CHAPTER 1

INTRODUCTION, MOTIVATION AND LITERATURE COVERAGE
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1.1 GENERAL

Reliability is an inter-disciplinary concept which deals with the degradation laws of physical elements and systems. However, reliability as a theory and distinct branch of research was founded only some forty years ago. The rapid growth of the theory took place due to increase in the complexity of equipments for space, military, professional and industrial use whose failure can be seriously hazardous to human life in addition to large investment of capital.

A valuable contribution has been made to the development of general theory of reliability by B.V. Gnedenko, R.E. Barlow, Proschan, S.M. Ross, Lamberson, Jorgenson, Shooman. Further the Operations Research Society of America identified this as an important area and published a special issue in 1984 on Reliability and Maintainability.

There are several books dealing with different aspects of Reliability theory. See [3,4,8,15]

The failure of a system increases cost and sometimes causes considerable damage to human lives. This has led to an increase in demand for higher reliability. So in order to increase the Reliability of a system the concept of Redundancy is used. Then it may suffer from the disadvantage of being a needlessly costly system. Thus an optimal
number of components are to be found-out by compromising with the conflicting virtues of high reliability and minimal system cost.

For these reasons, several workers sought to tackle various types of optimal problems for the given system, i.e., to search for the ways and means to render a system "Optimal" in a desired sense. However, the way in which the system can fail plays an important role while developing optimal reliability of complex systems which is hardly considered in the available models. **THIS FORMS IN FACT THE BASIC AIM OF THE THESIS.**

The present thesis aims to develop optimal 'Series - Parallel' systems, that is subject to two modes of failures.

We now present the brief review of earlier works having relevance to as well as motivating our work in the thesis.
1.2 A LITERATURE - COVERAGE

The earliest contribution in Reliability theory may traced to the pioneering work due to Lotka [9]. He developed a replacement policy for the equipment subject to random failures. Later for the same model Campbell [5], studied the preventive maintenance policies.

Barlow and Hunter [1], proposed periodic replacement or overhaul at times $T, 2T, 3T, ...$ (for same $T > 0$) and minimal repair if the system failed otherwise.

Barlow and Proschan [2], developed various replacement policies when life times have increasing failure rate.

Earlier works in the general theory of Reliability and replacement policy were excellently reported in Barlow and Proschan [2], Jorgenson et al [8], Shooman [15], Gnedenko et al [7].

Much work has been reported on Redundant systems. The earliest contribution to the Redundant systems may be due to Gaver [6]. He derived Mean Time to system failure and stationery availability for a two-unit Parallel Redundant system with exponential distributed life time and arbitrarily distributed repair times.

Parthasarathy [12], discussed the cost analysis of the two-unit cold standby system.
Nakagawa [11], proposed the optimal replacement period $T^*$, the optimal number of units $N^*$ and the optimal pair $(N^*, T^*)$ for a $N$-unit Parallel system. Later on Venugopal et al [18], derived optimal number of units $N^*$, optimal pair $(N^*, n^*)$ for N-unit Parallel system by incorporating important practical features in terms of maintenance cost and repair cost into the modelling cost. Further these results were extended in Min-Tsai Lai and John Yuan [10], to the systems subject to common cause failures.

Rami Reddy. C, and Vijaya Bhaskara Rao .D [13], dealt with N-unit Parallel system subject to two - modes of failure with repair provision. Under certain assumptions, they developed a procedure leading to the optimal number of units $N^*$ for the system based on cost considerations.

Rami Reddy.C, and Vijaya Bhaskara Rao .D [14], considers a system with two states: Operating and Failed. The system undergoes overhauls preventively at scheduled times and the successive overhaul times constitute non-decreasing Geometric Process (G.P.). They assumed that the failure rate increases with the number of overhauls. Also they assumed that the system undergoes only minimal repairs if it fails between overhauls.

For this model they developed a procedure which lead to the optimal number of overhauls $N^*$, and the optimal pair $(N^*, T^*)$ based on the cost considerations.

In the following section we list down some basics that are needed for the later work of the thesis.

1.3 BASICS

The fundamental concepts in Reliability theory are: **Reliability**, **Failure time distribution**, **Hazard rate** and **Mean Time Before Failure (MTBF)** of a system. Reliability may be variedly defined as follows:
(i) Reliability is the probability of a device giving satisfactory performance for a specified period of time under specified operating conditions.

