1. INTRODUCTION

Non-Newtonian Fluids are commonly divided into three broad groups,

1. Time dependent fluids,

2. Time dependent Non-Newtonian Fluids,

3. Visco-elastic Fluids.

Time independent fluids are those for which the rate of shear at a given point is solely dependent upon the instantaneous shear stress at that point. These materials are sometimes referred to as "Non-Newtonian viscous fluids" or alternatively as "Purely Viscous fluids".

In this group we have

(a) Fluids with yield stress

(b) Fluids without yield stress

The physical behaviour of fluids with a yield stress is usually explained in terms of an internal structure in three dimensions which is capable of preventing movement for values of shear stress less than the yield stress $\tau_y$.

Fluids without yield stress are of two kinds
The term “Pseudoplastic” and “Dilatant” are applied to slopes of the logarithmic flow curves of less than unity respectively. The apparent viscosity decreases with increasing shear rate in pseudoplastic fluids and increases in dilatant systems. At the extremes of very low and very high shear rate polymer solutions melts and most slurries of limited concentration are found to be Newtonian in shear and pseudoplasticity is restricted to an intermediate range of shear rates. Examples of pseudoplastic fluids may usually be found in the following materials: rubber solutions, polymer solutions or melts, greases, starch suspensions, cellulose, acetate solutions used in rayon manufacturing, soap, paper pulp, napalm, paints, biological fluids. Examples of dilatant fluids include the following: some corn flour/sugar solutions, some gum arabic/borax solutions, starch, potassium silicate, quick sand and wet beach sand.
Time-dependent Non-Newtonian fluids are those which show a decrease in viscosity or shear stress with time, under isothermal conditions and steady shear, are termed “thixotropic”. Those that show increases are termed “rheopectic” or “antithixotropic”. The latter behaviour is relatively uncommon. Thixotropic systems are of great commercial importance, especially in the paint and food industries. Only one recommended theoretical study of thixotropic systems in which the Eyring - Powell equation was applied to elucidated the flow mechanisms is available. Examples of thixotropic properties have been found in oil well drilling muds, printing inks, many food materials and that of rheopectic properties have been found in bentonite clay and suspensions. Visco-elastic fluids describe all fluid systems exhibiting normal stress effects. It may be shown (Oldroyd 8,9,10, Reiner13) that normal stress effects do not necessarily depend solely on fluid elasticity, but is generally the case, and the usual as well as the most outstanding and important examples of systems exhibiting normal stress effects are clearly elastic in nature. Examples of visco-elastic fluids include bitumans, flour dough, napalm and similar jellies, polymers, melts and nylon.
Three approaches have been used to study this type of fluid behaviour.

I. Fundamental(Tensor) equations which assume only that the stress components at a point determined by the first, second, ………….., nth gradients of velocity at that point or by the strains, the strain rates, and the total time derivatives of the elements of the stress tensor (Oldroys 8, 9, 10) have been solved for the forces required to produce a variety of assumed physical deformations of an element of incompressible fluid which was isotropic at rest. These phenomenological theories have been extensively discussed by Markovitz (4), Philippoff (12) and Jobling and Roberts (2). Generally no attempt has been made to relate the observed fluid behaviour to molecular characteristics. Infact Pao (11) has pointed out that this approach in reality assumes the form of the end result and then works back to what are really the true fluid properties.

II. An approach similar to the theory of rubber like elasticity in solids has been applied to polymeric melts and solutions, particularly by Lodge (3), Pao (11), Rouse (16) and Takemura (17). While these approaches in principle lead to a more direct understanding of the important molecular characteristics,
this possible advantage has not been exploited since these theories have been far less completely developed than those of Rivlin(14) and Oldroyd(8).

III. Experimentation to test the above theoretical predictions or to develop new theoretical concepts has been very limited and largely qualitative in nature. Infact it has been suggested that the major remaining progress will be due to such experimental studies.

