HALL EFFECTS ON UNSTEADY FLOW OF INCOMPRESSIBLE ELECTRICALLY CONDUCTING SECOND GRADE FLUID THROUGH A COMPOSITE MEDIUM IN A ROTATING PARALLEL PLATE CHANNEL BOUNDED BY A POROUS BED
3.1. Introduction:

In recent years, there has been a considerable interest in rotating hydro magnetic fluid flows due to possible applications to geophysical and astrophysical problems. An order of magnitude analysis shows that in the basis field equations, the Coriolis force is very significant as compared to the inertial force. Furthermore, it reveals that the Coriolis and magneto hydrodynamic forces are of comparable magnitude. It is generally admitted that a number of astronomical bodies (e.g. the Sun, Earth, Jupiter, magnetic Stars, pulsars) possess fluid interiors and (at least surface) magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydro magnetic spin-up. This problem of spin-up in magneto hydro dynamic rotating by many researchers notably Gilman and Benton [11], Benton and Loper [1], Chawala [2], Debnath [5, 6] and Singh [22]. In all these analyses, the effects of the Hall current are not considered. Therefore, the results in these investigations cannot be applied to the flow of ionized gases. This is because in an ionized gas where the density is low and/or the applied magnetic field is strong, the effect of Hall currents may be significant. Katagiri [14] has studied the effects of the Hall currents on the magneto hydro dynamic boundary layer flow past a semi-infinite plate. Gupta [12] and Pop and Soundalgekar [16] have investigated the effects of the Hall current on the steady hydro magnetic flow in an incompressible viscous non-rotating fluid. Debnath et al. [4] discussed its effects on unsteady flow in a rotating viscous fluid.

Despite the above studies, attention has hardly been focused to study the effects of the Hall current on unsteady hydro magnetic rotating non-Newtonian fluid flows. Such work seems to be important and useful partly for gaining a basic understanding of such
flows, and partly for possible applications of these fluids in chemical process industries, food and construction engineering, movement of biological fluids, in petroleum production and in power engineering. Another important field of application is the electromagnetic propulsion. Basically, an electromagnetic propulsion system consists of a power source, such as a nuclear reactor, plasma and a tube through which the plasma is accelerated by electromagnetic forces. The study of such systems, which is closely associated with magneto chemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, shear stress-shear rate relationship, thermal conductivity, electrical conductivity, and radiation. Some of these properties will undoubtedly be influenced by the presence of an external magnetic field which sets the plasma in hydrodynamic motion. Sarpkaya [21] discussed the steady flow of uniformly conducting incompressible fluid (Bingham plastic model and the power law model) between two parallel planes. The fluid considered is under the influence of a constant pressure gradient. Recently Hayat et.al (13) studied an exact solution of an oscillatory flow of a rotating second grade fluid under the influence of uniform transverse magnetic field taking hall current in account. The problem reduces to that of Singh [22]. Similar to Singh [22], it is also noted in a second-grade fluid that the claim of Ganapathy [10] namely: “that the solution of Mazumder [15] explains the correct resonance phenomenon in a rotating system” is incorrect. The primary purpose of the present paper is to study the effects of the Hall current on the unsteady flow of a rotating conducting second-grade fluid enclosed between two rigid non-conducting parallel plates. The hydro magnetic flow is generated in the uniformly rotating fluid system by non torsional oscillations of the lower plate. The exact solution of the velocity field is obtained by making use of
Laplace transform technique. Special attention is devoted to the physical nature of the solutions, and the structure of the various part of the solution depends upon the Hall parameter but is independent of the material parameter of the second-grade fluid.

In this chapter we study the effects of hall current on the unsteady flow of a rotating conducting second order fluid enclosed between two rigid in finite non-conducting parallel plate channel bounded below by a porous bed. The exact solution of the steady and the unsteady velocity field are evaluated and the time of decay of the transient velocity is discussed. The steady part of the solution which depends on the Hall parameter is analytically and computationally discussed with reference to the governing parameters.
3. 2. Formulation and Solution of the problem:

We consider the unsteady flow of an incompressible electrically conducting second grade fluid through a composite medium in a rigidly rotating parallel plate channel with upper plate bounding the clean fluid and the lower plate bounding below on sparsely packed porous bed subjected to a uniform transverse magnetic field normal to the channel and taking hall current into account. In the initial undisturbed state both the fluid and the plates are in rigid rotation with the same angular velocity $\Omega$ about the normal to the plates and at $t > 0$ the fluid is driven by a constant pressure gradient parallel to the channel walls and in addition the lower plate perform non-torsional oscillations in its own plane. In the equation of motion along x-direction the x-component current density $\mu_e J_x H$ and the x-component current density $-\mu_e J_x H$.

We choose a Cartesian system $0(x, y, z)$ such that the boundary walls are at $z=0$ and $z=1$, z-axis being the axis of rotation of the plates. The fluid medium consists of two zones namely zone 1 and zone 2. Zone 1 consists of clean fluid governed by equations of motion, which are derived using constitutive equations for the stresses in compressible second order fluid while zone 2 corresponds to the flow through porous bed governed by Brinkman’s equations. At the interface the fluid satisfies the continuity condition of velocity and shear stress. Since the plates extends to infinity along x and y directions, all the physical quantities except the pressure depend on z and t alone.

The constitutive equation for the stress $T$ in an incompressible fluid of second grade is given by

$$T(t) = -pl + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_t$$

(3.2.1)

Where $\mu$ is the dynamic viscosity $\alpha_1$, $\alpha_2$ are the normal stress moduli and the kinematical tensors $A_1$ and $A_2$ are defined through [Rivlin et.al. (20)].

$$A_1 = (\text{grad } V) + (\text{grad } V)^T,$$
\[ A_2 = \frac{dA_i}{dt} + A_i (\text{grad } V) + (\text{grad } V)^T A_1, \]  

(3.2.2)

Where \( V \) is the velocity, \( \text{grad} \) the gradient operator and \( \frac{d}{dt} \) the material time derivative.

