CHAPTER - 7

VERIFICATION OF MATHEMATICAL MODEL WITH EXPERIMENTAL RESULT

7.1. INTRODUCTION:

Verification of mathematical model is essential to establish its validity in the concerned situation. Mathematical modelling of flood propagation due to failure of river dike is a complex two-dimensional problem. Complexity arises not only in the mathematical simulation but in its verification as well. Measurement of real field data of flood propagation for such situation appears to be almost an impossible task. Therefore, verification of the developed model has been done with the help of experimental data generated in the Hydraulic Laboratory of the Civil Engineering Department of Assam Engineering College.

As the flow depth in the laboratory model is very small as compared to the real situation, calibrated value of resistance parameter has been used while checking validity of the mathematical model. A detail discussion on the calibration of resistance coefficient has been presented at the beginning of this chapter.

Comparison of computed and experimental results has been done first on the basis of position of wave front at different times and then
on the basis of variation of tip velocity with distance. Comparative plots of experimental and computed wave-fronts, at different times, have therefore been presented for different bed slopes and different bed roughness.

Plot showing variation of tip velocity with distance has then been presented for different combinations of bed slopes and bed roughness.

On the basis of above two comparisons, a critical assessment regarding accuracy of the proposed mathematical model has been made.

Finally some concluding remarks towards the validity of the proposed model has been drawn.

7.2 MODEL CALIBRATION:

7.2.1. Necessity of Model Calibration:

The proposed mathematical model contains the empirical constant 'n' (Manning's roughness coefficient), which as such, does not need any calibration. Standard value of 'n' for different bed materials are available in wide range\textsuperscript{6,14}. But as the flow depth in the laboratory model is significantly small, the standard values for resistance coefficients are not at all applicable for the laboratory model. Again flood in the proposed situation propagates two-dimensionally over a valley. Therefore, after emerging with a higher depth, through the opening, flood water attains considerably smaller depth within a very short time as it spreads over the valley immediately. Because of this
fact, use of single value of resistance coefficient is also not justified for the whole computation period. Experimental investigations have also revealed the fact that depth of flow influences the resistance parameter to a great extent in the situation under study. Therefore, calibration of resistance parameter has been considered essential for verification of the developed model using laboratory data.

7.2.2 Calibration of Roughness Coefficient:

Calibration of roughness coefficient has been done on the basis of wave tip positions over a horizontal bed. It has been observed that adoption of two average values for ‘n’, one for higher flow depth and the other for smaller flow depth, is essential for simulating the flood movement. Therefore, calibration has been carried out in two different phases. In the first phase, calibration for higher flow depth has been done and in the second phase ‘n’ value for smaller flow depth has been calibrated. Laboratory observations have revealed that flow depth in the model can be considered as higher, only for a duration of approximately one second after opening of the gate. Therefore, wave tip positions obtained up to a period of one second have been used for the purpose of calibrating ‘n’ value for higher flow depth. Wave tip positions obtained between one second and two seconds have been used for calibrating ‘n’ value for smaller flow depth. This ‘n’ value has been considered as average ‘n’ value for rest of the period as flow depths remain small in the subsequent times. However, as has already been discussed in the previous chapter, ‘n’ value of jute bed being
very sensitive to the flow depth, three 'n' values have been used for simulating flood movement over this bed. First 'n' value adopted is for initial one second. After elapse of one second two 'n' values have been used for the flow domain. In case of horizontal bed, flow depth in the region near the right boundary is comparatively higher than the region away from the boundary. This is due to the influence of 'wave reflection' from the right boundary. Because of this fact, two different 'n' values have been used for these two regions. 'n' value, calibrated for the region near the right boundary can be regarded as 'n' value for small depth while 'n' value of the region away from the boundary can be termed as 'n' value for very small depth.

7.2.3 Calibrated 'n' Values for Different Bed Roughness.

The 'n' values, thus calibrated for different bed roughness of the laboratory model, have been presented in tabular form in the table 7.1

<table>
<thead>
<tr>
<th>Roughness</th>
<th>P.C.C. Bed</th>
<th>Stone Chips Bed</th>
<th>Jute Fabric Bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Depth</td>
<td>Higher</td>
<td>Smaller</td>
<td>Higher</td>
</tr>
<tr>
<td>Time Since Gate Opening</td>
<td>&lt;1sec.</td>
<td>&gt;1sec</td>
<td>&lt;1sec</td>
</tr>
<tr>
<td>Computation Grid</td>
<td>all</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td>Calibrated 'n' Values</td>
<td>0.04</td>
<td>0.15</td>
<td>0.13</td>
</tr>
</tbody>
</table>
7.3 COMPARISON OF COMPUTED AND EXPERIMENTAL RESULTS:

Comparison of computed and experimental results has been done first on the basis of wave tip position and then on the basis of variation of tip velocity with distance.

