CHAPTER V

ALTERNATIVE MEASURE OF SPEED OF AGING AS A FUNCTION OF DEMOGRAPHIC COMPONENTS
5.1 Introduction

Previous chapter has been devoted to extend and develop of aging measures as a function of demographic components. Liao (1996) suggested such measures of aging as a function of crude birth rate (CBR), crude death rate (CDR) and migration rate. In this chapter, an attempt has been made to develop an alternative measure of aging as a function of demographic components. The basic idea of these alternative measures is to use fertility (adjusted with NRR and population growth) and mortality rates alternative to CBR and CDR respectively. Here life expectancy of a person and net reproduction rate (NRR) has been used as alternative to crude death rate and crude birth rates respectively. Age and sex specific alternative birth and death rates have also been calculated. Sex specific birth rate has been computed with sex ratio at birth. Actually life table mortality measures have been considered here. All the measures presented in chapter IV have been studied here considering alternative birth and death rates.

5.2 Proposed Alternative Aging Indices

Demographic measure varies among different sub-populations (age, sex, region, education level, economic status, marital status etc.). Nath and Deka (2006) suggested few improved aging indices considering birth and death rates for male, female and male-female simultaneously. Here an attempt has been made to expand these measures with respect to region (urban and rural) and sex specific alternative
birth rates and life expectancies. The indices presented here are for closed population.

5.2.1 Measures Related to Proportion of Aged 60 and Above ($P_{60}$)

5.2.1.1 Urban Population

i) Rate of change in the proportion of urban population aged 60 and over ($P_{60}^u$) at time $t$:

$$\frac{dP_{60}^u(t)}{dt} = \frac{N_{60}^u(t)}{N^u(t)} \left\{ \frac{1}{e_0^u(t)} - \frac{1}{e_{60}^u(t)} \right\} + \left\{ a_{60}^u(t) - b^u(t)^* \right\}$$

(5.1)

under usual notations and suffix 'u' stands for urban population.

ii) Rate of change in the proportion of urban male population aged 60 and over ($P_{60}^{um}$) at time $t$:

$$\frac{dP_{60}^{um}(t)}{dt} = \frac{N_{60}^{um}(t)}{N^{um}(t)} \left\{ \frac{1}{e_0^{um}(t)} - \frac{1}{e_{60}^{um}(t)} \right\} + \left\{ a_{60}^{um}(t) - b^{um}(t)^* \cdot \frac{N^v(t)}{N^{um}(t)} \right\}$$

(5.2)

under usual notations and suffix 'm' stands for male population.

Here, $b^{um}(t)^* = \frac{B^{um}(t)^*}{N^v(t)}$: birth rate of urban male child at time $t$.

iii) Rate of change in the proportion of urban female population aged 60 and over ($P_{60}^{uf}$) at time $t$:

$$\frac{dP_{60}^{uf}(t)}{dt} = \frac{N_{60}^{uf}(t)}{N^{uf}(t)} \left\{ \frac{1}{e_0^{uf}(t)} - \frac{1}{e_{60}^{uf}(t)} \right\} + \left\{ a_{60}^{uf}(t) - b^{uf}(t)^* \cdot \frac{N^v(t)}{N^{uf}(t)} \right\}$$

(5.3)

under usual notations and suffix 'f' stands for female population.

Here, $b^{uf}(t)^* = \frac{B^{uf}(t)^*}{N^v(t)}$: birth rate of urban female child at time $t$. 

157
iv) Rate of change in the proportion of urban population aged 60 and over ($P^u_{60}$) at
time $t$ (considering urban male-female birth and death rates simultaneously):

$$\frac{dP^u_{60}(t)}{dt} = \frac{N^u_{60}(t)}{N^u(t)} \left[ \alpha^u_{60}(t) - \frac{1}{e^u_{60}(t)} \right] + \frac{N^f_{60}(t)}{N^u(t)} \left[ \alpha^f_{60}(t) - \frac{1}{e^f_{60}(t)} \right]$$

$$+ \frac{N^f_{60}(t)}{N^u(t)} \left[ \frac{1}{e^m_{60}(t)} \cdot \frac{N^m_{60}(t)}{N^u(t)} + \frac{1}{e^f_{60}(t)} \cdot \frac{N^f_{60}(t)}{N^u(t)} - b^u_{60}(t) - b^f_{60}(t) \right]$$  (5.4)

under usual notations.

5.2.1.2 Rural Population

i) Rate of change in the proportion of rural population aged 60 and over ($P^r_{60}$) at
time $t$:

$$\frac{dP^r_{60}(t)}{dt} = \frac{N^r_{60}(t)}{N^r(t)} \left[ 1 - \frac{1}{e^r_{60}(t)} \right] + \left[ a^r_{60}(t) - b^r_{60}(t) \right]$$  (5.5)

under usual notations and suffix ‘R’ stands for rural population.

ii) Rate of change in the proportion of rural male population aged 60 and over
($P^m_{60}$) at time $t$:

$$\frac{dP^m_{60}(t)}{dt} = \frac{N^m_{60}(t)}{N^m(t)} \left[ 1 - \frac{1}{e^m_{60}(t)} \right] + \left[ a^m_{60}(t) - b^m_{60}(t) \right]$$  (5.6)

under usual notations.

Here, $b^m_{60}(t) = \frac{b^m_{60}(t)}{N^r(t)}$: birth rate of rural male child at time $t$.

iii) Rate of change in the proportion of rural female population aged 60 and over
($P^f_{60}$) at time $t$:

$$\frac{dP^f_{60}(t)}{dt} = \frac{N^f_{60}(t)}{N^f(t)} \left[ \frac{1}{e^f_{60}(t)} - \frac{1}{e^f_{60}(t)} \right] + \left[ a^f_{60}(t) - b^f_{60}(t) \right]$$  (5.7)
under usual notations.

Here, $b^f(t)^* = \frac{B^f(t)}{N_R(t)}$: birth rate of rural female child at time t.

iv) Rate of change in the proportion of rural population aged 60 and over ($P^R_{60}$) at time t (considering rural male-female birth and death rates simultaneously):

$$\frac{dP^R_{60}(t)}{dt} = \frac{N^m_{60}(t)}{N^R(t)} \left[ a^m_{60}(t) - \frac{1}{e^m_{60}(t)} \right] + \frac{N^f_{60}(t)}{N^R(t)} \left[ a^f_{60}(t) - \frac{1}{e^f_{60}(t)} \right]$$

$$+ \frac{N^R_{60}(t)}{N^R(t)} \left[ \frac{1}{e^m_{0}(t)} \cdot \frac{N^m_{60}(t)}{N^R(t)} + \frac{1}{e^f_{0}(t)} \cdot \frac{N^f_{60}(t)}{N^R(t)} - b^m(t)^* - b^f(t)^* \right]$$  \hspace{1cm} (5.8)

under usual notations.

5.2.1.3 Overall Population

i) Rate of change in the proportion of males age 60 and over ($P^m_{60}$) at time t:

$$\frac{dP^m_{60}(t)}{dt} = \frac{N^m_{60}(t)}{N^m(t)} \left[ \frac{1}{e^m_{60}(t)} - \frac{1}{e^m_{60}(t)} \right] + \left[ a^m_{60}(t) - b^m(t)^* \frac{N(t)}{N^m(t)} \right]$$ \hspace{1cm} (5.9)

under usual notations.

ii) Rate of change in the proportion of females age 60 and over ($P^f_{60}$) at time t:

$$\frac{dP^f_{60}(t)}{dt} = \frac{N^f_{60}(t)}{N^f(t)} \left[ \frac{1}{e^f_{60}(t)} - \frac{1}{e^f_{60}(t)} \right] + \left[ a^f_{60}(t) - b^f(t)^* \frac{N(t)}{N^f(t)} \right]$$ \hspace{1cm} (5.10)

under usual notations.

iii) Rate of change in the proportion of persons age 60 and over at time t:

$$\frac{dP_{60}(t)}{dt} = \frac{N_{60}(t)}{N(t)} \left[ \frac{1}{e_{60}(t)} - \frac{1}{e_{60}(t)} \right] + \left[ a_{60}(t) - b(t)^* \right]$$ \hspace{1cm} (5.11)

under usual notations.
iv) Rate of change in the proportion of persons age 60 and over, $P_{60}$, at time $t$ (considering male-female birth and death rate simultaneously):

$$\frac{dP_{60}(t)}{dt} = \frac{N^{m}_{60}(t)}{N(t)} \left[ a^{m}_{60}(t) - \frac{1}{e^{m}_{60}(t)} \right] + \frac{N^{f}_{60}(t)}{N(t)} \left[ a^{f}_{60}(t) - \frac{1}{e^{f}_{60}(t)} \right]$$

$$+ \frac{N_{60}(t)}{N(t)} \left[ \frac{1}{e^{m}_{60}(t)} \cdot \frac{N^{m}(t)}{N(t)} + \frac{1}{e^{f}_{60}(t)} \cdot \frac{N^{f}(t)}{N(t)} - b^{m}(t)^* - b^{f}(t)^* \right] \quad (5.12)$$

under usual notations.

v) Rate of change in the proportion of persons age 60 and over, $P_{60}$, at time $t$ (considering urban and rural birth and death rates simultaneously):

$$\frac{dP_{60}(t)}{dt} = \frac{N^{u}_{60}(t)}{N(t)} \left[ a^{u}_{60}(t) - \frac{1}{e^{u}_{60}(t)} \right] + \frac{N^{r}_{60}(t)}{N(t)} \left[ a^{r}_{60}(t) - \frac{1}{e^{r}_{60}(t)} \right]$$

$$+ \frac{N_{60}(t)}{N(t)} \left[ \frac{1}{e^{u}_{60}(t)} \cdot \frac{N^{u}(t)}{N(t)} + \frac{1}{e^{r}_{60}(t)} \cdot \frac{N^{r}(t)}{N(t)} - b^{u}(t)^* \cdot \frac{N^{u}(t)}{N(t)} - b^{r}(t)^* \cdot \frac{N^{r}(t)}{N(t)} \right] \quad (5.13)$$

under usual notations.

vi) Rate of change in the proportion of persons age 60 and over, $P_{60}$, at time $t$ (considering urban-rural and male-female birth and death rates simultaneously):

$$\frac{dP_{60}(t)}{dt} = \frac{N^{um}_{60}(t)}{N(t)} \left[ a^{um}_{60}(t) - \frac{1}{e^{um}_{60}(t)} \right] + \frac{N^{rf}_{60}(t)}{N(t)} \left[ a^{rf}_{60}(t) - \frac{1}{e^{rf}_{60}(t)} \right]$$

$$+ \frac{N^{ur}_{60}(t)}{N(t)} \left[ a^{ur}_{60}(t) - \frac{1}{e^{ur}_{60}(t)} \right] + \frac{N^{fu}_{60}(t)}{N(t)} \left[ a^{fu}_{60}(t) - \frac{1}{e^{fu}_{60}(t)} \right]$$

$$+ \frac{N_{60}(t)}{N(t)} \left[ \frac{1}{e^{um}_{60}(t)} \cdot \frac{N^{um}(t)}{N(t)} + \frac{1}{e^{rf}_{60}(t)} \cdot \frac{N^{rf}(t)}{N(t)} + \frac{1}{e^{ur}_{60}(t)} \cdot \frac{N^{ur}(t)}{N(t)} + \frac{1}{e^{fu}_{60}(t)} \cdot \frac{N^{fu}(t)}{N(t)} \right]$$

$$- \frac{N^{r}(t)}{N(t)} \left[ b^{um}(t)^* + b^{rf}(t)^* + b^{ur}(t)^* + b^{fu}(t)^* \right] \quad (5.14)$$

under usual notations.
5.2.2 Measures Related to Proportion of Age below 15 (P_{15})

5.2.2.1 Urban Population

i) Rate of change in the proportion of urban population under 15 (P_{15}^u) at time t:

\[
\frac{dP_{15}^u(t)}{dt} = \frac{N_{15}^u(t)}{N^u(t)} \left[ \frac{1}{e_0^u(t)} - \alpha_{15}^u(t) - d_{15}^u(t) \right] + \left[ b_{15}^u(t) - b^u(t) \right] \tag{5.15}
\]

under usual notations.