The unsteady hydro magnetic flow in a rotating co-ordinate system is governed by the equation of motion, continuity equation and the Maxwell equations in the form.

\[
\rho \left( \frac{\partial V}{\partial t} + (V \cdot \nabla) V + 2 \Omega \times V + \Omega \times (\Omega \times r) \right) = \nabla \cdot T + J \times B 
\]  

(3.2.3)

\[ \nabla \cdot V = 0 \]  

(3.2.4)

\[ \nabla \cdot B = 0 \]  

(3.2.5)

\[ \nabla \times B = \mu_m J \]  

(3.2.6)

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  

(3.2.7)

Where,

\( J \) is the current density, \( B \) is the total magnetic field, \( E \) is the total electric field, \( \mu_m \) is the magnetic permeability and \( r \) is radial co-ordinate given by \( r^2 = x^2 + y^2 \). When the strength of the magnetic field is very large, the generalized ohm's law is modified to include the hall current so that

\[
J + \frac{\omega_e \tau_e}{B_o} (J \times B) = \sigma \left[ E + V \times B + \frac{1}{e \eta_e} \nabla P_e \right] 
\]  

(3.2.8)

Where \( \omega_e \) is the cyclotron frequency of the electrons, \( \tau_e \) is the electron collision time, \( \sigma \) is the electrical conductivity, \( e \) is the electron charge and \( P_e \) is the electron pressure. The ion-slip and thermo electric effects are not included in equation (3.2.8).
Further it is assumed that $\omega, \tau_c \sim 0 \ (1)$ and $\omega, \tau_i << 1$, where $\omega_c$ and $\tau_i$ are the cyclotron frequency and collision time for ions respectively.

The unsteady hydro magnetic equations governing the incompressible electrically conducting second grade fluid in zone 1 under the influence of transverse magnetic field with reference to a frame rotating with a constant angular velocity $\Omega$ are

\[
\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial z^2} + \alpha_1 \frac{\partial^3 u}{\partial z^3} + \mu_e J_y H_0 u \tag{3.2.9}
\]

\[
\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \frac{\partial^2 v}{\partial z^2} + \alpha_1 \frac{\partial^3 v}{\partial z^3} - \mu_e J_x H_0 v \tag{3.2.10}
\]

The Brinkman-equations governing the flow through porous medium with respect to the rotating frame zone 2.

\[
\frac{\partial u_p}{\partial t} - 2\Omega v_p = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v_{\text{eff}} \frac{\partial^2 u_p}{\partial z^2} + \alpha_1 \frac{\partial^3 u_p}{\partial z^3} + \mu_e J_y^p H_0 u_p - \frac{v}{k} u_p \tag{3.2.11}
\]

\[
\frac{\partial v_p}{\partial t} + 2\Omega u_p = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v_{\text{eff}} \frac{\partial^2 v_p}{\partial z^2} + \alpha_1 \frac{\partial^3 v_p}{\partial z^3} - \mu_e J_x^p H_0 v_p - \frac{v}{k} v_p \tag{3.2.12}
\]

where $(u, v)$ and $(u_p, v_p)$ are velocity components along $O(x, y)$ directions respectively. $\rho$ the density of the fluid , $\sigma$ the conductivity of the medium, $\mu_e$ the magnetic permeability, $\nu$ the coefficient of kinematic viscosity, $\nu_{\text{eff}}$ the coefficient of effective kinematic viscosity, $k$ the permeability of the medium, $H_0$ is the applied magnetic field. Hence $u, v$ and $u_p, v_p$ are function of $z$ and $t$ alone and hence the respective equations of continuity are trivially satisfied. In equation (3.2.8) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

\[
J_x + m J_y = \sigma \mu_e H_0 v \tag{3.2.13}
\]

\[
J_y - m J_x = -\sigma \mu_e H_0 u \tag{3.2.14}
\]
Where \( m = \tau_e \omega_e \) is the hall parameter.

On solving equations (3.2.13) and (3.2.14) we obtain

\[
J_x = \frac{\sigma \mu_x H_0}{1 + m^2} (v + mu) \quad (3.2.15)
\]
\[
J_y = \frac{\sigma \mu_x H_0}{1 + m^2} (mv - u) \quad (3.2.16)
\]

Similarly we obtain,

\[
J_{x_p}^p = \frac{\sigma \mu_x H_0}{1 + m^2} (v_p + mu_p) \quad (3.2.17)
\]
\[
J_{y_p}^p = \frac{\sigma \mu_x H_0}{1 + m^2} (mv_p - u_p) \quad (3.2.18)
\]

Using the equations (3.2.15) and (3.2.16), the equations of the motion with reference to rotating frame in zone 1 are given by

\[
\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial z^2} + \frac{u_1}{\rho} \frac{\partial^2 u}{\partial z \partial t} + \frac{\sigma \mu_x H_0^2}{\rho(1 + m^2)} (mv - u) \quad (3.2.19)
\]
\[
\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \frac{\partial^2 v}{\partial z^2} + \frac{v_1}{\rho} \frac{\partial^2 v}{\partial z \partial t} - \frac{\sigma \mu_x H_0^2}{\rho(1 + m^2)} (v + mu) \quad (3.2.20)
\]

Using the equations (3.2.17) and (3.2.18), the equations of motion governing flow through a porous medium with respect to a rotating frame in zone 2 are given by

\[
\frac{\partial u_p}{\partial t} - 2\Omega v_p = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v_{eff} \frac{\partial^2 u_p}{\partial z^2} + \frac{u_1}{\rho} \frac{\partial^2 u_p}{\partial z \partial t} + \frac{\sigma \mu_x H_0^2}{\rho(1 + m^2)} (mv_p - u_p) - \frac{v}{k} u_p \quad (3.2.21)
\]
\[
\frac{\partial v_p}{\partial t} + 2\Omega u_p = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v_{eff} \frac{\partial^2 v_p}{\partial z^2} + \frac{v_1}{\rho} \frac{\partial^2 v_p}{\partial z \partial t} - \frac{\sigma \mu_x H_0^2}{\rho(1 + m^2)} (v_p + mu_p) - \frac{v}{k} v_p \quad (3.2.22)
\]

Let \( q = u + iv \), \( q_p = u_p + iv_p \) and \( \zeta = x - iy \)
Now combining equations (3.2.19) and (3.2.20), we obtain

\[ \frac{\partial q}{\partial t} + 2i\Omega q = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu \frac{\partial^2 q}{\partial \xi^2} + \frac{\alpha_1}{\rho} \frac{\partial^3 q}{\partial \xi^2 \partial t} - \frac{\sigma \mu_s H_0^2}{\rho(1 - im)} q \]  