(ii) Reliability is the integral of the distribution of probabilities of failure - free operation from the instant of switch-on to the first failure.

(iii) The Reliability, R(t) of a component is the probability that the component will not fail for a time 't'.

(iv) Reliability is the mean operating time for a given specimen between two failures.

(v) Reliability of a system is called its capacity for failure - free operation for a definite period of time under operating conditions and for minimum time lost for repair and preventive maintenance.

Since it is accepted that a several identical systems operating under given conditions which are similar, fail at different units of time, then it follows that a failure phenomenon can only be described in probabilistic terms. Thus the fundamental definition of Reliability must heavily depend on concepts from Probability theory.

Mathematically we can present the above ideas as follows:

Consider a component (or a unit or a system), whose life time is denoted by T, assumed to be non-negative, continuous and increasing random variable.

The probability of failure as a function of time 't' can be defined by

\[ F(t) = P(T \leq t), \quad t > 0. \] (1)

is called the failure time distribution where 't' is the random variable denoting the failure time. Also,

\[ f(t) = F'(t) \] (1a)
and it is assumed that $F(t)$ possesses continuous derivatives, so that

$$
F(t) = \int_0^t f(x) \, dx \tag{2}
$$

$F(t)$ is the failure distribution function (unreliability function).

Hence the Reliability function $R(t)$ is defined as

$$
R(t) = 1 - F(t) = P(T > t), \quad t > 0 \tag{3}
$$

Therefore, that the Reliability function is the probability that the unit survived up to time $t$. For this reason it is also called *Survival Function*.

The Hazard rate or failure rate denoted by $h(t)$ is defined as

$$
h(t) = \frac{f(t)}{R(t)} \tag{4a}
$$

The Reliability and hazard functions are related by

$$
R(t) = \exp \left[ - \int_0^t h(u) \, du \right] \tag{4b}
$$

We can also write it as

$$
f(t) = h(t) \exp \left[ - \int_0^t h(u) \, du \right] \tag{4c}
$$
The MTTF of the component is given by
\[ E(T) = \int_0^\infty t \cdot f(t) \, dt \] (5)

Alternatively:

Since \[ f(t) = -R'(t) \]
\[ E(t) = -\int_0^\infty t R'(t) \, dt, \]

\[ = -t R(t) \bigg|_0^\infty + \int_0^\infty R(t) \, dt \quad \text{(by integrating by-parts)} \]

\[ = \int_0^\infty R(t) \, dt \] (6)

a widely used result in Reliability theory.

A system may taken to be an arbitrary device consisting of components with given Reliabilities. We now elaborate the following systems which are useful for our present work.
Series System

The Series system is probably the most widely used model. In a Series system all the components must operate successfully, for the system to function. The block diagram of a n-unit Series system is

![Series Block Diagram](image)

For the Series model,

\[
R(t) = \prod_{i=1}^{n} R_i
\]  

(7)

Where \( R_i \) indicates the reliability of \( i \)th component. Note that the system's reliability rapidly decreases as the number of Series components increases and the Reliability will always be less than or equal to the least reliable component. So for a Series system

\[
R(t) \leq \min \{ R_i(t) \}
\]

for \( i \).
In particular, when all the components of the system are identical, that is having the same reliability implying

\[ R_i(t) = P(t) \quad i = 1, 2, \ldots, n \]

then

\[ R(t) = [ P(t) ] \]

Also

\[ F(t) = 1 - R(t), \]

**Parallel System**

A parallel system is a system that is not considered to have failed unless all units have failed. The block diagram of a \( n \)-unit parallel system is

![Parallel System Block Diagram](image)

Fig: Parallel System Block Diagram
If the i-th unit has Reliability $R_i(t)$, then the n-unit parallel system Reliability is given by:

$$R(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

Series-Parallel System

The Reliability block diagram for a Series-Parallel System is given below:

A system is called a series-parallel system of order (m,n) if the system comprising of m (≥ 2) identical parallel systems of order n, connected in series.

If the i-th component has reliability $R_i(t)$, then the system reliability $R(t)$ is given by:

$$R(t) = [1 - \prod_{i=1}^{n} (1 - R_i(t))]^m$$

In particular, if all the components have the same reliability function $R_i(t) = [1-F(t)]$, $F(t)$ being the common failure time distribution function, then the systems MTBF is given by

$$\mu_n^{(m)} = \int [1-F(t)^n]^m dt.$$
SELECTED REFERENCES