ii) Rate of change in the proportion of urban male population age below 15 (P_{15}^{um}) at time t:

\[
\frac{dP_{15}^{um}(t)}{dt} = \frac{N_{15}^{um}(t)}{N^{um}(t)} \left[ \frac{1}{e_0^{um}(t)} - \alpha_{15}^{um}(t) - d_{15}^{um}(t) \right] + \left[ b_{15}^{um}(t) - b^{um}(t) \cdot \frac{N^u(t)}{N^{um}(t)} \right] \tag{5.16}
\]

under usual notations.

iii) Rate of change in the proportion of urban female population age below 15 (P_{15}^{uf}) at time t:

\[
\frac{dP_{15}^{uf}(t)}{dt} = \frac{N_{15}^{uf}(t)}{N^{uf}(t)} \left[ \frac{1}{e_0^{uf}(t)} - \alpha_{15}^{uf}(t) - d_{15}^{uf}(t) \right] + \left[ b_{15}^{uf}(t) - b^{uf}(t) \cdot \frac{N^u(t)}{N^{uf}(t)} \right] \tag{5.17}
\]

under usual notations.

iv) Rate of change in the proportion of urban population age below 15 (P_{15}^u) at time t (considering urban male-female birth and death rates simultaneously):

\[
\frac{dP_{15}^u(t)}{dt} = \frac{N_{15}^{um}(t)}{N^u(t)} \left[ b_{15}^{um}(t) - \alpha_{15}^{um}(t) - d_{15}^{um}(t) \right] + \frac{N_{15}^{uf}(t)}{N^u(t)} \left[ b_{15}^{uf}(t) - \alpha_{15}^{uf}(t) - d_{15}^{uf}(t) \right] \]

\[
+ \frac{N_u^u(t)}{N^u(t)} \left[ \frac{1}{e_0^{um}(t)} N^{um}(t) + \frac{1}{e_0^{uf}(t)} N^{uf}(t) - b^{um}(t) - b^{uf}(t) \right] \tag{5.18}
\]

under usual notations.
5.2.2.2 Rural Population

i) Rate of change in the proportion of rural population age below 15 ($P_{15}^R$) at time $t$:

$$\frac{dP_{15}^R(t)}{dt} = \frac{N_{15}^R(t)}{N^R(t)} \left[ \frac{1}{e_0^R(t)} - a_{15}^R(t) - d_{15}^R(t) \right] + \left[ b_{15}^R(t)^* - b^R(t)^* \right]$$  \hspace{1cm} (5.19)

under usual notations.

ii) Rate of change in the proportion of rural male population age below 15 ($P_{15}^{Rm}$) at time $t$:

$$\frac{dP_{15}^{Rm}(t)}{dt} = \frac{N_{15}^{Rm}(t)}{N^{Rm}(t)} \left[ \frac{1}{e_0^{Rm}(t)} - a_{15}^{Rm}(t) - d_{15}^{Rm}(t) \right] + \left[ b_{15}^{Rm}(t)^* - b^{Rm}(t)^* \cdot \frac{N^R(t)}{N^{Rm}(t)} \right]$$  \hspace{1cm} (5.20)

under usual notations.

iii) Rate of change in the proportion of rural female population age below 15 ($P_{15}^{Rf}$) at time $t$:

$$\frac{dP_{15}^{Rf}(t)}{dt} = \frac{N_{15}^{Rf}(t)}{N^{Rf}(t)} \left[ \frac{1}{e_0^{Rf}(t)} - a_{15}^{Rf}(t) - d_{15}^{Rf}(t) \right] + \left[ b_{15}^{Rf}(t)^* - b^{Rf}(t)^* \cdot \frac{N^R(t)}{N^{Rf}(t)} \right]$$  \hspace{1cm} (5.21)

under usual notations.

iv) Rate of change in the proportion of rural population age below 15 ($P_{15}^R$) at time $t$ (considering rural male-female birth and death rates simultaneously):

$$\frac{dP_{15}^R(t)}{dt} = \frac{N_{15}^{Rm}(t)}{N^R(t)} \left[ b_{15}^{Rm}(t)^* - a_{15}^{Rm}(t) - d_{15}^{Rm}(t) \right] + \frac{N_{15}^{Rf}(t)}{N^R(t)} \left[ b_{15}^{Rf}(t)^* - a_{15}^{Rf}(t) - d_{15}^{Rf}(t) \right]$$

$$\hspace{2cm} + \frac{N^R(t)}{N^R(t)} \left[ \frac{1}{e_0^{Rm}(t)} \cdot \frac{N^{Rm}(t)}{N^R(t)} + \frac{1}{e_0^{Rf}(t)} \cdot \frac{N^{Rf}(t)}{N^R(t)} - b^{Rm}(t)^* - b^{Rf}(t)^* \right]$$  \hspace{1cm} (5.22)

under usual notations.

162
5.2.2.3 Overall Population

i) Rate of change in the proportion male children under 15 ($P_{m15}$) at time $t$:

\[
\frac{dP_{m15}(t)}{dt} = \frac{N_{m15}(t)}{N(t)} \left[ \frac{1}{e_0^m(t)} - a_{m15}(t) - d_{m15}(t) \right] + \left[ b_{m15}(t)^* - b^m(t)^* \frac{N(t)}{N_m(t)} \right] \quad (5.23)
\]

under usual notations.

ii) Rate of change in the proportion female children under 15 ($P_{f15}$) at time $t$:

\[
\frac{dP_{f15}(t)}{dt} = \frac{N_{f15}(t)}{N(t)} \left[ \frac{1}{e_0^f(t)} - a_{f15}(t) - d_{f15}(t) \right] + \left[ b_{f15}(t)^* - b^f(t)^* \frac{N(t)}{N_f(t)} \right] \quad (5.24)
\]

under usual notations.

iii) Rate of change in the proportion of persons aged below 15 at time $t$:

\[
\frac{dP_{15}(t)}{dt} = \frac{N_{15}(t)}{N(t)} \left[ \frac{1}{e_0(t)} - a_{15}(t) - d_{15}(t) \right] + \left[ b_{15}(t)^* - b(t)^* \right] \quad (5.25)
\]

under usual notations.

iv) Rate of change in the proportion children under 15, $P_{15}$, at time $t$ (considering male-female birth and death rates simultaneously):

\[
\frac{dP_{15}(t)}{dt} = \frac{N_{15}(t)}{N(t)} \left[ b_{15}^m(t)^* - a_{15}^m(t) - d_{15}^m(t) \right] + \frac{N_{15}(t)}{N(t)} \left[ b_{15}^f(t)^* - a_{15}^f(t) - d_{15}^f(t) \right] + \frac{N_{15}(t)}{N(t)} \left[ \frac{1}{e_0^m(t)} - \frac{1}{e_0^f(t)} \right] \left[ b^m(t)^* - b^f(t)^* \right] \quad (5.26)
\]

under usual notations.

v) Rate of change in the proportion children under 15, $P_{15}$, at time $t$ (considering urban and rural birth and death rates simultaneously):

\[
\frac{dP_{15}(t)}{dt} = \frac{N_{15}(t)}{N(t)} \left[ b_{15}^u(t)^* - a_{15}^u(t) - d_{15}^u(t) \right] + \frac{N_{15}(t)}{N(t)} \left[ b_{15}^r(t)^* - a_{15}^r(t) - d_{15}^r(t) \right]
\]
under usual notations.

vi) Rate of change in the proportion children under 15, \( P_{15} \), at time \( t \) (considering urban-rural, male-female birth and death rates simultaneously):

\[
\frac{dP_{15}(t)}{dt} = N_{15}(t) \left[ \frac{1}{e_{15}^{m}(t)} \cdot N_{15}^{m}(t) + \frac{1}{e_{15}^{f}(t)} \cdot N_{15}^{f}(t) - b_{15}(t)^* \cdot N_{15}^{f}(t) - b_{15}(t)^* \cdot N_{15}^{m}(t) \right]
\]

(5.27)

under usual notations.

5.2.3 Measures Related to Aged-Child Ratio (R)

5.2.3.1 Urban Population

i) Rate of change in the aged-child ratio of urban population (\( R_{u} \)) at time \( t \):

\[
\frac{dR_{u}(t)}{dt} = \frac{N_{u}^{m}(t)}{N_{u}(t)} \left[ a_{15}^{m}(t) + \frac{1}{e_{15}^{m}(t)} \cdot N_{15}^{m}(t) - \frac{1}{e_{15}^{m}(t)} \cdot N_{15}^{m}(t) \right]
\]

(5.29)

under usual notations.

ii) Rate of change in the aged-child ratio of urban male population (\( R_{u}^{m} \)) at time \( t \):

\[
\frac{dR_{u}^{m}(t)}{dt} = \frac{N_{u}^{m}(t)}{N_{15}^{m}(t)} \left[ a_{15}^{m}(t) + \frac{1}{e_{15}^{m}(t)} \cdot N_{15}^{m}(t) - \frac{1}{e_{15}^{m}(t)} \cdot N_{15}^{m}(t) \right]
\]

(5.30)

under usual notations.
iii) Rate of change in the aged-child ratio of urban female population \( (R_{uf}) \):

\[
\frac{dR_{uf}(t)}{dt} = \frac{N_{60f}(t)}{N_{15f}(t)} \left[ a_{15f}(t) + d_{15f}(t) - b_{15}(t) \right] + \left[ a_{60f}(t) - \frac{1}{e_{60f}(t)} \right] \]

under usual notations.

iv) Rate of change in the aged-child ratio of urban population \( (R_u) \) at time \( t \) (considering urban male-female birth and death rates simultaneously):

\[
\frac{dR_u(t)}{dt} = \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}(t) + d_{15}(t) - b_{15}(t) \right] + \frac{N_{60}(t)}{N_{50}(t)} \left[ a_{60f}(t) - \frac{1}{e_{60f}(t)} \right] + \frac{N_{60f}(t)}{N_{50f}(t)} \left[ a_{60f}(t) - \frac{1}{e_{60f}(t)} + \frac{N_{60}(t)}{N_{50}(t)} \right]

under usual notations.

5.2.3.2 Rural Population

i) Rate of change in the aged-child ratio of rural population \( (R_R) \) at time \( t \):

\[
\frac{dR_R(t)}{dt} = \frac{N_{60R}(t)}{N_{15R}(t)} \left[ a_{15R}(t) + d_{15}(t) - b_{15}(t) \right] + \left[ a_{60R}(t) - \frac{1}{e_{60R}(t)} \right]

under usual notations.

ii) Rate of change in the aged-child ratio of rural male population \( (R_{Rm}) \) at time \( t \):

\[
\frac{dR_{Rm}(t)}{dt} = \frac{N_{60Rm}(t)}{N_{15Rm}(t)} \left[ a_{15Rm}(t) + d_{15Rm}(t) - b_{15Rm}(t) \right] + \left[ a_{60Rm}(t) - \frac{1}{e_{60Rm}(t)} \right]

under usual notations.

iii) Rate of change in the aged-child ratio of rural female population \( (R_{Rf}) \) at time \( t \):

\[
\frac{dR_{Rf}(t)}{dt} = \frac{N_{60Rf}(t)}{N_{15Rf}(t)} \left[ a_{15Rf}(t) + d_{15Rf}(t) - b_{15Rf}(t) \right] + \left[ a_{60Rf}(t) - \frac{1}{e_{60Rf}(t)} \right]

under usual notations.
iv) Rate of change in the aged-child ratio of rural population ($R^R$) at time $t$ (considering rural male-female birth and death rates simultaneously):

$$
\frac{dR^R(t)}{dt} = \frac{N^R_{60}(t)}{N^R_{15}(t)} \left[ a^m_{15}(t) + d^m_{15}(t) - b^m_{15}(t) \right] + \frac{N^R_{60}(t)}{N^R_{15}(t)} \left[ a^f_{15}(t) + d^f_{15}(t) - b^f_{15}(t) \right]
$$

under usual notations.

