta \text{ with } \beta \]

\[ G = 3 \times 10^3, \sigma_1 = 2, \alpha = 2, X = \pi / 4, \beta = 50, \text{ Pe } = 35, R = 50 \]

\[ \beta = 1.5 \ 0.5 \ 0.3 \ 0.1 \ -1.5 \ -0.5 \ -0.3 \ -0.1 \]
Figures 13-19 indicate the behaviour of the nondimensional temperature $\theta$ with variation in the governing parameters. We observe that the temperature exhibits a slight increasing tendency for an increase in positive $G$ while a small decrement is observed for an increase in $|G|(G<0)$. In general, the variation of $\theta$ with reference to $G$ is found only at the second order perturbation. From figures 15a, 15b, 16 and 18 it may be noted that $\theta$ increases with $\sigma, \alpha$ and $M$. The growth of $\theta$ with $\alpha$ is rapid compared to other parameters. $\theta$ however decays with an increase in $R$ as may be seen from figs.17a and 17b. A similar observation may be made in fig.19 that irrespective of dilation constriction $\theta$ decays with the amplitude $|\beta|$. 

The shear stress and the Nusselt number have been evaluated for variations in different governing parameters $G, \hat{M}, \sigma_1, \alpha$ and $\beta$ and are tabulated in tables.1-4. It is found that the shear stress at both the walls increases in magnitude with $G(<0)$ while it decreases with $G(>0)$. The shear stress at both the walls decreases with increase in $\hat{M}$ and $\sigma_1$ for all $G$. For fixed $G, \hat{M}, \sigma_1$ and $\alpha$ the shear stress on the upper plate decreases with increase in positive $\beta$ and
### Table 1

**Shear Stress (τ) at the Upper Plate**  \( \eta = 1 \)

\( R = 50, \, P_E = 35, \, X = \pi/4 \)

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3 \times 10^6 )</td>
<td>-0.2698</td>
<td>0.2301</td>
<td>-1.3185</td>
<td>0.0706</td>
<td>1.8496</td>
<td>1.1606</td>
<td>-0.9988</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>0.2150</td>
<td>0.7149</td>
<td>-1.0098</td>
<td>0.4457</td>
<td>1.9373</td>
<td>1.2991</td>
<td>-0.0279</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>0.4332</td>
<td>0.9331</td>
<td>-0.8709</td>
<td>0.6146</td>
<td>1.9768</td>
<td>1.3614</td>
<td>0.4089</td>
</tr>
<tr>
<td>0</td>
<td>0.4574</td>
<td>0.9574</td>
<td>-0.8555</td>
<td>0.6333</td>
<td>1.9812</td>
<td>1.3883</td>
<td>0.4574</td>
</tr>
<tr>
<td>( -10^3 )</td>
<td>0.4817</td>
<td>0.9816</td>
<td>-0.8400</td>
<td>0.6521</td>
<td>1.9856</td>
<td>1.3752</td>
<td>0.5060</td>
</tr>
<tr>
<td>( -10^2 )</td>
<td>0.6999</td>
<td>1.1998</td>
<td>-0.7011</td>
<td>0.8209</td>
<td>2.0225</td>
<td>1.4375</td>
<td>0.9423</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>1.1848</td>
<td>1.6847</td>
<td>-0.3924</td>
<td>1.1960</td>
<td>2.1129</td>
<td>1.5759</td>
<td>1.9138</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{M} )</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( b )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
<td>-0.5</td>
<td>-0.3</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
TABLE-2

SHEAR STRESS ($\gamma$) AT THE LOWER PLATE $\gamma = -1$

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\times10^4$</td>
<td>-0.8515</td>
<td>-0.3515</td>
<td>0.72547</td>
<td>-0.8114</td>
<td>-1.1135</td>
<td>-0.8617</td>
<td>-1.5805</td>
</tr>
<tr>
<td>$10^5$</td>
<td>-0.3666</td>
<td>0.1333</td>
<td>1.0344</td>
<td>-0.4363</td>
<td>-1.0257</td>
<td>-0.7233</td>
<td>-0.6096</td>
</tr>
<tr>
<td>$10^6$</td>
<td>-0.1484</td>
<td>0.3515</td>
<td>1.1735</td>
<td>-0.2615</td>
<td>-0.9863</td>
<td>-0.6610</td>
<td>-0.1727</td>
</tr>
<tr>
<td>0</td>
<td>-0.1241</td>
<td>0.3757</td>
<td>1.1888</td>
<td>-0.2487</td>
<td>-0.9819</td>
<td>-0.6541</td>
<td>-0.1241</td>
</tr>
<tr>
<td>$-10^6$</td>
<td>-0.0999</td>
<td>0.4000</td>
<td>1.2042</td>
<td>-0.2300</td>
<td>-0.9775</td>
<td>-0.6472</td>
<td>-0.0756</td>
</tr>
<tr>
<td>$-10^7$</td>
<td>0.1182</td>
<td>0.6182</td>
<td>1.3431</td>
<td>-0.0611</td>
<td>-0.9380</td>
<td>-0.5849</td>
<td>0.3612</td>
</tr>
<tr>
<td>$-3\times10^7$</td>
<td>0.6031</td>
<td>1.1030</td>
<td>1.6518</td>
<td>0.3139</td>
<td>-0.8502</td>
<td>-0.4464</td>
<td>1.3321</td>
</tr>
</tbody>
</table>