(3.2.23)

and combining equations (3.2.23) and (3.2.24), we obtain

\[ \frac{\partial q_p}{\partial t} + 2i\Omega q_p = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu_{eff} \frac{\partial^2 q_p}{\partial \xi^2} + \frac{\alpha_{p1}}{\rho} \frac{\partial^3 q_p}{\partial \xi^2 \partial t} - \frac{\sigma \mu_{sp} H_0^2}{\rho(1 - im)} q_p - \frac{\nu}{k} q_p \]  

(3.2.24)

The boundary and initial conditions are

\[ q_p = a e^{i\omega t} + b e^{-i\omega t}, \quad z = 0 \]  

(3.2.25)

\[ q = 0, \quad t \neq 0, \quad z = 1 \]  

(3.2.26)

\[ q = 0, \quad q_p = 0, \quad t \leq 0, \quad \text{for all } z \]  

(3.2.27)

The interfacial conditions are

\[ \begin{align*}
q &= q_p, \\
\nu \frac{dq}{dz} &= \nu_{ef} \frac{dq_p}{dz}
\end{align*} \]  

at \( z = h \)  

(3.2.28)

We introduce the following non dimensional variables are

\[ z^* = \frac{z - \xi}{\Delta}, \quad q^* = \frac{q}{q_p}, \quad \xi^* = \frac{\xi}{\Delta}, \quad \frac{t^*}{\omega} = \frac{t \Delta}{\nu}, \quad \frac{\omega^*}{\nu} = \frac{\omega \Delta^2}{\nu}, \quad \frac{P^*}{\rho \nu^2} = \frac{P \Delta^2}{\rho \nu^2}, \quad h^* = \frac{h}{\Delta} \]

Using non dimensional variables the governing equations are (dropping asterisks in all forms)

\[ \frac{\partial q}{\partial t} + 2iE^{-1} q = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 q}{\partial \xi^2} + \alpha \frac{\partial^3 q}{\partial \xi^2 \partial t} - \frac{M^2}{(1 - im)} q \]  

(3.2.29)

\[ \frac{\partial q_p}{\partial t} + 2iE^{-1} q_p = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 q_p}{\partial \xi^2} + \alpha \frac{\partial^3 q_p}{\partial \xi^2 \partial t} - \frac{M^2}{(1 - im)} q_p - D^{-1} q_p \]  

(3.2.30)

where,
\[ M^2 = \frac{\sigma \mu e^{2} d^2}{\rho v} \] is the Hartmann number

\[ D^{-1} = \frac{j^2}{k} \] is the Inverse Darcy Parameter

\[ E = \frac{v}{\Omega l^2} \] is the Eckmann number

\[ \alpha = \frac{\alpha_1}{\rho l^3} \] is the second grade fluid parameter

\[ m = \tau_e \alpha_e \] is the hall parameter.

The corresponding initial and boundary conditions are

\[ q_p = a e^{i\omega t} + b e^{-i\omega t}, \quad t > 0, \quad z = 0 \quad (3.2.31) \]

\[ q = 0, \quad t > 0, \quad z = 1 \quad (3.2.32) \]

\[ q = q_p = 0, \quad t \leq 0, \quad \text{for all } z \quad (3.2.33) \]

The interfacial conditions are

\[ \begin{align*}
q &= q_p, \\
\frac{dq}{dz} &= \beta \frac{dq_p}{dz}, \\
\end{align*} \quad z = h \quad (3.2.34) \]

Taking Laplace transforms of equations (3.2.29) and (2.2.302) using initial condition (3.2.24) the governing equations in terms of the transformed variable in zone 1 reduces to

\[ (1 + sa) \frac{d^2 \tilde{q}}{dz^2} - \left( \frac{M^2}{1 - im} + 2iE^{-1} + s \right) \tilde{q} = -\frac{P}{s} \quad (3.2.35) \]
The relevant transformed boundary condition is

\[ \bar{q} = 0, \quad z = l, \]  

(3.2.36)

Likewise the governing equation in zone 2 is

\[ \frac{d^2 \bar{q}_p}{dz^2} + \left( \frac{M^2}{(1 - im)} + 2iE^{-1} + D^{-1} + s \right) \bar{q}_p = -\frac{p}{s} \]  

(3.2.37)

the corresponding transformed condition is

\[ \bar{q}_p = \frac{a}{s - i\omega} + \frac{b}{s + i\omega}, \quad z = 0 \]  

(3.2.38)

The transformed interfacial conditions are

\[ \bar{q} = \bar{q}_p, \quad z = h \]  

(3.2.39)

\[ \frac{d\bar{q}}{dz} = \beta \frac{d\bar{q}_p}{dz}, \quad z = h \]  

(3.2.40)

Solving equation (3.2.35) subjected to the condition (3.2.36), we get

\[ A \cosh \lambda_1 + B \sinh \lambda_1 + \frac{p}{\lambda_1^2 s(1 + s\alpha)} = 0 \]  

(3.2.41)

Solving (3.2.37) subjected to the conditions (3.2.38)

\[ C + \frac{p}{\lambda_2^2 s(\beta + s\alpha)} = \frac{a}{s - i\omega} + \frac{a}{s + i\omega} \]  

(3.2.42)

Making use of the interfacial conditions (3.2.39) and (3.2.40)

\[ \begin{align*} 
A \cosh \lambda_1 h + B \sinh \lambda_1 h + \frac{p}{\lambda_1^2 s(1 + s\alpha)} & = C \cosh \lambda_1 h + D \sinh \lambda_1 h + \frac{p}{\lambda_2^2 s(\beta + s\alpha)} \\
A \lambda_1 \sinh \lambda_1 h + B \lambda_1 \cosh \lambda_1 h & = \beta \left[ C \lambda_2 \sinh \lambda_2 h + D \lambda_2 \cosh \lambda_2 h \right] 
\end{align*} \]  

(3.2.43) 

(3.2.44)