5.2.3.3 Overall Population

i) Rate of change in the male aged-child ratio ($R^m$) at time $t$:

$$
\frac{dR^m(t)}{dt} = \frac{N^m_{60}(t)}{N^m_{15}(t)} \left[ a^m_{15}(t) + d^m_{15}(t) - \frac{1}{e^m_{60}(t)} \right] + \left[ a^m_{60}(t) - b^m_{15}(t) \right]
$$

under usual notations.

ii) Rate of change in the female aged-child ratio ($R^f$) at time $t$:

$$
\frac{dR^f(t)}{dt} = \frac{N^f_{60}(t)}{N^f_{15}(t)} \left[ a^f_{15}(t) + d^f_{15}(t) - \frac{1}{e^f_{60}(t)} \right] + \left[ a^f_{60}(t) - b^f_{15}(t) \right]
$$

under usual notations.

iii) Rate of change in the aged-child ratio at time $t$:

$$
\frac{dR(t)}{dt} = \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}(t) + d_{15}(t) - \frac{1}{e_{60}(t)} \right] + \left[ a_{60}(t) - b_{15}(t) \right]
$$

under usual notations.

iv) Rate of change in the aged-child ratio ($R$) at time $t$ (considering male-female age-specific birth and death rate simultaneously):

$$
\frac{dR(t)}{dt} = \frac{N_{60}(t) N^m_{60}(t)}{N_{15}(t) N^m_{15}(t)} \left[ a^m_{15}(t) + d^m_{15}(t) - b^m_{15}(t) \right] + \frac{N_{60}(t) N^f_{60}(t)}{N_{15}(t) N^f_{15}(t)} \left[ a^f_{15}(t) + d^f_{15}(t) - b^f_{15}(t) \right]
$$

166
under usual notations.

v) Rate of change in the aged-child ratio (R) at time t (considering urban and rural birth and death rates simultaneously):

\[
\frac{dR(t)}{dt} = \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}(t) + d_{15}(t) - b_{15}(t) \right] N_{15}^{R}(t) + \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}^{R}(t) + d_{15}^{R}(t) - b_{15}^{R}(t) \right] N_{15}^{R}(t)
\]

+ \frac{N_{60}^{m}(t)}{N_{15}(t)} \left[ a_{60}^{m}(t) - \frac{1}{e_{60}^{m}(t)} \right] + \frac{N_{60}^{f}(t)}{N_{15}(t)} \left[ a_{60}^{f}(t) - \frac{1}{e_{60}^{f}(t)} \right] \tag{5.41}

under usual notations.

vi) Rate of change in the aged-child ratio (R) at time t (considering urban-rural, male female birth and death rates simultaneously):

\[
\frac{dR(t)}{dt} = \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}(t) + d_{15}(t) - b_{15}(t) \right] N_{15}^{R}(t) + \frac{N_{60}(t)}{N_{15}(t)} \left[ a_{15}^{R}(t) + d_{15}^{R}(t) - b_{15}^{R}(t) \right] N_{15}^{R}(t)
\]

+ \frac{N_{60}^{m}(t)}{N_{15}(t)} \left[ a_{60}^{m}(t) - \frac{1}{e_{60}^{m}(t)} \right] + \frac{N_{60}^{f}(t)}{N_{15}(t)} \left[ a_{60}^{f}(t) - \frac{1}{e_{60}^{f}(t)} \right] \tag{5.42}

under usual notations.
5.2.4 Measures Related to Median Age (Md)

5.2.4.1 Urban Population

i) Rate of change in the median age of urban population (Md) at time t:

\[
\frac{dP_{Md}(t)}{dt} = \frac{N_u(t)}{N^u(t)} \left[ \frac{1}{e^u_0(t)} \left( \alpha^u_{Md}(t) - \alpha^u_{Md}(t) \right) - \left( b^u_{Md}(t)^* - b^u_0(t) \right) \right] \]

(5.43)

and,

\[
\frac{dM_{Md}(t)}{dt} = \frac{dP_{Md}(t)}{P_{Md}\_w(t)}
\]

(5.44)

under usual notations.

ii) Rate of change in the median age of urban male population (Md\_m) at time t:

\[
\frac{dP_{Md\_m}(t)}{dt} = \frac{N_{Md\_m}(t)}{N\_m(t)} \left[ \frac{1}{e^m_0(t)} \left( \alpha^m_{Md\_m}(t) - \alpha^m_{Md\_m}(t) \right) - \left( b^m_{Md\_m}(t)^* - b^m_0(t) \right) \right] N^m(t)
\]

(5.45)

and

\[
\frac{dM_{Md\_m}(t)}{dt} = \frac{dP_{Md\_m}(t)}{P_{Md\_m\_w}(t)}
\]

(5.46)

under usual notations.

iii) Rate of change in the median age of urban female population (Md\_f) at time t:

\[
\frac{dP_{Md\_f}(t)}{dt} = \frac{N_{Md\_f}(t)}{N\_f(t)} \left[ \frac{1}{e^f_0(t)} \left( \alpha^f_{Md\_f}(t) - \alpha^f_{Md\_f}(t) \right) - \left( b^f_{Md\_f}(t)^* - b^f_0(t) \right) \right] N^f(t)
\]

(5.47)

and

\[
\frac{dM_{Md\_f}(t)}{dt} = \frac{dP_{Md\_f}(t)}{P_{Md\_f\_w}(t)}
\]

(5.48)

under usual notations.
iv) Rate of change in the median age of urban population (Md\textsuperscript{u}) at time t (considering urban male-female birth and death rates simultaneously):

The expression for \( \frac{dP_{Md}^u(t)}{dt} \) is same as \( \frac{dP_{Md}^u(t)}{dt} \) replacing suffix 15 by Md.

\[
\frac{dP_{Md}^u(t)}{dt} = \frac{N_{Md}^u(t)}{N^u(t)} \left[ b_{Md}^u(t) - a_{Md}^u(t) - d_{Md}^u(t) \right] + \frac{N_{Md}^f(t)}{N^u(t)} \left[ b_{Md}^f(t) - a_{Md}^f(t) - d_{Md}^u(t) \right] \\
+ \frac{N_{Md}^m(t)}{N^u(t)} \left[ \frac{1}{e_{e}^u(t)} \cdot N_{Md}^m(t) + \frac{1}{e_{f}^u(t)} \cdot N_{Md}^f(t) - b_{Md}^u(t) - b_{Md}^u(t) \right]
\]

(5.49)

and

\[
\frac{dMd^u(t)}{dt} = -\left( w_dP_{Md}^u(t) \right) P_{Md-w}^u(t)
\]

(5.50)

under usual notations.

5.2.4.2 Rural Population

i) Rate of change in the median age of rural population (Md\textsuperscript{R}) at time t:

\[
\frac{dP_{Md}^R(t)}{dt} = \frac{N_{Md}^R(t)}{N^R(t)} \left[ \frac{1}{e_{e}^R(t)} - a_{Md}^R(t) - d_{Md}^R(t) \right] + \left[ b_{Md}^R(t) - b^R(t)^* \right]
\]

(5.51)

and

\[
\frac{dMd^R(t)}{dt} = -\left( w_dP_{Md}^R(t) \right) P_{Md-w}^R(t)
\]

(5.52)

under usual notations.

ii) Rate of change in the median age of rural male population (Md\textsuperscript{Rm}) at time t:

\[
\frac{dP_{Md}^{Rm}(t)}{dt} = \frac{N_{Md}^{Rm}(t)}{N^{Rm}(t)} \left[ \frac{1}{e_{0}^{Rm}(t)} - a_{Md}^{Rm}(t) - d_{Md}^{Rm}(t) \right] + \left[ b_{Md}^{Rm}(t) - b^{Rm}(t)^* \right] \frac{N^R(t)}{N^{Rm}(t)}
\]

(5.53)

and
\[
\frac{dM_{d}^{Rm}(t)}{dt} = - \left( \frac{w_{d}P_{Md}^{Rw}(t)}{P_{Md-w}(t)} \right)
\]

under usual notations.

iii) Rate of change in the median age of rural female population (Md_{Rf}) at time \( t \):

\[
\frac{dP_{Md}^{Rf}(t)}{dt} = \frac{N_{Md}^{Rf}(t)}{N^{Rf}(t)} \left[ \frac{1}{e_{0}^{Rf}(t) \alpha_{Md}^{Rf}(t) - d_{Md}^{Rf}(t)} \right] + \left[ b_{Md}^{Rf}(t)^{*} - b_{Md}^{Rf}(t)^{*} \right] \frac{N^{Rf}(t)}{N^{Rf}(t)}
\]

(5.55)

and

\[
\frac{dM_{d}^{Rf}(t)}{dt} = - \left( \frac{w_{d}P_{Md}^{Rf}(t)}{P_{Md-w}(t)} \right)
\]

under usual notations.

iv) Rate of change in the median age of rural population (Md_{R}) at time \( t \) (considering rural male-female birth and death rates simultaneously):

The expression for \( \frac{dP_{Md}^{R}(t)}{dt} \) is same as \( \frac{dP_{Md}^{Rf}(t)}{dt} \) replacing suffix 15 by Md.

\[
\frac{dP_{Md}^{R}(t)}{dt} = \frac{N_{Md}^{R}(t)}{N^{R}(t)} \left[ b_{Md}^{Rm}(t)^{*} - \alpha_{Md}^{Rm}(t) - d_{Md}^{Rm}(t) \right] + \frac{N_{Md}^{Rf}(t)}{N^{R}(t)} \left[ b_{Md}^{Rf}(t)^{*} - \alpha_{Md}^{Rf}(t) - d_{Md}^{Rf}(t) \right]
\]

+ \frac{N_{Md}^{R}(t)}{N^{R}(t)} \left[ \frac{1}{e_{0}^{Rm}(t) \alpha_{Md}^{Rm}(t) - d_{Md}^{Rm}(t)} \right] \frac{N_{Md}^{Rm}(t)}{N^{R}(t)} + \frac{1}{e_{0}^{Rf}(t) \alpha_{Md}^{Rf}(t) - d_{Md}^{Rf}(t)} \frac{N_{Md}^{Rf}(t)}{N^{R}(t)} b_{Md}^{Rm}(t)^{*} - b_{Md}^{Rf}(t)^{*}
\]

(5.57)

and

\[
\frac{dM_{d}^{R}(t)}{dt} = - \left( \frac{w_{d}P_{Md}^{R}(t)}{P_{Md-w}(t)} \right)
\]

(5.58)

under usual notations.
5.2.4.3 Overall Population

i) Rate of change in proportion of age median and below \( P_{md}^m \) for male at time \( t \):

\[
\frac{dP_{md}^m(t)}{dt} = \frac{N_{md}^m(t)}{N^m(t)} \left[ \frac{1}{e_0^m(t)} - \alpha_{md}^m(t) - d_{md}^m(t) \right] + \left[ \frac{b_{md}^m(t)}{N^m(t)} - b^m(t)^* \right] \quad (5.59)
\]

under usual notations.

The rate of change in median age of male population \( (Md^m) \) at time \( t \) can be calculated from the relation \( \frac{dP_{md}^m(t)}{dt} \) and their relation is

\[
\frac{dMd^m(t)}{dt} = -w \frac{dP_{md}^m(t)}{dt} \quad (5.60)
\]

under usual notations.

ii) Rate of change in proportion of age median and below \( P_{md}^f \) for female:

\[
\frac{dP_{md}^f(t)}{dt} = \frac{N_{md}^f(t)}{N^f(t)} \left[ \frac{1}{e_0^f(t)} - \alpha_{md}^f(t) - d_{md}^f(t) \right] + \left[ \frac{b_{md}^f(t)}{N^f(t)} - b^f(t)^* \right] \quad (5.61)
\]

under usual notations.

