Let \( \{X_n, n \geq 1\} \) be a sequence of independent identically distributed random variables with a common distribution function \( F \), defined over a probability space \((\Omega, \mathcal{F}, P)\). Let \( S_n = \sum_{k=1}^{n} X_k, n \geq 1 \). Let \( \{A_n, n \geq 1\} \) and \( \{B_n, n \geq 1\} \) be sequence of real constants \( (B_n \to \infty, \text{as } n \to \infty) \). Set \( Z_n = B_n^{-1} S_n - A_n, n \geq 1 \). If \( \{Z_n, n \geq 1\} \) for suitable choices of \( \{A_n\} \) and \( \{B_n\} \), converges weakly, then it is well known that the limit law is Stable (see GNEDENKO and KOLMOGOROV) is said to belong to the domain of attraction of the limiting Stable law. It is also known that every Stable law belong to its own domain of attraction.

In the theory of probability, we have three classical limit theorems. Viz (i) Laws of large numbers (ii) Central limit theorems and (iii) Laws of the iterated logarithm based on weak convergence or Central limit theorem.

When \( B_n = n \) and \( A_n = 0 \), the laws of large numbers tells that the sequence \( \{Z_n, n \geq 1\} \) converges to a degenerate random variable, under any modes of convergence i.e, convergence in probability or almost sure convergence.

Secondly, central limit theorem tells that under what conditions on \( \{X_n, n \geq 1\} \), there exists \( \{A_n\} \) and \( \{B_n\} \), such that a senquence \( \{Z_n\} \) converges (weakly) to a non-degenerate random variable, particularly, normal variable.

Finally, we recall that, limit of any sequence of real numbers may or may not exist. But, we know that the upper limit infimum always exists.
Therefore, it is natural to ask that what happens in between the law of large numbers and central limit theorem, when limit does not exists, i.e., whether non-trivial limit behaviour is obtainable or not? Hence, the study relates to these type of problems, we call it as "LAW OF ITERATED LOGARITHM" (LIL).

In 1924, Prof. A.YA. Khinchine is the first person to study these type of problems. In fact, he developed this in number theory. Later Prof. A.N. Kolmogorov (1929) developed for bounded random variables with finite variance. In 1941, Hartman and Wintner established that the second moment do indeed suffice for the existence, of this LIL for partial sums and we call Hartman and Wintner type LIL as classical law of iterated logarithm. For complete literature one can refer to Stout, W [12].

For any sequence \(\{X_n, n \geq 1\}\) of random variables the sum \(X_n + X_{n+1} + \cdots + X_{n+j-1}\) is called the forward delayed sum and \(X_{n-j+1} + \cdots + X_n\) is called the backward delayed sum. Lai, T.L. [10] was the first to consider the LIL for delayed sums obtained by cutting off some initial observations (which depend on \(n\)) from the sum \(S_n\). He shown that these results can be entirely different from the laws of iterated logarithm for \(S_n\). These type of LIL attracted to many probabilists and one can refer to Csorgo, M and Revesz, P [3] for complete coverage of this field for the situation of variance finite.

When the random variables \(X_n\)'s are indipendently distributed with a common symmetric distribution function, Chover, J[3] obtained a law of iterated logarithm for the sequence of partial sums \(\{S_n\}\) and he established these results based on the probabilities of the random variable \(X_1\).
In 1986, Allan Gut established the classical LIL for geometrically fast subsequences of partial sums and the same is extended to Weiner processes by Vasudeva and Savitha [13]. Motivated by the work of Allan Gut, we study the non-trivial limit behaviour for partial sums and delayed sums. i.e., we obtain iterated logarithm laws for partial sums and delayed sums. In particular, for at least (at most) geometrically fast subsequences, we establish limit supremum and limit infimum results.

In next chapter, we prove law of iterated logarithm for partial sums and in chapter III, we give law of iterated logarithm for delayed sums.