Solving equations (3.2.41), (3.2.42), (3.2.43) and (3.2.44), we obtain the constants A, B, C and D involved in the variable and are substituting in the following equations
\[
\tilde{q} = A \cosh \lambda_2 z + B \sinh \lambda_2 z + \frac{P}{\lambda_1^2 s(1 + s\alpha)}
\]  
(3.2.45)

\[
\tilde{q}_p = C \cosh \lambda_2 z + D \sinh \lambda_2 z + \frac{P}{\lambda_2^2 s(\beta + s\alpha)}
\]  
(3.2.46)

where,

\[
A_1 = \left\{ \frac{a}{s - i\omega} + \frac{b}{s + i\omega} - \frac{P}{\lambda_2^2 s(\beta + s\alpha)} \right\} \cosh \lambda_2 h + \lambda_2^2 \frac{P}{\lambda_1^2 s(1 + s\alpha)} \sinh \lambda_1 h + \\
+ \frac{P}{s} \left( \frac{1}{\lambda_2^2 (\beta + s\alpha)} - \frac{1}{\lambda_1^2 (1 + s\alpha)} \right) \lambda_1 \beta \cosh \lambda_2 h - \\
\left\{ \frac{\lambda_2}{\lambda_1} \beta \left( \frac{a}{s - i\omega} + \frac{b}{s + i\omega} - \frac{P}{\lambda_2^2 s(\beta + s\alpha)} \right) \sinh \lambda_2 h + \lambda_2 \frac{P}{\lambda_1^2 s(1 + s\alpha)} \sinh \lambda_1 h \right\} \sinh \lambda_2 h
\]

\[
A_2 = \frac{\lambda_2}{\lambda_1} \beta \cosh \lambda_2 h \cdot \sinh(\lambda_1 (1 - h)) + \sinh \lambda_2 h \cdot \sinh(\lambda_1 (1 - h))
\]

\[
A = \frac{A_1 \sinh \lambda_1}{A_2}
\]

\[
B = \frac{1}{\sinh \lambda_1} \left\{ - \frac{P}{\lambda_1^2 s(1 + s\alpha)} \cosh \lambda_1 \cdot \sinh \lambda_1 \cdot \frac{A_1}{A_2} \right\}
\]

\[
C = \frac{a}{s - i\omega} + \frac{a}{s + i\omega} - \frac{P}{\lambda_2^2 s(\beta + s\alpha)}
\]

\[
D = \frac{1}{\sinh \lambda_2 h} \left\{ \sinh(\lambda_1 (1 - h)) \cdot \frac{A_1}{A_2} - \left( \frac{a}{s - i\omega} + \frac{b}{s + i\omega} - \frac{P}{\lambda_2^2 s(\beta + s\alpha)} \right) \cosh \lambda_2 h - \right. \\
\left. \frac{P}{\lambda_1^2 s(1 + s\alpha)} \sinh \lambda_1 h - \frac{P}{\lambda_2^2 (\beta + s\alpha)} - \frac{1}{\lambda_1^2 (1 + s\alpha)} \right\}
\]
Taking inverse Laplace transforms to the equations (3.2.45) and (3.2.46) on both sides. We obtain

\[
q = \left\{ -\frac{P}{c^2 \beta} + \frac{P \text{Cosh}(c_2 h)}{c_2 c_7} - \frac{P \text{Sinh}(c_2 h) \text{Cosh}(c_1 h)}{c_2 c_7 \text{Sinh}(c_1)} + \right. \\
+ \frac{P c_2 \beta \text{Cosh}(c_2 h) \text{Sinh}(c_1)}{c_1^2 c_7 \text{Sinh}(c_1)} - \frac{P \beta c_2 \text{Cosh}(c_2 h)}{c_1^2 c_7} \right\} \text{Sinh}(c_1(1 - z)) - \\
- \frac{P \text{Sinh}(c_2 z)}{c_1^2 c_7 \text{Sinh}(c_1)} + \frac{P^2}{c_1^2} + a \left\{ \frac{\beta c_4 \text{Sinh}(c_3(1 - z))}{c_9} \right\} e^{i \omega t} + b \left\{ \frac{\beta c_6 \text{Sinh}(c_4(1 - z))}{c_9} \right\} e^{-i \omega t} + \\
\left. + \left\{ \frac{P \beta D^{-1/2} (1 - h) \text{Cosh}(D^{-1/2} h)}{c_1^2 (c_1^2 \alpha + 1)c_{10}} + \frac{P \text{Sinh}(D^{-1/2} h)}{c_1^2 (c_1^2 \alpha + 1)c_{10}} - \frac{p}{c_1^2 (c_1^2 \alpha + 1)} \right\} (1 - z) e^{- \left( \frac{M^2 + 2 \beta \alpha}{1 - \beta \alpha} \right)t \right\}
\]

(3.2.47)

and

\[
q_p = -\frac{P \text{Cosh}(c_2 z)}{c_1^2 \beta} - \frac{P \text{Cosh}^2(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_2 c_7 \text{Sinh}(c_2 h)} + \\
+ \frac{P \beta c_2 \text{Cosh}(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_1^2 c_7 \text{Sinh}(c_1) \text{Sinh}(c_2 h)} - \\
- \frac{P \beta c_2 \text{Cosh}(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_1^2 c_7 \text{Sinh}(c_2 h)} + \\
\frac{P \text{Cosh}(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_2 c_7 \text{Sinh}(c_2 h)} + \\
\frac{P \text{Sinh}(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_2 c_7} - \\
- \frac{P \beta c_2 \text{Cosh}(c_2 h) \text{Sinh}(c_1(1 - h)) \text{Sinh}(c_2 z)}{c_1 c_7 \text{Sinh}(c_1)} + \\
\frac{P \text{Cosh}(c_2 z) \text{Sinh}(c_2 z)}{c_1^2 \beta \text{Sinh}(c_2 h)} - \frac{P \text{Sinh}(c_1 h) \text{Sinh}(c_2 z)}{c_1^2 \text{Sinh}(c_1) \text{Sinh}(c_2 h)} - \frac{P \text{Sinh}(c_2 z)}{c_1^2 \beta \text{Sinh}(c_2 h)} + \frac{P}{c_1^2 \beta} \\
+ a \left\{ \text{Cosh}(c_4 z) + \frac{\beta c_4 \text{Cosh}^2(c_4 h) \text{Sinh}(c_3(1 - h)) \text{Sinh}(c_4 z)}{c_4 \text{Sinh}(c_4 h)} \right\}
\]