The rate of change in median age of male population \( (Md^f) \) at time \( t \) can be calculated from the relation \( \frac{dP_{md}^f(t)}{dt} \) and their relation is

\[
\frac{dMd^f(t)}{dt} = -w \frac{dP_{md}^f(t)}{dt} \quad (5.62)
\]

under usual notations.

iii) Rate of change in proportion of age median and below for person at time \( t \):

\[
\frac{dP_{md}(t)}{dt} = \frac{N_{md}(t)}{N(t)} \left[ \frac{1}{e_0(t)} - \alpha_{md}(t) - d_{md}(t) \right] + \left[ \frac{b_{md}(t)}{N(t)} - b(t)^* \right] \quad (5.63)
\]

under usual notations.

171
The rate of change in Median (Md) can be solved from $\frac{dP_{\text{Md}}(t)}{dt}$ and for this Liao (1996) used the following relation

$$ \frac{dM_d(t)}{dt} \approx \frac{w dP_{\text{Md}}(t)}{P_{\text{Md}-w}} \quad (5.64) $$

where $w$ is the width of the age group, particularly the one immediately below median and $P_{\text{Md}-w}$ is the proportion of the population that falls into the age immediately below median.

iv) Rate of change in the proportion of age median and below ($P_{\text{Md}}$) for person at time $t$ (considering male-female birth and death rates simultaneously):

The rate of change in the proportion of median and below for person ($P_{\text{Md}}$) considering male-female birth and death rates simultaneously can be derived by the same procedure of that $\frac{dP_{\text{s}}(t)}{dt}$.

$$ \frac{dP_{\text{Md}}(t)}{dt} = \frac{N_{\text{Md}}^m(t)}{N(t)} \left[ b_m^m(t) - a_m^m(t) - d_m^m(t) \right] + \frac{N_{\text{Md}}^f(t)}{N(t)} \left[ b_m^f(t) - a_m^f(t) - d_m^f(t) \right] + \frac{N_{\text{Md}}(t)}{N(t)} \left[ \frac{1}{e^m(t)} \cdot \frac{N^m(t)}{N(t)} + \frac{1}{e^f(t)} \cdot \frac{N^f(t)}{N(t)} - b^m(t)^* - b^f(t)^* \right] \quad (5.65) $$

under usual notations.

And the rate of change of median age is

$$ \frac{dM_d(t)}{dt} = \frac{-w \cdot \frac{dP_{\text{Md}}(t)}{dt}}{P_{\text{Md}-w}} \quad (5.66) $$

v) Rate of change in the proportion of age median and below ($P_{\text{Md}}$) for person at time $t$ (considering urban and rural birth and death rates simultaneously):

$$ \frac{dP_{\text{Md}}(t)}{dt} = \frac{N_{\text{Md}}^u(t)}{N(t)} \left[ b_m^u(t) - a_m^u(t) - d_m^u(t) \right] + \frac{N_{\text{Md}}^r(t)}{N(t)} \left[ b_m^r(t) - a_m^r(t) - d_m^r(t) \right] $$
\[ + \frac{N_{ms}(t)}{N(t)} \left[ \frac{1}{e_v(t)} \cdot \frac{N^n(t)}{N(t)} + \frac{1}{e^s(t)} \cdot \frac{N^s(t)}{N(t)} - b^n(t) \cdot \frac{N^u(t)}{N(t)} - b^s(t) \cdot \frac{N^g(t)}{N(t)} \right] \] (5.67)

and

\[ \frac{dMd(t)}{dt} = -\left( \omega dP_{ms}(t) \right) \quad \frac{P_{ms}}{P_{ms-\nu}(t)} \] (5.68)

Under usual notations.

vi) Rate of change in the proportion of age median and below (PMd) for person at time t (considering urban-rural, male-female birth and death rates simultaneously):

\[ \frac{dP_{ms}(t)}{dt} = \frac{N_{ms}(t)}{N(t)} \left[ \left( \frac{1}{e_v(t)} - a_{ms}(t) - d_{ms}(t) \right) + \left[ \frac{1}{e_0(t)} - \left( b^n(t) + b^s(t) \right) \right] \right] \]

\[ = \frac{N_{ms}(t)}{N(t)} \left[ b_{ms}(t) - a_{ms}(t) - d_{ms}(t) \right] + \frac{N_{ms}(t)}{N(t)} \left[ \frac{1}{e_0(t)} - \left( b^n(t) + b^s(t) \right) \right] \]

\[ = \frac{N_{ms}^{um}(t)}{N(t)} \left[ b_{ms}^{um}(t) - a_{ms}^{um}(t) - d_{ms}^{um}(t) \right] + \frac{N_{ms}^{uf}(t)}{N(t)} \left[ b_{ms}^{uf}(t) - a_{ms}^{uf}(t) - d_{ms}^{uf}(t) \right] \]

\[ + \frac{N_{ms}^{bn}(t)}{N(t)} \left[ b_{ms}^{bn}(t) - a_{ms}^{bn}(t) - d_{ms}^{bn}(t) \right] + \frac{N_{ms}^{bf}(t)}{N(t)} \left[ b_{ms}^{bf}(t) - a_{ms}^{bf}(t) - d_{ms}^{bf}(t) \right] \]

\[ + \frac{N_{Ms}(t)}{N(t)} \left[ \frac{1}{e_v(t)} \cdot \frac{N^n(t)}{N(t)} + \frac{1}{e^s(t)} \cdot \frac{N^s(t)}{N(t)} + \frac{1}{e_v(t)} \cdot \frac{N^u(t)}{N(t)} + \frac{1}{e^s(t)} \cdot \frac{N^g(t)}{N(t)} \right] \]

\[ \frac{dMd(t)}{dt} = -\left( \omega dP_{ms}(t) \right) \quad \frac{P_{ms}}{P_{ms-\nu}(t)} \] (5.70)

under usual notations.
5.2.5 Measures Related to Mean Age ($A_p$)

5.2.5.1 Urban Population

i) Rate of change in the mean age of urban population ($A_p^u$) at time $t$:

$$\frac{dA_p^u(t)}{dt} = 1 - \frac{1}{e^u_0(t)} \left[ A_p^u(t) - A_p^u(t) \right] - \left[ b^u(t)^* - A_p^u(t) \right]$$

(5.71)

under usual notations.

ii) Rate of change in the mean age of urban male population ($A_p^{um}$) at time $t$:

$$\frac{dA_p^{um}(t)}{dt} = 1 - \frac{1}{e^{um}_0(t)} \left[ A_p^{um}(t) - A_p^{um}(t) \right] - \left[ b^{um}(t)^* - A_p^{um}(t) \right]$$

(5.72)

under usual notations.

iii) Rate of change in the mean age of urban female population ($A_p^{uf}$) at time $t$:

$$\frac{dA_p^{uf}(t)}{dt} = 1 - \frac{1}{e^{uf}_0(t)} \left[ A_p^{uf}(t) - A_p^{uf}(t) \right] - \left[ b^{uf}(t)^* - A_p^{uf}(t) \right]$$

(5.73)

under usual notations.

iv) Rate of change in the mean age of urban population ($A_p^u$) at time $t$ (considering urban male-female birth and death rates simultaneously):

$$\frac{dA_p^u(t)}{dt} = 1 - A_p^u(t) \left[ b^{um}(t)^* + b^{uf}(t)^* \right] + \frac{1}{e^{um}_0(t)} \left[ A_p^u(t) - A_p^{um}(t) \cdot \frac{N^{um}(t)}{N^u(t)} \right] + \frac{1}{e^{uf}_0(t)} \left[ A_p^u(t) - A_p^{uf}(t) \cdot \frac{N^{uf}(t)}{N^u(t)} \right]$$

(5.74)

under usual notations.
5.2.5.2 Rural Population

i) Rate of change in the mean age of rural population \( A^R_p \) at time \( t \):

\[
\frac{dA^R_p(t)}{dt} = 1 - \frac{1}{e^R(t)} \left[ A^R_D(t) - A^R_p(t) \right] - \left[ b^R(t)^* - A^R_p(t) \right]
\]

(5.75)

under usual notations.

ii) Rate of change in the mean age of rural male population \( A^R_{pm} \) at time \( t \):

\[
\frac{dA^R_{pm}(t)}{dt} = 1 - \frac{1}{e^R_{pm}(t)} \left[ A^R_{pm}(t) - A^R_p(t) \right] - \left[ b^R_{pm}(t)^* - A^R_p(t) \right]
\]

(5.76)

under usual notations.

iii) Rate of change in the mean age of rural female population \( A^R_{pf} \) at time \( t \):

\[
\frac{dA^R_{pf}(t)}{dt} = 1 - \frac{1}{e^R_{pf}(t)} \left[ A^R_{pf}(t) - A^R_p(t) \right] - \left[ b^R_{pf}(t)^* - A^R_p(t) \right]
\]

(5.77)

under usual notations.

iv) Rate of change in the mean age of rural population \( A^R_p \) at time \( t \) (considering rural male-female birth and death rates simultaneously):

\[
\frac{dA^R_p(t)}{dt} = 1 - A^R_p(t) \left[ b^R_{pm}(t)^* + b^R_{pf}(t)^* \right] + \frac{1}{e^R(t)} \left[ A^R_D(t) - A^R_p(t) \right] \cdot \left( \frac{N^R_{pm}(t)}{N^R(t)} \right) + \frac{1}{e^R(t)} \left[ A^R_D(t) - A^R_p(t) \right] \cdot \left( \frac{N^R_{pf}(t)}{N^R(t)} \right)
\]

(5.78)

under usual notations.
5.2.5.3 Overall population

i) Rate of change in the Average age of male ($A^m_p$) at time $t$:

$$\frac{dA^m_p(t)}{dt} = 1 - \frac{1}{e^m_o(t)} \left[ A^m_p(t) - A^m_p(t) \right] - b^m(t) \ast A^m_p(t)$$  \hspace{1cm} (5.79)

under usual notations.

ii) Rate of change in the Average age of female ($A^f_p$) at time $t$:

$$\frac{dA^f_p(t)}{dt} = 1 - \frac{1}{e^f_o(t)} \left[ A^f_p(t) - A^f_p(t) \right] - b^f(t) \ast A^f_p(t)$$  \hspace{1cm} (5.80)

under usual notations.

iii) Rate of change in the Average age of person ($\bar{A}_p$) at time $t$:

$$\frac{d\bar{A}_p(t)}{dt} = 1 - \frac{1}{e_o(t)} \left[ A_o(t) - A_p(t) \right] - b(t) \ast \bar{A}_p(t)$$  \hspace{1cm} (5.81)

iv) Rate of change in the Average age of person ($\bar{A}_p$) at time $t$ (considering male-female birth and death rates simultaneously):

$$\frac{d\bar{A}_p(t)}{dt} = 1 - A_p(t) \left[ b^m(t) \ast + b^f(t) \ast \right] + \frac{1}{e^m_o(t)} \left[ A_p(t) - A^m_p(t) \frac{N^m(t)}{N(t)} \right]$$  

$$+ \frac{1}{e^f_o(t)} \left[ A_p(t) - A^f_p(t) \frac{N^f(t)}{N(t)} \right]$$

\hspace{1cm} (5.82)

under usual notations.

v) Rate of change in the Average age of person ($\bar{A}_p$) at time $t$ (considering urban and rural birth and death rates simultaneously):

$$\frac{d\bar{A}_p(t)}{dt} = 1 - A_p(t) b(t) \ast - \frac{1}{e_o(t)} \left[ A_o(t) - A_p(t) \right]$$

$$= 1 - A_p(t) b(t) \ast + \frac{1}{e_o(t)} \left[ A_p(t) - A_o(t) \right]$$

176
\begin{align}
\frac{dA_p(t)}{dt} &= 1 - A_p(t).b(t) + \frac{1}{e_0(t)} \left[ A_p(t) - A_D(t) \right] \frac{N^r(t)}{N(t)} \\
&= 1 - A_p(t) \left[ b^m(t) + b^f(t) \right] \frac{N^r(t)}{N(t)} + \left[ b^m(t) + b^f(t) \right] \frac{N^r(t)}{N(t)} \\
&\quad + \frac{1}{e^{mm}_0(t)} \left[ A_p(t) - A^{mm}_D(t) \right] \frac{N^{mm}(t)}{N(t)} + \frac{1}{e^{mf}_0(t)} \left[ A_p(t) - A^{mf}_D(t) \right] \frac{N^{mf}(t)}{N(t)} \\
&\quad + \frac{1}{e^{mm}_0(t)} \left[ A_p(t) - A^{mm}_D(t) \right] \frac{N^{mm}(t)}{N(t)} + \frac{1}{e^{mf}_0(t)} \left[ A_p(t) - A^{mf}_D(t) \right] \frac{N^{mf}(t)}{N(t)}
\end{align}

under usual notations.

vi) Rate of change in the Average age of person \( A_p \) at time \( t \) (considering urban-rural, male-female birth and death rates simultaneously):

\begin{align}
\frac{dA_p(t)}{dt} &= 1 - A_p(t).b(t) + \frac{1}{e_0(t)} \left[ A_p(t) - A_D(t) \right] \\
&= 1 - A_p(t) \left[ b^m(t) + b^f(t) \right] \frac{N^r(t)}{N(t)} + \left[ b^m(t) + b^f(t) \right] \frac{N^r(t)}{N(t)} \\
&\quad + \frac{1}{e^{mm}_0(t)} \left[ A_p(t) - A^{mm}_D(t) \right] \frac{N^{mm}(t)}{N(t)} + \frac{1}{e^{mf}_0(t)} \left[ A_p(t) - A^{mf}_D(t) \right] \frac{N^{mf}(t)}{N(t)} \\
&\quad + \frac{1}{e^{mm}_0(t)} \left[ A_p(t) - A^{mm}_D(t) \right] \frac{N^{mm}(t)}{N(t)} + \frac{1}{e^{mf}_0(t)} \left[ A_p(t) - A^{mf}_D(t) \right] \frac{N^{mf}(t)}{N(t)}
\end{align}

under usual notations.