FOR LEGEND AS IN TABLE-I
<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^6$</td>
<td>-3.4864</td>
<td>-7.8133</td>
<td>-7.9565</td>
<td>-3.3432</td>
<td>-2.7633</td>
<td>-2.4499</td>
<td>-1.3816</td>
</tr>
<tr>
<td>$-10^6$</td>
<td>-3.3855</td>
<td>-7.7499</td>
<td>-7.9464</td>
<td>-3.2630</td>
<td>-2.7500</td>
<td>-2.4218</td>
<td>-1.4197</td>
</tr>
<tr>
<td>$-3 \times 10^6$</td>
<td>-3.3351</td>
<td>-7.7182</td>
<td>-7.9413</td>
<td>-3.2219</td>
<td>-2.7434</td>
<td>-2.4077</td>
<td>-1.4286</td>
</tr>
</tbody>
</table>

FOR LEGEND AS IN TABLE-I
### TABLE-4

**AVERAGE NUSSELT NUMBER (N) AT THE LOWER PLATE $\gamma = -1$**

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^6$</td>
<td>3.4864</td>
<td>7.8133</td>
<td>7.9565</td>
<td>3.3432</td>
<td>2.7633</td>
<td>2.4499</td>
<td>1.3818</td>
</tr>
<tr>
<td>$10^6$</td>
<td>3.4360</td>
<td>7.7816</td>
<td>7.9514</td>
<td>3.3042</td>
<td>2.7567</td>
<td>2.4358</td>
<td>1.5307</td>
</tr>
<tr>
<td>$10^4$</td>
<td>3.4133</td>
<td>7.7673</td>
<td>7.9491</td>
<td>3.2857</td>
<td>2.7537</td>
<td>2.4295</td>
<td>1.5078</td>
</tr>
<tr>
<td>0</td>
<td>+3.4108</td>
<td>7.7657</td>
<td>7.9489</td>
<td>3.2836</td>
<td>2.7333</td>
<td>2.4288</td>
<td>1.5052</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>+3.4082</td>
<td>7.7642</td>
<td>7.9486</td>
<td>3.2815</td>
<td>2.7530</td>
<td>2.4281</td>
<td>1.5027</td>
</tr>
<tr>
<td>$-10^6$</td>
<td>3.3855</td>
<td>7.7499</td>
<td>7.9464</td>
<td>3.2630</td>
<td>2.7500</td>
<td>2.4218</td>
<td>1.4197</td>
</tr>
<tr>
<td>$-3 \times 10^6$</td>
<td>3.3351</td>
<td>7.7182</td>
<td>7.9413</td>
<td>3.2219</td>
<td>2.7434</td>
<td>2.4077</td>
<td>1.4286</td>
</tr>
</tbody>
</table>

*FOR SEE LEGEND AS IN TABLE-I*
increases with increases in negative $\beta$. The reverse is true at the lower plate. Also the amplitude of the prescribed non-uniform temperature increases $\tau$ for $G<0$ and decreases for $G>0$. The reverse is true at the lower plate.

Tables 3 & 4 show the variation the average Nusselt number for all $G, \sigma, \beta, \hat{M}$ and $\alpha$. It is observed that the average Nusselt number at the upper wall is negative and at the lower wall it is positive. For fixed $\sigma, \beta, \hat{M}$ and $\alpha$, it increases in magnitude with $G>0$ and decreases with $G<0$. The Nusselt number at both the walls increases in magnitude with an increase in $\sigma, \hat{M}$ and $\beta$ fixing the other parameters. Also it is found that the amplitude of the prescribed non-uniform temperature decreases the average Nusselt number.