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The shear stresses on the upper plate and lower plate are given by

\[ \tau_U = \left( \frac{dq}{dz} \right)_{z=1} \quad \text{and} \quad \tau_L = \left( \frac{dq_p}{dz} \right)_{z=0} \]

(These forms are mentioned in the appendix)
3.3. Associated boundary layer discussion:-

The solutions for the combined velocity $q$ and $q_p$ consists of three kinds of terms namely, 1. Steady state, 2. The quasi-steady state terms associated with non-torsional oscillations in the boundary, 3. The transient terms involving exponentially varying time dependence. We shall now discuss the interplay between coriolis force, Darcy force and Lorentz force in presence of hall currents. From the velocity expressions (3.2.47) and (3.2.48), it follows that the transient components in the velocity in the clean fluid region decays in the dimensionless time $t > \frac{1 + m^2}{[M^4 + (M^2m + 2E^{-1})^2]^\frac{1}{2}}$, while in the porous bed, these transient components decay in the time i.e.,

$$t > \max\left\{\frac{1 + m^2}{[M^4 + (M^2m + 2E^{-1})^3]^\frac{1}{3}}, \frac{1 + m^2}{[(M^2 + D^{-1}(1 + m^2))^2 + (M^2m + 2E^{-1})^2]^\frac{1}{2}}\right\}$$

The decay time for hydro magnetic transient velocity ($M^2 \neq 0$) is always less than the decay time in the absence of any magnetic field ($M^2=0$) in both regions. The decay time of the transient velocity enhances in the presence of hall current in comparison to in the case of near presence of magnetic field. Also the decay time increases with increase in the hall parameter $m$. This is true with reference to the transient terms in the velocity in porous bed. Thus in a composite medium in the presence of clean fluid the time of decay of the transient velocity continues to be higher although the time of decay involving the porous parameter is comparatively less. When the transient terms decay the steady and oscillatory solutions in the clean fluid and porous bed are given by (ie., when the transient terms decay in the limit as $t \to \infty$)
\[(q)_{\text{steady}} = \left\{ \frac{P}{c_2} + \frac{P \cosh(c_2 h)}{c_2 c_7} - \frac{P \sinh(c_2 h) \cosh(c_1 h)}{c_2 c_7 \sinh(c_1)} \right\} + \]
\[
\left\{ \frac{P c_2}{c_2^2 c_7 \sinh(c_1)} \beta \cosh(c_2 h) \sinh(c_1 h) \right\} \sinh(c_1 (1 - z)) - \]
\[
- \frac{P \sinh(c_1 z)}{c_2^2 c_7 \sinh(c_1)} + \frac{P}{c_2^2}
\]

\[(3.3.1)\]

\[(q)_{\text{Oscillatory}} = a \left\{ \beta c_4 \sinh(c_4 (1 - z)) \right\} e^{i\omega t} + b \left\{ \beta c_6 \sinh(c_6 (1 - z)) \right\} e^{-i\omega t}
\]

\[(3.3.2)\]

\[(q_p)_{\text{steady}} = \frac{P \cosh(c_2 z)}{c_2^2} - \frac{P \cosh^2(c_2 h) \sinh(c_1 (1 - h)) \sinh(c_2 z)}{c_2 c_7 \sinh(c_2 h)} + \]
\[
+ \frac{P \beta c_4 \cosh(c_2 h) \sinh(c_4 h) \sinh(c_1 (1 - h)) \sinh(c_2 z)}{c_2^2 c_7 \sinh(c_1) \sinh(c_2 h)} - \]
\[
- \frac{P \cosh(c_2 h) \sinh(c_1 (1 - h)) \sinh(c_2 z)}{c_2 c_7 \sinh(c_2 h)} + \]
\[
+ \frac{P \sinh(c_2 h) \sinh(c_1 (1 - h)) \sinh(c_2 z)}{c_2 c_7} - \]
\[
- \frac{P \beta c_4 \cosh(c_4 h) \sinh(c_4 (1 - h)) \sinh(c_4 h)}{c_2^2 c_7 \sinh(c_2 h)} + \]
\[
+ \frac{P \sinh(c_2 h) \sinh(c_2 z)}{c_2^2 \beta \sinh(c_2 h)} - \frac{P \sinh(c_2 h) \sinh(c_2 z)}{c_2^2 \sinh(c_2 h)} - \frac{P \sinh(c_2 z)}{c_2^2 \beta} + \frac{P}{c_2^2 \beta}
\]

\[(3.3.3)\]

\[(q_p)_{\text{Oscillatory}} = a \left\{ \cosh(c_4 z) + \beta c_4 \cosh^2(c_4 h) \sinh(c_4 (1 - h)) \sinh(c_4 z) \right\}
\]

\[c_4 \sinh(c_4 h)\]
\begin{align} 
\beta d_4 \text{Sinh}(c_4 h) \text{Sinh}(c_3 (1 - h)) \text{Sinh}(c_4 z) & - \frac{\text{Cosh}(c_4 h) \text{Sinh}(c_4 z)}{\text{Sinh}(c_4 h)} \\
\text{e}^{i\omega t} & \\
+ b \left( \text{Cosh}(c_5 z) + \frac{\beta c_6 \text{Cosh}^2(c_5 h) \text{Sinh}(c_5 (1 - h)) \text{Sinh}(c_6 z)}{c_5 \text{Sinh}(c_5 h)} \right) \\
- \frac{\beta c_5 \text{Sinh}(c_5 h) \text{Sinh}(c_5 (1 - h)) \text{Sinh}(c_6 z)}{c_5} & - \frac{\text{Cosh}(c_5 h) \text{Sinh}(c_6 z)}{\text{Sinh}(c_6 h)} \\
\text{e}^{-i\omega t} & \\
(3.3.4) & 
\end{align}

In the presence of hall current a boundary layer formed near the upper plate and the thickness of this layer is comparatively larger than corresponding thickness of layer in the absence of hall current and it gradually increases with increase in \( m \).