5.2.6 Literature on Calculating Alternative Birth and Death Rates

The alternative death rates have been calculated from life table i.e., life expectancy of person. The rates are:

\begin{equation}
l(t) = \frac{1}{e_0(t)}
\end{equation}

Where \( e_0(t) \) is the life expectancy at birth at time \( t \). Here, \( l(t) \) is also known as life table death rate. And

\begin{equation}
l_{oo}(t) = \frac{1}{e_{oo}(t)}
\end{equation}

177
Where \( e_{60}(t) \) is the remaining life expectancy of a person aged 60 at time \( t \).

Alternative birth rates have been calculated from its functional relationship with \( \text{NRR} \), \( r \) and \( T \). The relations are stated below:

A fundamental equation of population dynamics is

\[
    r(t) = b(t) - d(t) \tag{5.87}
\]

In which \( r(t) \) is the "instantaneous" per capita growth rate, \( b(t) \) is the per capita birth rate, and \( d(t) \) is the per capita death rate.

Bertram and Murray (1997) proposed the following relationship

\[
    e^{r(t)} = 1 + b(t) - d(t) \tag{5.88}
\]

We will use \( d(t) = \frac{1}{e_{60}(t)} \). Now we need to find out the value of \( r(t) \), intrinsic rate at time \( t \). For this we use an approximation formula given by Coale (1955) and the approximation is

\[
    \text{NRR} = e^{r(t)T} \tag{5.89}
\]

Where \( \text{NRR} \) is the net reproduction rate, \( r \) is the intrinsic increase and \( T \) is the mean length of generation.

The formula in equation (5.89) is exact and it is originally found in Dublin and Lotka's (1925) article "On the True Rate of Natural Increase".

Again, \( T \) is very close to the average age of childbearing of female population for stationary population (Keyfitz, 1971). Here, \( T \) is assuming as average age of childbearing of female population and this is calculated from the age-specific fertility rate of Bangladesh female population. Therefore,

\[
    e^{r(t)} = \frac{\text{NRR}}{T} \tag{5.90}
\]
Therefore, birth rate \( b(t) \) has been calculated from equation (5.88) after putting the value of \( e^{t_0} \) from equation (5.90) and \( d(t) \) from equation (5.85).

Coale (1955) pointed out that neither rates nor life table values should be considered as predictions—especially when the fertility and mortality rates are highly volatile—but both are useful adjunct to crude vital rates.

Since Preston et al. (1989) and Liao (1996) presented aging measures as a function of fertility and mortality i.e., crude birth rate (CBR) and crude death rate (CDR), hence we can use the alternative birth and death rates for aging measures as a function of fertility and mortality.

After calculating \( b(t) \) for nation, urban and rural areas we have computed sex specific birth rate adjusting with the sex ratio at birth. Similarly age specific birth rate for example, birth rate for person under fifteen and person age at or below median, which are not actually the demographic birth rates but used for calculating the rate of change aging measures, have been obtained.

Therefore our alternative aging measures can be treated as a measure of aging which is a function of fertility and life expectancy of a person. Since both fertility (function of NRR and \( r \)) and life expectancy are the improved measures of CBR and CDR hence we can expect our proposed measure will be better theoretically.

### 5.3 Materials of the Study

The measures discussed above have been applied to the Bangladesh census population of 1981 and 2001 to measure the speed of aging process and a comparison is made among these measures. The census data have been collected from the Bangladesh Bureau of Statistics (BBS, 2003). Various demographic rates
have also been collected from other BBS publication (BBS, 2004). Sex specific birth rates have been calculated using the sex ratio at birth. Age specific death rates and average age at death have been computed from the distribution of age specific death rate by age, sex and locality, 1980 and 2001. Normally various rates are expressed in terms of per 100 or per 1000 populations. However, for the ease of calculation these rates are presented per unit population in our study.

5.4 Results and Discussion

This chapter intends to measure of population aging as a function of fertility and life expectancy, alternative to fertility and mortality measures used in previous chapter, of a person. The rate of change of conventional aging measures alternative to those presented in chapter IV have been proposed and applied to observe the greying process of Bangladesh population for two census years 1981 and 2001. We subdivide the entire population by region (urban-rural) and sex. Overall as well as group wise aging measures for each partitioning of the population have been computed for two census periods. A comparison has been made among the different formulae proposed here. Further, the discrepancy has also been observed between the results of this chapter with those of previous chapter. This study tries to find out the best method of measuring rate of change of aging process with respect to the demographic components. In this chapter all the measures have been calculated for closed population of Bangladesh. Going to the discussion of aging process with results of this chapter we first throw light on fertility and life expectancy status of the Bangladesh population at 1981 and 2001.
5.4.1 Fertility Pattern

Though fertility pattern over these two census periods (1981 and 2001) has been discussed in chapter IV, an additional effort has been given here to display this pattern with alternative measures computed in this chapter. Alternative crude birth rate was 41.67 per thousand persons in 1981 and in 2001 it reached down to 23.12 (Table 5.1). The rates in both two years are higher than those of crude birth rate (CBR). The decreasing rate over the two decades with alternative birth rate is higher (55.48 percent) than those of CBR (45 percent). Like CBR, a huge urban-rural gap is found with the alternative measures for both 1981 and 2001. These measures indicate a wider urban-rural gap than CBR. Again, sex specific birth rate is different and male births are outnumbered than female birth. Sex specific birth rate is decreasing and the decreasing rate in male birth is higher than those of female birth with alternative measures which is opposite to measure with CBR.

Net reproduction rate (NRR) is a powerful measure of fertility behaviour. NRR declined from 1.89 to 1.23 between 1981 and 2001 (Table 5.2). Urban fertility situation is better than that of rural with respect to NRR for both the time period though the decreasing rate of NRR in urban area is slower (60 percent) than that of rural (71 percent) between 1981 and 2001. With this process we see that the urban NRR is below replacement level with negative population growth rate in 2001 (Table 5.2). Again, the average child bearing age of female population is about 28 years in both 1981 and 2001.

5.4.2 Life Expectancy Pattern

Life expectancy at birth of a person in Bangladesh was 54.8 and 64.2 years at 1981 and 2001 respectively. The corresponding life expectancy at age sixty was
16.10 and 16.44 years (Table 5.1). Over the two decades average longevity of a person has been increased by 10 years whereas remaining life expectancy at age sixty increased slightly. Average longevity of an urban person is higher than that of rural in both 1981 and 2001. Life expectancy at age sixty shows the similar pattern in urban and rural areas. A narrow gender gap is found in life expectancy at birth for both the years. In 1981, male life expectancy is higher than that of female but female life expectancy exceeds to their male counterpart at 2001. The behaviour of the life expectancy at age sixty is almost similar to that of life expectancy at birth with respect to sex. From this discussion, we can conclude that Bangladesh has earned positive gain in life expectancy at birth over the period 1981-2001 though the gain in life expectancy at older age is not satisfactory.

5.4.3 Aging through Alternative Demographic Components

In this chapter, rate of change of various conventional aging measures have been computed with alternative birth rate and life expectancy of a person. Like previous chapter (Chapter IV) four types of formulae have been applied to obtain the rate of change of aging measures of Bangladesh population for 1981 and 2001. These measures indicate per year change i.e., 1980-1981 and 2000-2001. The basic considerations of these formulae are:

(a) First: Overall life expectancy and alternative birth rate of population (equations 5.11, 5.25, 5.39, 5.64 and 5.81).

(b) Second: Life expectancy and alternative birth rates for male and female population simultaneously (equations 5.12, 5.26, 5.40, 5.66 and 5.82).

(c) Third: Life expectancy and alternative birth rates for urban and rural population simultaneously (equations 5.13, 5.27, 5.41, 5.68 and 5.83).
(d) Fourth: Life expectancy and alternative birth rates for region (urban-rural) and sex (male-female) simultaneously (equations 5.14, 5.28, 5.42, 5.70 and 5.84).

Here aging process of Bangladesh with sub-division of population with respect to region and sex has been observed under the above mentioned considerations for 1980-1981 and 2000-2001. A comparative discussion has been made among the different formulae with alternative measures and results with previous chapter (chapter IV). Again, a discussion for proposing the best method of calculating rate of change of conventional aging measures has also been performed.

5.4.3.1 Aging Process of Bangladesh Population

Various alternative measures of the rate of change in aging process for Bangladesh population have been presented in Table 5.3. The per year change of proportion of person aged 60 or more ($P_{60}$) according to the first, second, third and fourth formulae were - 0.00118, - 0.00105, - 0.00246 and - 0.00111 respectively at 1981. The corresponding figures were - 0.00042, - 0.00039, - 0.00046 and - 0.00027 in 2001. Four formulae show the same direction of decreasing peak aging with slightly different values. The current values are smaller than those of calculated in chapter IV having the same direction. All the measures show a declining peak aging process having a very slow pace and in 2001 it is almost zero. Therefore, it can be said that the greying process of Bangladesh population is more or less stable at 2001 according to the peak aging measures with alternative approach.
Rates of change of base aging \( \left( \frac{dp_{15}(t)}{dt} \right) \) in 1981 were - 0.02021, - 0.00988, - 0.01734 and - 0.02048 according to the four formulae under consideration. In 2001, the corresponding figures were - 0.01604, - 0.01977, - 0.00292 and - 0.01639. All the measures indicate a decreasing young population for both 1981 and 2001 with different magnitudes. In 1981, formula with the fourth consideration showed the highest decreasing rate while formula with the third consideration showed the fastest decreasing process in 2001. Decreasing rate was slower in 2001 than those of 1981 with respect to the first and fourth formulae but reverse picture was found with the second and third formulae between these two time periods with similar pattern of those of chapter IV. Again, alternative measures show slower decreasing process than those of calculated in chapter IV for both 1981 and 2001 in most of the cases with a very few exceptions. From the study it is evident that Bangladesh population is becoming aging according to aging at base.

According to the alternative rate of change of aged-child ratio (R), we observe an increasing aging process in the country at both 1981 and 2001. The values of these measures were 0.0027, 0.00265, 0.00292 and 0.00293 corresponding to four formulae at 1981. In 2001, the corresponding values were 0.00529, 0.00520, 0.00584 and 0.00581. All the formulae under consideration indicate more or less similar aging process at both 1981 and 2001 where aging process was faster in 2001 than those of 1981. Alternative measures show a faster aging process in 1981 than those of measures in chapter IV at the same year. On the other hand, in 2001 the pattern was opposite i.e., at that time alternative measures indicate a slower process than those of measures in previous chapter.
(chapter IV). Considering all the rate of change measures of aged-child ratio (R), it can be concluded that the Bangladesh population is greying over time.