2. CONSTITUTIVE EQUATIONS:

Oldroyd(8) in 1950 was the first to outline a method for formulating constitutive equations which would be valid for large deformations. Equation

\[ \tau + \lambda_1 \frac{d\tau}{dt} = \mu^* \left( \frac{d\gamma}{dt} + \lambda_2 \frac{d^2\gamma}{dt^2} \right) \] ........................(2.1)

was taken as the basis for the theory, where \( \mu^* \) is viscosity. \( \lambda_1, \lambda_2 \) are "relaxation times". The relaxation times obviously have the physical significance that if the motion is suddenly stopped, the shear stress decays as \( \exp(-t/\lambda_1) \) and if the stress is removed the rate of strain decays as \( \exp(-t/\lambda_2) \).
This equation reduces to the Newtonian fluid if $\lambda_1=0=\lambda_2$, and to the Maxwell fluid if $\lambda_2=0$ as special cases.

Maxwell's equation was successful in describing the behaviour of flour doughs. It proved inadequate however to correlate the observation of Oldroyd, Strawbridge and Toms (18) on polymethylmethacrylate in various solvents. Walters (19) has performed an analysis comparable to that of Oldroyd for liquids that at low shear rates are characterized by a more general relationship than equation (2.1)

The effects of changes in shear rate with time upon the stresses in a fluid were also incorporated into the constitutive equations of Rivilin and Ericksen (15). In their theory the components of stress at time $t$ in an element of material depend only on the gradients of displacement, velocity, acceleration, second acceleration, .........., $(n-1)th$ acceleration, in the element at time $t$. Rivilin (14) has solved some spherical problems in this theory, which
has been applied theoretically to creeping flow around a sphere by Caswell and Schwarz(1).

The theories of Noll(6) and of Green and Rivlin(14,15) make allowance for fluid “memory” by assuming that at time $t$ the stress state of the fluid depends on its kinematic state at all times up to and including $t$. With the assumption that Noll’s “simple fluid” has a short lived memory and is moving slowly. Coleman and Noll(6-7) evolved an approximation scheme giving the same stress equations as those obtained for Rivlin - Ericksen fluids.

The type of visco-elastic fluid depend solely on the stress, rate of strain constitutive relation. Based on this constitutive relation, we categorise them into four fluids viz., Walters-liquid ‘A’, liquid ‘B’, Kuvshinski fluid, and Rivlin-Ericksen fluid.

The constitutive equation governing the Rivlin-Ericksen fluid has been developed by Rivlin and Ericksen(15). It is given by
\[
T = -P I + \phi_1 A + \phi_2 B + \phi_3 A^2
\]

where

\[I = || \delta_{ij} ||, \quad \delta_{ij} \text{ is the Kronecker delta}\]

\[A = || a_{ij} ||, \quad a_{ij} = 1/2[ u_{ij} + u_{ji} ] \text{ is the deformation tensor}\]

\[B = || b_{ij} ||, \quad b_{ij} = a_{ij} + a_{ji} + 2 V_{mi} V_{mj} \text{ is the visco-elastic tensor}\]

\[a_{\eta}, a_{\eta_i} \text{ are the acceleration gradients}\]

\[V_{mi}, V_{mj} \text{ are velocity gradients}\]

\[\phi_1, \phi_2 \text{ and } \phi_3 \text{ are material constants called the coefficient of viscosity, visco-elasticity and cross-viscosity respectively, which are considered to be constants.}\]

The equation of continuity is

\[
\frac{\partial \rho}{\partial t} + \rho V_{ij} = 0
\]
The equation of linear momentum is

\[ T_{ij} + \rho ( f_i - a_i ) = 0 \]

where \( \rho \) is the density of the fluid.

\[ a_i = ( \partial V_i / \partial t ) + V_j \nabla_j \]  

\( V_j \) is the acceleration vector  

\( f_i \) is the body force per unit mass

and \( T_{ij} \) is the component of the stress.

For incompressible fluids we have

\[ V_{i,i} = 0. \]

The visco-elastic equations are obtained by substituting for \( T_{ij} \) from the constitutive equation in the above linear momentum equation. In deriving the equations governing the flow phenomena of visco-elastic fluid, we assume that the fluid is isotropic, homogenous, incompressible and posses visco-elastic nature.