\[
[\therefore a_1^{-1} = \frac{1}{\sqrt{2}} \left( \frac{(M^4 + (M^2 m + 2E^{-1} \beta^2) (1 + m^2))^\frac{1}{2}}{1 + m^2} + \frac{M^2}{1 + m^2} \right)^\frac{1}{2}]
\]

Near the lower plate in the presence of hall current the thickness of the layer is more than the thickness of the layer in the absence of hall current and thickness gradually increases with increase in \( m \).

\[
[\therefore a_2^{-1} = \frac{1}{\sqrt{2}} \left( \frac{(M^2 + D^{-1} (1 + m^2))^\frac{1}{2} + (M^2 m + 2E^{-1} \beta^2 (1 + m^2))^\frac{1}{2}}{\beta (1 + m^2)} \right)^\frac{1}{2}]
\]

The similar analysis holds good for the oscillatory velocity terms in the presence of hall current.
3.4. Results and Discussion:

The computational analysis has been carried out to discuss the behaviour of velocity components in both clean and porous regions as well as shear stresses in the rotating parallel plate channel with reference to variations in the governing parameters namely viz. $E$ the Ekman number, $M$ the Hartmann number, $D^{-1}$ the inverse Darcy parameter, $\alpha$ the second grade fluid parameter and $m$ the hall parameter. The clean fluid region in the rotating parallel plate channel bounded by the upper plate $z=1$. This is at relative rest with reference to the rotating frame. The porous bed is bounded by lower plate $z=0$ which executes non-torsional oscillations in its own plane with reference to rotating frame. This analysis has also been taken up in two cases: 1. when the thickness of the porous bed in the composite medium is relativity small, 2. the thickness is relatively large.

We may note in general that the thickness of the porous bed plays a significant role deciding the flow features in the composite medium. The profiles are drawn for the velocity components in the composite medium with the expressions for $u$ and $v$ chosen from the clean fluid region while the relative expressions for velocity component $u_p$ and $v_p$ are chosen from the porous region.

The figures (1-5) represent the profiles for the velocity component $u$ along the imposed pressure gradient while the fig (6-10) represent the profiles for $v$ in the case when the thickness of the porous bed is small ($h=0.3$). We observe that the magnitude of $u$ and $v$ enhances with $E$ in the clean fluid region ($0.4 \leq z \leq 1$) and in the porous bed the magnitude of $u$ reduces while the magnitude of $v$ enhances with $E$. At the interface ($0.3 \leq z \leq 0.4$) the magnitude of $u$ enhances and the magnitude of $v$ reduces with
increase in $E$. This shows that $u$ and $v$ undergoes the transitional behaviour at the interface of the clean fluid and porous region (fig. 1 and 6). However the resultant velocity enhances with increase in $E$ in the entire flow region. The figures (2 and 7) correspond to the behaviour of $u$ and $v$ with reference to the Hartmann number $M$. It is interesting to note that while the magnitude of $v$ reduces with increase in $M$ entire flow region, the corresponding the magnitude of $u$ reduces in the clean fluid region and enhances in the porous bed. However the resultant velocity reduces in the clean fluid region and increases in the porous medium. Lesser the permeability of the porous bed lower the magnitude of $v$ in the entire flow region. In contrast the magnitude of $u$ enhances in the clean fluid region and reduces in the porous bed with increase in $D^{-1}$ (fig. 3 and 5). However the resultant velocity reduces throughout the fluid region with increase in the inverse Darcy parameter $D^{-1}$. Thus the permeability of the porous bed influences the flow field. The influence of second grade fluid parameter $\alpha$ on the flow field may be noticed from (fig. 4 and 9). We find that the magnitude of $u$ increases with increase in $\alpha$ everywhere in the flow field. Where as the magnitude of $v$ increases outside the interface and reduces within the increase in $\alpha$. It is interesting to note that the effect of hall parameter on the flow field is to enhances the magnitude of the velocity components in the entire region for increase in $m$ (fig. 5 and 10)

The fig (11-20) depict the velocity profiles when the thickness of the porous bed sufficiently large ($h=0.6$). In contrast to the case of smaller thickness of the bed the magnitude of $u$ enhances everywhere in the flow field with increase in the Ekman number $E$ (fig. 11). However for similar variation in $E$, the magnitude of $v$ increases in the porous bed and reduces in the clean fluid region (fig. 16). The resultant velocity enhances
throughout the fluid region with increase in the Ekman number $E$. The reversal behavior with increase in the Hartmann number may be noticed with magnitude of $v$ reducing everywhere in the flow field (fig. 17), while the magnitude of $u$ reduces in the porous bed and enhances in the clean fluid region (fig. 12). Lesser the permeability of the porous bed lower the magnitude of $u$ except in the vicinity of the upper plate (fig. 13). The figure (18) shows that an increase in $D^{-1}$ reduces the velocity $v$ in the porous bed and enhances in the clean fluid region. An increase in the second grade fluid parameter $\alpha$ enhances $u$ and reduces $v$ in the entire fluid region (fig. 14 and 19). The influence of hall parameter $m$ on the flow field may be noticed in the fig (15 and 20). We find that enhances and retards with increase in the hall parameter $m$.

The shear stresses on the upper and lower plates are tabulated in the tables (I-IV). We observe that the stresses in the either plates enhances with Ekman number $E$, second grade fluid parameter $\alpha$ and the hall parameter $m$. However the stresses $\tau_x$ on the upper plate and $\tau_y$ on the lower plate enhances with Hartmann number $M$. The stresses $\tau_x$ on the lower plate and $\tau_y$ on the upper plate reduces with increase in $M$. It is interesting to note that these stresses on the upper and lower plates reduces with increase in $D^{-1}$ and hence lower the permeability of the porous bed lower the stresses on the boundary.
The velocity profiles when the lower plate execute non-torsional oscillations

Fig. 1: The velocity profile for $u$ with $E$.
$a=b=1, \omega = \frac{\pi}{2}, \alpha = 0.25, D^{-1}=2000, M=2, m=1$

Fig. 2: The velocity profile for $u$ with $M$.
$a=b=1, \omega = \frac{\pi}{2}, \alpha = 0.25, D^{-1}=2000, E=0.01, m=1$
Fig. 3: The velocity profile for $u$ with $D^{-1}$

$a=b=1$, $\omega = \frac{\pi}{2}$, $\alpha = 0.25$, $M=2$, $E=0.01$, $m=1$

Fig. 4: The velocity profile for $u$ with $\alpha$.