Alternative rate of change of median age were - 0.05, 0.26, 0.40 and 0.29 according to four formulae under consideration respectively. In 2001, the figures were, 0.43, 0.55, 0.51 and 0.43 respectively. Most of the measures show an increasing aging process in both 1981 and 2001 though the difference in magnitude among various formulae of this process is mentionable. The increasing rate was significantly higher in 2001 than 1981. Again, the alternative measures indicate a slower increasing aging process than those of calculated in chapter IV at both 1981 and 2001. According to the rate of change of median age of population (both previous and alternative) it is evident that the Bangladesh population is becoming aged over time.

Like the measures of chapter IV, the rate of change of mean age (A_p) with alternative measures give confusing results. Among the four formulae (a, b, c, d; p.182-183) some of them show increasing aging process but some of them indicate decreasing process at both 1981 and 2001. Only the fourth formula, considering sex and location specific fertility and life expectancy simultaneously, show an increasing aging process at both 1981 and 2001 and those are consistent with those measure of median. According to this measure, aging process is faster at 2001 than that of 1981.

With the above discussions on alternative rate of change of conventional aging measures we observe that all of them indicate an upward aging process of Bangladesh population except for those of regarding on the peak aging. The results in this chapter are consistent with those of chapter IV in direction having slightly
different in magnitudes where alternative measures indicate a relatively slow pace of aging process.

### 5.4.3.1.1 Gender Differences

Alternative rate of change of conventional aging measures at national level with respect to sex for 1981 and 2001 have been presented in Table 5.4. According to the measure of peak aging \( \frac{dp_{m}^{a}(t)}{dt} \) for male and female population, we observe a decreasing aging process at both 1980-1981 and 2000-2001 but in 2001 the values were almost zero. Therefore, the greying process of Bangladesh with respect to sex is more or less stable in terms of speed of peak aging measure with alternative approach.

The values of the rate of change of proportion of young male and female population, \( \frac{dp_{m}^{y}(t)}{dt} \) and \( \frac{dp_{f}^{y}(t)}{dt} \), were -0.02179 and -0.0181 respectively at 1981. The corresponding figures were -0.01682 and -0.0148 in 2001. In both the years young population was decreasing with respect to sex and young male decreasing rate was higher than that of their female counterpart. The decreasing process is slower in 2001 than 1981. Again, the alternative measures indicate a slower aging process at base than those of calculated in chapter IV though the direction of the process is same. So, with these measures we see a clear base aging process with respect to sex in Bangladesh population where male aging is faster than female aging.

Alternative measures of velocity in aged-child ratio \( \frac{dR(t)}{dt} \) with respect to sex also indicate an increasing aging process. The values of these measures were
0.00371 and 0.00164 for male and female respectively at 1981. The corresponding values were 0.00556 and 0.00483 at 2001. The aging process is faster in 2001 than 1981. Alternative measures show a faster aging process at 1981 than those of chapter IV but in 2001 we see a reversed process.

The speed of changing median age \( \left( \frac{dM(t)}{dt} \right) \) with alternative measures indicates an increasing aging process of Bangladesh with respect to sex. In 1981, the values were 0.36 and 0.30 for male and female respectively. The corresponding figures were 0.30 and 0.58 in 2001. Female aging was faster (almost double) than that of male in 2001 though the picture was opposite in 1981. More speed in female aging was observed in 2001 compared to 1981 but the pattern of speed in male aging was different between these two time periods. Again, the alternative measures indicate a slower process of aging with respect to sex than those of computed in chapter IV but the direction in the gender gap on the process is same with both the methods (previous and alternative).

In 2001, we observe positive aging process for male and female population of Bangladesh with the alternative measures of rate of change of mean age \( \left( \frac{dA_{\mu}(t)}{dt} \right) \). These measures indicate slightly decreasing aging process with respect to sex at 1981. The gender gap is not visible with these measures. Surprisingly a huge gap is found between alternative measure in mean age and measures presented in chapter IV where alternative measures show a very slow aging process. From the alternative rate of change of average age (median and mean) it can be said that both male and female population of Bangladesh are greying over time.
5.4.3.2 Urban Aging

Alternative rate of change of conventional aging measures for urban population of Bangladesh have been presented in Table 5.5. Various alternative measures have been calculated here under two considerations. The first one is based on the life expectancy (at birth and at age 60) and alternative birth rates for urban population while the second one is based on simultaneous considerations of these demographic components for male and female. In our study, we name these two considerations as method-1 (equations 5.1, 5.15, 5.29, 5.44, 5.71) and method-2 (equations 5.4, 5.18, 5.32, 5.50, 5.74) respectively. These two methods are basically alternative measures of Liao (1996) and Nath-Deka (2006) respectively for urban population.

Speed of peak aging in urban areas \( \left( \frac{dp_{15}^u(t)}{dt} \right) \) in 1981 were - 0.00016 and 0.00013 with method-1 and method-2 respectively. The corresponding values in 2001 were 0.00103 and 0.00161. These indicate a gradually increasing aging process in 2001 though in 1981 the process was almost stable. Method-2 indicates slightly faster aging process than method-1. Again, alternative measures show faster aging process than those of chapter IV. Therefore, it can be concluded that the Bangladesh urban population is greying gradually over time though national aging at peak is more or less stable.

According to the rate of change of urban base aging with alternative measure \( \left( \frac{dp_{15}^u(t)}{dt} \right) \) we observe significantly decrease in urban young population at both 1981 and 2001. The values of these measures for method-1 and method-2 were - 0.02069 and - 0.0261 respectively at 1981 with corresponding values...
- 0.0223 and - 0.0454 in 2001. Method-2 reveals faster base aging process than method-1 in 2001 though there is no significant difference between these two methods in 1981. The decreasing young population was faster in 2001 than that of 1981 with both the methods. Again, the alternative measures show a faster base aging in 2001 than those of chapter IV though in 1981 the process was opposite.

The alternative measures of aging velocity in urban population with aged-child ratio \( \left( \frac{dR^*(t)}{dt} \right) \) for 1981 were 0.00543 and 0.00544 with method-1 and method-2 respectively. The corresponding figures in 2001 were 0.01212 and 0.01261. These measures indicate a clear urban population aging in Bangladesh where year 2001 experienced faster aging process than 1981. Method-1 and method-2 express almost similar aging process. Again, alternative measures reveal slightly faster urban aging than those of computed in chapter IV in 2001 but the scenario was opposite in 1981.

The speed of change of urban median age \( \left( \frac{dM^u(t)}{dt} \right) \) express the same aging process as \( \left( \frac{dR^*(t)}{dt} \right) \). Median ages were increased by 0.66 and 0.73 year with alternative method-1 and method-2 respectively at 1980-1981. The corresponding figures were 0.84 and 1.11 at 2000-2001. Alternative measures show a slower urban aging process than those in chapter IV except for 2001 with method-2.

The alternative rate of change of mean age of urban population \( \left( \frac{dA^u(t)}{dt} \right) \) supports the aging process with those measures of median age for both 1981 and
2001 except for the measure with method-1 in 1981. Increasing rate of mean age was lower than median age. Again, alternative measures regarding with the mean age show a slower urban aging process than those of chapter IV in most of the cases. According to the speed of changing median and mean age, it is evident that urban population of Bangladesh is greying over time.

Combining all the alternative rate of change of conventional aging measures we find a clear urban aging process though magnitude of the process is different among various measures.

5.4.3.2.1 Gender Differences

Table 5.6 represents the alternative rate of change of conventional aging measures for urban male and female population. Alternative old-age aging measures \( \left( \frac{dP_{oe}(t)}{dt} \right) \) with respect to sex show the similar pattern of overall urban population i.e., almost stable in 1981 and slightly increasing in 2001. Male elderly increasing rate is higher than that of female. Again, the alternative measures show a faster peak aging process than those of computed in chapter IV. From these results, it can be said that urban male and female population are greying over time with a very slow pace.

Urban young male and female are significantly decreasing over time with the alternative rate of change of proportion of children under fifteen \( \left( \frac{dP_{15}(t)}{dt} \right) \). The values of these measures for male and female population were - 0.01943 and - 0.02245 respectively in 1981 with corresponding figures - 0.02161 and - 0.03302 in 2001. Female young decreasing rate is slightly higher than those of male in both
the years. The decreasing process is faster in 2001 than that of 1981. Again, the alternative measures with respect to sex show a slower base aging process than those of chapter IV in 1981 whereas in 2001 the picture is opposite i.e., urban base aging process is comparatively high with alternative measures. From the above discussions we get a clear indication of urban base aging.

The speed of changing aged-child ratio in urban areas with respect to sex, \( \frac{dR^{males}}{dt} \) and \( \frac{dR^{females}}{dt} \), were 0.00719 and 0.00372 respectively in 1981 with the alternative approach. The corresponding figures were 0.0137 and 0.0104 in 2001. These measures indicate a clear urban aging process with respect to sex in Bangladesh where male aging is faster than that of female in both 1981 and 2001. Again, alternative measures show a faster aging process for urban male and female persons than those of computed in chapter IV at 2001 but in 1981 the pace was reversed.

We get a clear indication of urban aging with respect to sex in both 1981 and 2001 with the alternative rate of change of median age. The values of \( \frac{dMd^{males}}{dt} \) and \( \frac{dMd^{females}}{dt} \) were 0.72 and 0.47 in 1981 with 0.82 and 0.89 in 2001. Urban female aging was faster than that of male in 2001 but it was opposite in 1981. Again, the alternative measures show a slower aging process in urban areas with respect to sex in both 1981 and 2001 than those of chapter IV. Another alternative measure of changing the mean age of population also supports the claim with that of median age. These measures exhibit a slower urban aging process with respect to sex than those of measures regarding on the median age at both 1981 and 2001. These measures indicate a faster urban male aging than that of female for
both 1981 and 2001. Again, the aging process is faster in 2001 than that of 1981 with these measures. Alternative measures regarding on the mean age show the similar pattern with those of computed in chapter IV. From the discussions on urban aging with respect to sex, it can be concluded that the urban male and female population are greying over time with no visible gender gap in this process.

5.4.3.3 Rural Aging

Rural aging of Bangladesh has also been studied with alternative approach in this chapter. These measures are presented in Table 5.7. Alternative measures have been computed by the two methods, method-1 and method-2, similar with those of mentioned earlier in the urban aging process.

Rural old-age aging measures with method-1 and method-2 indicate slow decreasing rate in 1981 and almost stable in 2001 though the sign of measures in this year is negative. The peak aging in rural areas is not clear with alternative measures.

The values of the rate of change of proportion of young in rural areas, \(\frac{dp_y(t)}{dt}\), with alternative measures were -0.0206 and -0.00091 in 1981 for method-1 and method-2 respectively. In 2001, the corresponding figures were -0.0149 and -0.0135. In both the years, 1981 and 2001, we observe a clear decreasing rate of young rural population and it is faster in 2001 than 1981. Method-1 indicates a faster decreasing process than that of method-2 for both the years. Again, the alternative measures show a slower base aging than those of chapter IV in 1981 but in 2001 the picture is opposite. From these measures, it can be concluded that the rural population is becoming aging over time.
Aged-child ratio in rural areas showed an increasing trend in 1980-1981 and 2000-2001 with alternative measures. The yearly increase in the aged-child ratio was 0.0025 for both method-1 and method-2 in 1981 with figures 0.0041 and 0.0040 in 2001. Again, the alternative measures indicate a slower aging process in 2001 than those of chapter IV though this was opposite in 1981. From these results, it can be concluded that the rural population is becoming greying according to this alternative measure.

The yearly increase in rural median age for method-1 and method-2 in 1981 were 0.29 and 0.20 respectively with alternative measures. The corresponding values in 2001 were 0.42 and 0.48. Per year increasing rate was remarkably faster (almost double) in 2001 than those of 1981. Again, the alternative measures reveal a slower increasing median ages of rural population than those of computed in chapter IV. The rate of change of mean age of rural population \( \frac{dA^E_p(t)}{dt} \) with alternative measures was confusing in both 1981 and 2001. Method-1 indicates the decreasing mean age in 1981 and 2001 whereas method-2 reveals the increasing mean age in both the years. Per year increasing rate of the mean age was slower than that of median age with method-2. Combining the alternative rate of change measures of median and mean age, we get a clear aging process of rural Bangladesh.

5.4.3.3.1 Gender Differences

Various alternative measures for the rural population with respect to sex for 1981 and 2001 have been presented in Table 5.8. According to the measure of peak aging it is found that the proportion of rural male and female elderly was
decreasing in both 1981 and 2001 where this decreasing tendency is almost negligible at 2001. There was no visible gender gap in 1981 but in 2001 this gap is mentionable. Alternative measures show more stable peak aging process of rural population with respect to sex than those of chapter IV. From these results, we can conclude that the peak aging process of Bangladesh rural population with respect to sex is not clear.

Yearly changes in the proportion of rural male and female young population with alternative measures in 1981 were -0.0231 and -0.0180 respectively. The corresponding values of 2001 were -0.0163 and -0.0136. There is a faster young male population decreasing than that of female. The decreasing rate was slower in 2001 for both young male and female population than that of 1981. Again, the alternative measures indicate a slower base aging process of rural population with respect to sex than those of measures in chapter IV in both 1981 and 2001. With these measures regarding on the base aging it is evident that the rural aging with respect to sex is obvious though its peak aging process is not clear.

According to the rate of change of aged-child ratio \( \left( \frac{dR(t)}{dt} \right) \) with alternative approach, it is observed that aging process of rural male and female population was increasing in both 1981 and 2001. The values of these measures for rural male and female population were 0.0035 and 0.0016 respectively in 1981 with corresponding figures 0.0041 and 0.0040 in 2001. The male aging was faster than that of female for both the years having narrow gender gap in 2001 compared to 1981. Again, the alternative measures indicate a faster aging process of rural population with respect to sex in 1981 than those of chapter IV with an opposite
direction in 2001. We observed a clear rural aging process with respect to sex with these alternative measures.

Per year increase of the median age of rural male and female population with alternative measures in 1981 were 0.28 and 0.25 respectively. The corresponding values were 0.422 and 0.424 in 2001. The gender gap is not visible in both the years with these measures. The rural aging process was faster in 2001 than that of 1981 with these alternative measures. Again, the alternative measure of speed of changing of median age indicates a slower rural aging process with respect to sex in both 1981 and 2001 than that of chapter IV. Alternative rate of change of mean age for rural male and female population gave inconsistent results with those of median age in 1981 and consistent in 2001 though the magnitude was very low in that year. Combining all the alternative rate of change of conventional aging measures with respect to sex it can be concluded that both the male and female population of rural Bangladesh are greying over time.

5.4.3.4 Urban-Rural Differences

Alternative rate of change of conventional aging measures for urban and rural population have been presented in Tables 5.5 to 5.8. Mentionable urban-rural gap has been observed in both 1981 and 2001. Urban old-age aging process was almost stable in 1981 whereas that of rural was somewhat decreasing. The trend of increasing peak aging process was observed in the period 1981-2001 for both urban and rural areas where the rural aging was almost stable and urban aging was slightly upwarding. Like overall urban-rural aging, the gap between urban and rural with respect to sex is visible in both 1981 and 2001.
A noticeable urban-rural gap was found in base aging process at 2001 according to the alternative measures where urban aging was faster than that of rural though the gap between these two was negligible at 1981. These results are somewhat different from those of presented in chapter IV, where no visible locality (urban-rural) gap was found at both 1981 and 2001. The decreasing rate in proportion of urban female population was faster than that of rural in 1981 but an opposite direction was found in urban-rural male population. Both urban young male and female population decreasing rate was faster than that of rural in 2001. There is a huge gap between the results of base aging in urban and rural areas with the alternative approaches in this chapter and those of measures presented in chapter IV. Therefore, it is observed that the two rate of change measures of base aging, alternative and previous chapter (chapter IV), capture the fertility behaviour in the aging process differently.

Alternative measures of change in the speed of aged-child ratio indicate a wide urban-rural gap in the aging process for both 1981 and 2001 where urban aging was remarkably faster than that of rural. Sex specific measures also support the claim.

Per year increase in the median and mean age with alternative measures show a wide urban-rural gap in the aging process at both 1981 and 2001 where faster urban aging process than that of rural is mentionable. This is also true for sex-specific measures. These results are partially different with those of chapter IV where the gap is narrow in 2001. From the above discussions it can be concluded that the dimensions of urban and rural aging process is different.
5.4.3.5 Comparison between Alternative and Existing Measures

A close look of the measures of Liao and Nath-Deka with their corresponding alternative measures indicates the same direction of changing aging process having different magnitudes (Table 5.9). Though the speed of peak aging process is almost stable for both existing and alternative Liao and Nath-Deka measures, alternative measures show slightly faster aging process. Again, both the alternative approaches of Liao and Nath-Deka indicate the slower base aging process though the aging at base is increasing over the year using all the measures.

More speed of increasing aging process of the Bangladesh population has been found with alternative to Liao and Nath-Deka at 1981 in terms of measure of the aged-child ratio. But measures at 2001 reflect an opposite picture. Again, the alternative measures represent a slow pace of increasing median age at both 1981 and 2001. The measures regarding on the rate of change of mean age of population are confusing with all the measures (existing and alternative ones). Only alternative measure of mean age to Nath-Deka gives somewhat accepting result consistent with the rate of change of median age for the year 2001.

An empirical study finds that the yearly linear rate of change of aging at base, aged-child ratio and median age over the year 1981-2001 are - 0.0036, 0.0017 and 0.22 respectively. Alternative measures have a narrow gap with these yearly rate of change compared to those of measured with Liao and Nath-Deka’s approach. So, we can claim that these are good alternative measures. Therefore, use of both the measures, existing and alternative, will be helpful for better understanding of the aging process.

5.4.3.6 Comparison among Alternative Indices

We have proposed several alternative approaches of conventional aging measures in this chapter. These are alternative index to Liao, alternative index to Preston and Himes et al., alternative index to Nath and Deka, alternative index to
improved-1 and improved-2 indices those are presented in chapter IV. All indices, whose alternative measures proposed here, have been explained in chapter IV. The results are presented in Table 5.10. Alternative improved-1 index considers simultaneous effect of urban-rural life expectancies and alternative birth rates (based the third consideration discussed in 'c'; p.182) whereas improved-2 index considers those of with respect to sex (based the third consideration discussed in 'd'; p.183). The entire alternative measures exhibit slightly decreasing peak aging process in both 1981 and 2001 where the decreasing rate is almost negligible in the later year. The magnitudes of this process are more or less same with the alternative measures except for alternative to improved-1 index at 1981 and alternative to improved-2 index at 2001. Alternative to improved-2 index indicates the slowest decreasing rate in 2001. This index gives more consistent peak aging result than that of alternative improved-1 index.

Alternative measures of base aging show the different aging process among the formulae at both 1981 and 2001 though the direction (decreasing proportion of young) is same for all the approaches. In 1981, the alternative to Nath and Deka’s formula showed a very slow decreasing rate while alternative improved-2 and Liao’s measures are close to each other and these are slightly different with the alternative improved-1 measure. Variations are poor among the different alternative measures except for improved-1 in 2001. Here, alternative improved-2 approach is close to Liao’s measure like 1981.

The speed of aging is faster with alternative improved-1 and improved-2 measures than those of Liao and Nath-Deka’s measure in both 1981 and 2001 regarding on the rate of change of aged-child ratio. In both the years, improved-1
and improved-2 measures are close to each other while Liao and Nath-Deka’s
methods are almost same.

All the alternative measures regarding on the median age indicate an
upward aging process in Bangladesh at both 1981 and 2001 except for alternative
to Liao’s method in 1981. Alternative improved-1 and improved-2 measures show
a faster aging process than those of Liao and Nath-Deka’s measures in 1981. In
2001, the alternative improved-2 method shows the same speed as that of Liao and
alternative improved-1 methods are almost same as that of Nath-Deka’s approach.

Only alternative improved-2 index shows the reliable aging process in both
1981 and 2001 with the measure related to the rate of change of mean age. All
other alternative measures of the rate of change of mean age (Preston et al., Nath-
Deka, and improved-1) show the confusing aging process while Nath-Deka’s
approach shows an acceptable aging process in 2001. From the results it is clear
that all the alternative measures show the same direction of Bangladesh population
aging except for the measures regarding on the mean age. Like previous chapter
(chapter IV), only the alternative improved-2 index gives consistent measures with
other alternative approaches. So, we can claim that improved-2 is the best
alternative approach of measuring speed of aging.

Considering combined aging measures, aged-child ratio and median age,
we observed that the alternative measures show a slower aging process than those
of chapter IV in both 1981 and 2001 except for measures with the aged-child ratio
in 1981. From the analysis it is found that these extended measures are good
alternative to existing measures of aging velocity and alternative approaches show
somewhat different pace of changing aging process. Since level of changing aging
process is different among the alternative measures hence it is recommended to consult all the measures carefully and to take decision on the basis of the combined idea on these measures. So, the alternative measures will be helpful for better understanding of the demographic aging process. The analysis also reveals that Bangladesh population is greying over time with a mentionable gender and urban-rural gap. The urban population is becoming aging with more pace than those of rural. Male demographic aging is faster than that of female at both 1981 and 2001. Consideration of this type of population aging process of the country will add some weights on taking aging policy. Therefore special care is needed to interpret speed of aging through demographic components such fertility, mortality and life expectancy of a person.
Table 5.1: Life Expectancy, Alternative Birth and Other Rates in Bangladesh: 1981-2001

<table>
<thead>
<tr>
<th>Year</th>
<th>Locality</th>
<th>Sex</th>
<th>Life expectancy, alternative birth and other rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(b(t))</td>
</tr>
<tr>
<td>1981</td>
<td>National</td>
<td>Male</td>
<td>0.0210</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0206</td>
<td>54.50</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0417</td>
<td>54.80</td>
</tr>
<tr>
<td></td>
<td>1981 Urban</td>
<td>Male</td>
<td>0.0136</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0132</td>
<td>60.50</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0268</td>
<td>60.30</td>
</tr>
<tr>
<td></td>
<td>1981 Rural</td>
<td>Male</td>
<td>0.0218</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0214</td>
<td>53.90</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0432</td>
<td>54.30</td>
</tr>
<tr>
<td></td>
<td>1981 National</td>
<td>Male</td>
<td>0.0121</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0110</td>
<td>64.50</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0232</td>
<td>64.20</td>
</tr>
<tr>
<td>2001</td>
<td>Urban</td>
<td>Male</td>
<td>0.0034</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0031</td>
<td>66.70</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0065</td>
<td>66.40</td>
</tr>
<tr>
<td></td>
<td>2001 Rural</td>
<td>Male</td>
<td>0.0147</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>0.0134</td>
<td>64.10</td>
</tr>
<tr>
<td></td>
<td>Person</td>
<td>0.0281</td>
<td>63.20</td>
</tr>
</tbody>
</table>