$a=b=1$, $\omega = \frac{\pi}{2}$, $M=2$, $D^{-1}=2000$, $E=0.01$, $m=1$
Fig. 5: The velocity profile for $u$ with $m$.

$a=b=1, \omega = \frac{\pi}{2}, M=2, D^{-1}=2000, E=0.01, \alpha=0.25$

Fig. 6: The velocity profile for $v$ with $E$.

$a=b=1, \omega = \frac{\pi}{2}, \alpha = 0.25, D^{-1}=2000, M=2, m=1$
Fig. 7: The velocity profile for \( v \) with \( M \).
\[ a=b=1, \omega = \frac{\pi}{2}, \alpha = 0.25, M^{-1}=2000, E=0.01, m=1 \]

Fig. 8: The velocity profile for \( v \) with \( D^{-1} \)
\[ a=b=1, \omega = \frac{\pi}{2}, \alpha = 0.25, M=2, E=0.01, m=1 \]
Fig. 9: The velocity profile for $v$ with $\alpha$.

$a = b = 1$, $\omega = \frac{\pi}{2}$, $M = 2$, $D^{-1} = 2000$, $E = 0.01$, $m = 1$

Fig. 10: The velocity profile for $v$ with $m$.

$a = b = 1$, $\omega = \frac{\pi}{2}$, $M = 2$, $D^{-1} = 2000$, $E = 0.01$, $\alpha = 0.25$
Fig. 11: The velocity profile for $u$ with $E$.

$a=b=1$, $\omega = \frac{\pi}{4}$, $\alpha = 0.25$, $D^{-1}=2000$, $M=2$, $m=1$

Fig. 12: The velocity profile for $u$ with $M$.

$a=b=1$, $\omega = \frac{\pi}{4}$, $\alpha = 0.25$, $D^{-1}=2000$, $E=0.01$, $m=1$
Fig. 13: The velocity profile for $u$ with $D^{-1}$
$a = b = 1$, $\omega = \frac{\pi}{4}$, $\alpha = 0.25$, $M = 2$, $E = 0.01$, $m = 1$

Fig. 14: The velocity profile for $u$ with $\alpha$.
$a = b = 1$, $\omega = \frac{\pi}{4}$, $M = 2$, $D^{-1} = 2000$, $E = 0.01$, $m = 1$
Fig. 15: The velocity profile for $u$ with $m$.
$a=b=1, \omega = \frac{\pi}{4}, M=2, D^{-1}=2000, E=0.01, \alpha=0.25$

Fig. 16: The velocity profile for $v$ with $E$.
$a=b=1, \omega = \frac{\pi}{4}, \alpha = 0.25, D^{-1}=2000, M=2, m=1$
Fig. 17: The velocity profile for $v$ with $M$.
$a=b=1, \omega = \frac{\pi}{4}, \alpha = 0.25, D^{-1}=2000, E=0.01, m=1$

Fig. 18: The velocity profile for $v$ with $D^{-1}$
$a=b=1, \omega = \frac{\pi}{4}, \alpha = 0.25, M=2, E=0.01, m=1$
Fig. 19: The velocity profile for $v$ with $\alpha$.

$a=b=1, \ \omega = \frac{\pi}{2}, M=2, D^{-1}=2000, E=0.01, m=1$

Fig. 20: The velocity profile for $v$ with $m$.

$a=b=1, \ \omega = \frac{\pi}{2}, M=2, D^{-1}=2000, E=0.01, \alpha=0.25$
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The shear stresses ($\tau_x$) on the upper plate

$$a=b=1, \ t=1, \ \omega=\frac{\pi}{4}$$
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The shear stresses (τy) on the upper plate

\[ a=b=1, \ t=1, \ \omega = \frac{\pi}{4} \]
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The shear stresses ($\tau_x$) on the lower plate

$$a = b = 1, \ t = 1, \ \omega = \frac{\pi}{4}$$
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The shear stresses ($\tau_y$) on the lower plate

$$a=b=1, \ t=1, \ \omega = \frac{\pi}{4}$$
3.5. Conclusions:

1. The thickness of the porous bed plays a significant role deciding the flow features in the composite medium.

2. When the thickness of the porous bed is small (h=0.3), the resultant velocity enhances with increase in $E$ in the entire flow region.

3. The resultant velocity reduces in the clean fluid region and increases in the porous medium.

4. Lesser the permeability of the porous bed lower the magnitude of $v$ in the entire flow region. In contrast the magnitude of $u$ enhances in the clean fluid region and reduces in the porous bed with increase in $D^{-1}$. However the resultant velocity reduces throughout the fluid region with increase in the inverse Darcy parameter $D^{-1}$.

5. The permeability of the porous bed influences the flow field.

6. The resultant velocity increases with increase in $\alpha$.

7. The effect of hall parameter on the flow field is to enhance the magnitude of resultant velocity in the entire region for increase in $m$.

8. When the thickness of the porous bed sufficiently large, The resultant velocity enhances throughout the fluid region with increase in the Ekman number $E$. The reversal behavior with increase in the Hartmann number may be noticed with magnitude of resultant velocity reducing everywhere in the flow field.

9. Lesser the permeability of the porous bed lower the magnitude of $u$ except in the vicinity of the upper plate. an increase in $D^{-1}$ reduces the velocity $v$ in the porous bed and enhances in the clean fluid region.
10. An increase in the second grade fluid parameter $\alpha$ enhances $u$ and reduces $v$ in the entire fluid region.

11. The influence of hall parameter $m$ on the flow field may be noticed, we find that enhances and retards with increase in the hall parameter $m$.

12. The stresses in the either plates enhances with Ekman number $E$, second grade fluid parameter $\alpha$ and the hall parameter $m$. However the stresses $\tau_x$ on the upper plate and $\tau_y$ on the lower plate enhances with Hartmann number $M$.