Table 5.2: Some Fertility Measures of Bangladesh: 1981-2001

<table>
<thead>
<tr>
<th>Year</th>
<th>Locality</th>
<th>Fertility measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>NRR</td>
</tr>
<tr>
<td>1981</td>
<td>National</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>Urban</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Rural</td>
<td>1.96</td>
</tr>
<tr>
<td>2001</td>
<td>National</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Urban</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Rural</td>
<td>1.39</td>
</tr>
</tbody>
</table>
Table 5.3: National Level Alternative Rate of Change Aging Measures for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>Year</th>
<th>Person-1</th>
<th>Person-2</th>
<th>Person-3</th>
<th>Person-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dp_{60}(t)}{dt}$</td>
<td>1981</td>
<td>-0.00118</td>
<td>-0.00105</td>
<td>-0.00246</td>
<td>-0.00111</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.00042</td>
<td>-0.00039</td>
<td>-0.00046</td>
<td>-0.00027</td>
</tr>
<tr>
<td>$\frac{dp_{15}(t)}{dt}$</td>
<td>1981</td>
<td>-0.02021</td>
<td>-0.00988</td>
<td>-0.01737</td>
<td>-0.02048</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.01604</td>
<td>-0.01977</td>
<td>-0.02916</td>
<td>-0.01639</td>
</tr>
<tr>
<td>$\frac{dR(t)}{dt}$</td>
<td>1981</td>
<td>0.00270</td>
<td>0.00265</td>
<td>0.00292</td>
<td>0.00293</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.00529</td>
<td>0.00520</td>
<td>0.00584</td>
<td>0.00581</td>
</tr>
<tr>
<td>$\frac{dp_{M4}(t)}{dt}$</td>
<td>1981</td>
<td>0.00132</td>
<td>-0.00705</td>
<td>-0.01077</td>
<td>-0.00778</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.00835</td>
<td>-0.01059</td>
<td>-0.00992</td>
<td>-0.01033</td>
</tr>
<tr>
<td>$\frac{dMd(t)}{dt}$</td>
<td>1981</td>
<td>-0.05</td>
<td>0.26</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.43</td>
<td>0.55</td>
<td>0.51</td>
<td>0.43</td>
</tr>
<tr>
<td>$\frac{dA_p(t)}{dt}$</td>
<td>1981</td>
<td>-0.50</td>
<td>-0.09</td>
<td>-0.74</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.15</td>
<td>0.23</td>
<td>-0.05</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Note: 1. measures considering overall alternative birth rates and life expectancies of population.
2. measures considering male-female alternative birth rates and life expectancies of population.
3. measures considering urban-rural alternative birth rates and life expectancies of population.
4. measures considering regional (urban-rural) and sex (male-female) alternative birth rates and life expectancies of population simultaneously.

Table 5.4: National Level Alternative Measures with respect to Sex for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>1981</th>
<th></th>
<th>2001</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>$\frac{dp_{60}(t)}{dt}$</td>
<td>-0.00115</td>
<td>-0.00121</td>
<td>-0.00054</td>
<td>-0.00032</td>
</tr>
<tr>
<td>$\frac{dp_{15}(t)}{dt}$</td>
<td>-0.02179</td>
<td>-0.01810</td>
<td>-0.01682</td>
<td>-0.01488</td>
</tr>
<tr>
<td>$\frac{dR(t)}{dt}$</td>
<td>0.00371</td>
<td>0.00164</td>
<td>0.00556</td>
<td>0.00483</td>
</tr>
<tr>
<td>$\frac{dp_{M4}(t)}{dt}$</td>
<td>-0.00867</td>
<td>-0.00768</td>
<td>-0.00588</td>
<td>-0.01097</td>
</tr>
<tr>
<td>$\frac{dMd(t)}{dt}$</td>
<td>0.36</td>
<td>0.30</td>
<td>0.30</td>
<td>0.58</td>
</tr>
<tr>
<td>$\frac{dA_p(t)}{dt}$</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table 5.5: Urban Level Alternative Measures for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>1981</th>
<th></th>
<th>2001</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Person-1</td>
<td>Person-2</td>
<td>Person-1</td>
<td>Person-2</td>
</tr>
<tr>
<td>$\frac{dp_{80}(t)}{dt}$</td>
<td>- 0.00016</td>
<td>0.00013</td>
<td>0.00103</td>
<td>0.00161</td>
</tr>
<tr>
<td>$\frac{dp_{15}(t)}{dt}$</td>
<td>- 0.02069</td>
<td>- 0.02610</td>
<td>- 0.02226</td>
<td>- 0.04536</td>
</tr>
<tr>
<td>$\frac{dR(t)}{dt}$</td>
<td>0.00543</td>
<td>0.00544</td>
<td>0.01212</td>
<td>0.01261</td>
</tr>
<tr>
<td>$\frac{dp_{Md}(t)}{dt}$</td>
<td>- 0.01353</td>
<td>- 0.01515</td>
<td>- 0.01882</td>
<td>- 0.02478</td>
</tr>
<tr>
<td>$\frac{dMd(t)}{dt}$</td>
<td>0.66</td>
<td>0.73</td>
<td>0.84</td>
<td>1.11</td>
</tr>
<tr>
<td>$\frac{dA_p(t)}{dt}$</td>
<td>- 0.05</td>
<td>0.33</td>
<td>0.27</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Note: 1 measures considering overall alternative birth rates and life expectancies of urban population
2 measures considering male-female alternative birth rates and life expectancies of urban population simultaneously

Table 5.6: Urban Level Alternative Measures with respect to Sex for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>1981</th>
<th></th>
<th>2001</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>$\frac{dp_{80}(t)}{dt}$</td>
<td>0.00017</td>
<td>- 0.00055</td>
<td>0.00128</td>
<td>0.00074</td>
</tr>
<tr>
<td>$\frac{dp_{15}(t)}{dt}$</td>
<td>- 0.01943</td>
<td>- 0.02245</td>
<td>- 0.02161</td>
<td>- 0.02302</td>
</tr>
<tr>
<td>$\frac{dR(t)}{dt}$</td>
<td>0.00719</td>
<td>0.00372</td>
<td>0.01367</td>
<td>0.01036</td>
</tr>
<tr>
<td>$\frac{dp_{Md}(t)}{dt}$</td>
<td>- 0.01444</td>
<td>- 0.01227</td>
<td>- 0.01735</td>
<td>- 0.02055</td>
</tr>
<tr>
<td>$\frac{dMd(t)}{dt}$</td>
<td>0.72</td>
<td>0.47</td>
<td>0.82</td>
<td>0.89</td>
</tr>
<tr>
<td>$\frac{dA_p(t)}{dt}$</td>
<td>0.26</td>
<td>0.25</td>
<td>0.35</td>
<td>0.34</td>
</tr>
</tbody>
</table>
### Table 5.7: Rural Level Alternative Measures for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>1981</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Person-1</td>
<td>Person-2</td>
</tr>
<tr>
<td>( \frac{dp_{60}(t)}{dt} )</td>
<td>-0.00130</td>
<td>-0.00117</td>
</tr>
<tr>
<td>( \frac{dp_{15}(t)}{dt} )</td>
<td>-0.02056</td>
<td>-0.00913</td>
</tr>
<tr>
<td>( \frac{dR(t)}{dt} )</td>
<td>0.00251</td>
<td>0.00252</td>
</tr>
<tr>
<td>( \frac{dP_{ma}(t)}{dt} )</td>
<td>-0.00766</td>
<td>-0.00533</td>
</tr>
<tr>
<td>( \frac{dMd(t)}{dt} )</td>
<td>0.29</td>
<td>0.20</td>
</tr>
<tr>
<td>( \frac{dAp(t)}{dt} )</td>
<td>-0.54</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Note: 1. Measures considering overall alternative birth rates and life expectancies of rural population
2. Measures considering male-female alternative birth rates and life expectancies of rural population simultaneously

### Table 5.8: Rural Level Alternative Measures with respect to Sex for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>1981</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>( \frac{dp_{60}(t)}{dt} )</td>
<td>-0.00136</td>
<td>-0.00124</td>
</tr>
<tr>
<td>( \frac{dp_{15}(t)}{dt} )</td>
<td>-0.02310</td>
<td>-0.01799</td>
</tr>
<tr>
<td>( \frac{dR(t)}{dt} )</td>
<td>0.00351</td>
<td>0.00158</td>
</tr>
<tr>
<td>( \frac{dP_{ma}(t)}{dt} )</td>
<td>-0.00789</td>
<td>-0.00559</td>
</tr>
<tr>
<td>( \frac{dMd(t)}{dt} )</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td>( \frac{dAp(t)}{dt} )</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
</tbody>
</table>
Table 5.9: Existing and Alternative Measures of Speed of Aging

<table>
<thead>
<tr>
<th>Measures</th>
<th>Year</th>
<th>Liao</th>
<th>Alternative to Liao</th>
<th>Nath and Deka</th>
<th>Alternative to Nath and Deka</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{dp_{\omega}(t)}{dt})</td>
<td>1981</td>
<td>-0.00302</td>
<td>-0.00118</td>
<td>-0.00458</td>
<td>-0.00105</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.00077</td>
<td>-0.00042</td>
<td>-0.00058</td>
<td>-0.00039</td>
</tr>
<tr>
<td>(\frac{dp_{\omega}(t)}{dt})</td>
<td>1981</td>
<td>-0.02713</td>
<td>-0.02021</td>
<td>-0.02695</td>
<td>-0.00988</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.02271</td>
<td>-0.01604</td>
<td>-0.03239</td>
<td>-0.01977</td>
</tr>
<tr>
<td>(\frac{dR(t)}{dt})</td>
<td>1981</td>
<td>0.00056</td>
<td>0.00270</td>
<td>0.00064</td>
<td>0.00265</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.00704</td>
<td>0.00529</td>
<td>0.00717</td>
<td>0.00520</td>
</tr>
<tr>
<td>(\frac{dp_{Md}(t)}{dt})</td>
<td>1981</td>
<td>-0.00558</td>
<td>0.00132</td>
<td>-0.01224</td>
<td>-0.00705</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.01587</td>
<td>-0.00835</td>
<td>-0.01348</td>
<td>-0.01059</td>
</tr>
<tr>
<td>(\frac{dMd(t)}{dt})</td>
<td>1981</td>
<td>0.21</td>
<td>-0.05</td>
<td>0.46</td>
<td>0.26366</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.82</td>
<td>0.43</td>
<td>0.70</td>
<td>0.54810</td>
</tr>
<tr>
<td>(\frac{dA_{p}(t)}{dt})</td>
<td>1981</td>
<td>-0.13</td>
<td>-0.50</td>
<td>0.13</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.36</td>
<td>-0.15</td>
<td>0.08</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 5.10: Different Alternative Measures for 1981 and 2001

<table>
<thead>
<tr>
<th>Measures</th>
<th>Year</th>
<th>Liao</th>
<th>Nath and Deka</th>
<th>Improved-1</th>
<th>Improved-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{dp_{\omega}(t)}{dt})</td>
<td>1981</td>
<td>-0.00118</td>
<td>-0.00105</td>
<td>-0.00246</td>
<td>-0.00111</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.00042</td>
<td>-0.00039</td>
<td>-0.00046</td>
<td>-0.00027</td>
</tr>
<tr>
<td>(\frac{dp_{\omega}(t)}{dt})</td>
<td>1981</td>
<td>-0.02021</td>
<td>-0.00988</td>
<td>-0.01737</td>
<td>-0.02048</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.01604</td>
<td>-0.01977</td>
<td>-0.02916</td>
<td>-0.01639</td>
</tr>
<tr>
<td>(\frac{dR(t)}{dt})</td>
<td>1981</td>
<td>0.00270</td>
<td>0.00265</td>
<td>0.00292</td>
<td>0.00293</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.00529</td>
<td>0.00520</td>
<td>0.00584</td>
<td>0.00581</td>
</tr>
<tr>
<td>(\frac{dp_{Md}(t)}{dt})</td>
<td>1981</td>
<td>0.00132</td>
<td>-0.00705</td>
<td>-0.01077</td>
<td>-0.00778</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.00835</td>
<td>-0.01059</td>
<td>-0.00992</td>
<td>-0.01033</td>
</tr>
<tr>
<td>(\frac{dMd(t)}{dt})</td>
<td>1981</td>
<td>-0.05</td>
<td>0.26</td>
<td>0.40</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>0.43</td>
<td>0.55</td>
<td>0.51</td>
<td>0.43</td>
</tr>
<tr>
<td>(\frac{dA_{p}(t)}{dt})</td>
<td>1981</td>
<td>-0.50</td>
<td>-0.09</td>
<td>-0.74</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>-0.15</td>
<td>0.23</td>
<td>-0.05</td>
<td>0.97</td>
</tr>
</tbody>
</table>

205