13. The stresses $\tau_x$ on the lower plate and $\tau_y$ on the upper plate reduce with increase in $M$. It is interesting to note that these stresses on the upper and lower plates reduces with increase in $D^{-1}$ and hence lower the permeability of the porous bed lower the stresses on the boundary.
References:


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Appendix:

\[
\beta = \frac{v_{\text{eff}}}{v}
\]

\[
\lambda_1 = \frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + s\right)}{1 + sa}, \quad \lambda_2 = \frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + D^{-1} + s\right)}{\beta + sa}
\]

\[
c_1 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1}\right)}{1 + i\omega a}}, \quad c_2 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + D^{-1}\right)}{\beta}}
\]

\[
c_3 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + i\omega\right)}{1 + i\omega a}}, \quad c_4 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + D^{-1} + i\omega\right)}{\beta + i\omega a}}
\]

\[
c_5 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} - i\omega\right)}{1 - i\omega a}}, \quad c_6 = \sqrt{\frac{\left(\frac{M^2}{1 - \text{im}} + 2iE^{-1} + D^{-1} - i\omega\right)}{\beta - i\omega a}}
\]

\[
c_7 = c_5 \beta \cosh(c_2 h) \sinh(c_1 (1 - h)) + c_4 \sinh(c_2 h) \cosh(c_1 (1 - h))
\]

\[
c_8 = c_5 \beta \cosh(c_4 h) \sinh(c_1 (1 - h)) + c_4 \sinh(c_4 h) \cosh(c_1 (1 - h))
\]

\[
c_9 = c_6 \beta \cosh(c_4 h) \sinh(c_1 (1 - h)) + c_5 \sinh(c_6 h) \cosh(c_1 (1 - h))
\]

\[
c_{10} = D^{-1/2} \beta (1 - h) \cosh(D^{-1/2} h) + \sinh(D^{-1/2} h)
\]
\[
\tau_u = \left\{ \frac{P}{c_i^2 c_7} \frac{P \cosh(c_2 h)}{c_2 c_7} \frac{P \sinh(c h) \cosh(c h)}{c_7 \sinh(c_i)} + \frac{P c_2 \beta \cosh(c_2 h) \sinh(c h)}{c_i^2 c_7 \sinh(c_i)} - \frac{P \beta c_2 \cosh(c_2 h)}{c_i^2 c_7} \right\} (-c_i) \\
- \frac{P c_i \cosh(c_i) + a \left\{ \frac{\beta c_4 (-c_i)}{c_8} \right\} e^{i \alpha t} + b \left\{ \frac{\beta c_6 (-c_i)}{c_9} \right\} e^{-i \alpha t}}{c_i^2 \sinh(c_i)} \right.
\]
\[
\left. - \left\{ \frac{P \beta D^{-1/2} (1 - h) \cosh(D^{-1/2} h)}{c_i^2 (c_i^2 + 1) c_8} + \frac{P \sinh(D^{-1/2} h)}{c_i^2 (c_i^2 + 1) c_8} - \frac{e^{-i \alpha t}}{e^{-i \alpha t}} \right\} \right]
\]

and
\[
\tau_L = \left\{ \frac{P c_2 \cosh^2(c_2 h)}{c_i c_7 \sinh(c_i) \sinh(c_i)} + \frac{P \beta c_2 \cosh(c_2 h) \sinh(c h)}{c_i c_7 \sinh(c_i) \sinh(c_i)} - \frac{P \beta c_2 \cosh(c_2 h)}{c_i c_7 \sinh(c_i) \sinh(c_i)} \right\} (-c_i) + \left\{ \frac{P \beta c_2 \cosh(c_2 h) \sinh(c_i (1 - h))}{c_i \c_5 \sinh(c_i)} \right\} \sinh(c_i (1 - h)) + \left\{ \frac{P c_2 \cosh(c_2 h)}{c_i^2 \beta \sinh(c_i) \sinh(c_i)} - \frac{P c_2 \cosh(c_2 h)}{c_i^2 \beta \sinh(c_i) \sinh(c_i)} \right\}
\]
\[
+ a \left\{ \frac{\beta c_4 \cosh^2(c h) \sinh(c_i (1 - h))}{c_i \sinh(c_i)} \right\} e^{i \alpha t} \left\{ \frac{\beta c_6 \cosh^2(c h) \sinh(c_i (1 - h))}{c_i \sinh(c_i)} \right\} e^{-i \alpha t}
\]
\[
+ b \left\{ \frac{\beta c_6 \cosh^2(c_2 h) \sinh(c_i (1 - h))}{c_i \sinh(c_i)} \right\} e^{i \alpha t} \left\{ \frac{\beta c_6 \cosh^2(c_2 h) \sinh(c_i (1 - h))}{c_i \sinh(c_i)} \right\} e^{-i \alpha t}
\]
\[ + \left\{ \begin{array}{c}
\frac{P \beta D^{-1} \cosh(D^{-1/2}h)}{c_1^2 (c_1^2 \alpha + 1) c_{10} \sinh(D^{-1/2}h)} + \\
\frac{P D^{-1/2}}{c_1^2 (c_1^2 \alpha + 1) c_{10} \sinh(D^{-1/2}h)} + \\
ed^2 \frac{P \beta D^{-1} \cosh(D^{-1/2}h)}{c_1^2 (c_1^2 \alpha + 1) c_{10} \sinh(D^{-1/2}h)} - \\
ed^2 \frac{P D^{-1} \cosh(D^{-1/2}h)}{c_1^2 (c_1^2 \alpha + 1) c_{10} \sinh(D^{-1/2}h)} + \\
\frac{P D^{-1/2}}{c_1^2 (c_1^2 \alpha + 1) \sinh(D^{-1/2}h)} - \\
ed^{2-1} \frac{P D^{-1/2}}{c_1^2 (c_1^2 \alpha + 1) \sinh(D^{-1/2}h)} \right\} e^{M^2 \frac{t}{\hbar \omega + 2D^{-1} + D^{-1} \nu t}} \\
+ \left\{ \frac{PC \cosh(D^{-1/2}h)}{\hbar c_1^2 (\beta + c_2^2 \alpha)} + \frac{P}{\hbar c_2^2 (\beta + c_2^2 \alpha)} \right\} e^{M^2 \frac{t}{\hbar \omega + 2D^{-1} + D^{-1} \nu t}} \right\} 
\]