7.3.1 Comparison of Computed and Experimental Wave Front:

Position of wave fronts, at different times, provide information regarding area submerged at different times after opening of the gate. A continuous plot of wave front also gives a clear idea regarding propagation of flood wave over the valley. As modelling of flood propagation is the prime objective of this study, comparison of computed wave tips with the experimental results have been chosen as the basis for assessment of the proposed model.

In Fig. 7.1 and Fig. 7.2 comparison of wave tips for horizontal and adverse (1:100) P. C. C bed has been shown respectively. Fig. 7.3 and Fig. 7.4 have shown the same comparison in case of horizontal and adverse (1:100) stone chips bed. Comparison of wave tip position in case of horizontal and adverse (1:100) jute fabric bed have been presented in Fig. 7.5 and Fig. 7.6 respectively.

7.3.1.1 Assessment on The Basis of Comparison of Wave Tip:

The above comparative plots have shown that the positions of computed wave fronts are in very good agreement with the corresponding experimental wave fronts. However, a little difference between computed and experimental wave fronts has been observed, which is expected because of the following facts.
FIG. 7.1 COMPARATIVE PLOT OF WAVE TIPS FOR HORIZONTAL RCC. BED

TIME ELAPSE (in sec) = 4.00
GRID SIZE = 200mm x 200mm

INDEX

COMPUTED

OBSERVED
FIG. 7.2: COMPARATIVE PLOT OF WAVE TIPS FOR ADVERSE PCC BED

GRID SIZE = 200mm x 200mm

TIME ELAPSED (in sec) = 4.50

TIME TIP TIME (sec)

1/2 1 3/4 4

INDEX

COMPUTED OBSERVED

GATE
TIME ELAPSE (in sec) = 5.00
GRID SIZE = 200mm x 200mm

INDEX
COMPUTED
OBSERVED
GATE G

FIG 7.3: COMPARATIVE PLOT OF WAVE TIPS FOR HORIZONTAL STONE CHIPS BED
Fig 7.4: Comparative plot of wave tips for adverse stone chips bed
FIG. 7.5: COMPARATIVE PLOT OF WAVE TIPS FOR HORIZONTAL JUTE BED

TIME ELAPSED (in sec) = 400
GRID SIZE = 200mm x 200mm

INDEX

COMPUTED OBSERVED
FIG 7.6: COMPARATIVE PLOT OF WAVE TIPS FOR ADVERSE JUTE BED
1) Computations are done at the grid points of the flow field only. Therefore, position of wave front at any instant of time is obtained by joining the extreme grid points where flood wave has already reached. Thus propagation of computed wave front, in real sense, is not continuous; rather it jumps from the present grid points to the subsequent grid points. This process, of course, can be made almost continuous by simply increasing the number of grid points in the computation domain. But increase of grid points results in a considerable increase of computation time in case of two-dimensional solution by the computer. Therefore, number of grids both in $x$ and $y$ directions has been decided on the basis of practical significance and available computing facilities. Because of this fact, existence of a little difference between the computed and experimental wave front is obvious.

2) In the mathematical model, average value for roughness coefficient has been used. This may also cause a little error in the computed wave front.

3) Although utmost care has been taken while adjusting the slope in the laboratory model, chances of slight error cannot be denied in such small laboratory model.

4) As the flow depth in the experimental model is very small, minor undulations of the bed, if any, may also influence the flood movement to some extent.

5) Scale effect due to small depth in the laboratory model is obvious due to surface tension and other parameters.
In spite of above drawbacks the maximum error has been observed as less than one grid distance. Therefore, on the basis of above comparison, the developed model can be assessed to be quite satisfactory for simulating flood movement in such two-dimensional flow situation.

7.3.2 Comparison of Variation of Tip Velocity With Distance:

Exact determination of tip velocity at any point in a desired direction is not possible from the experimental observations. But as already discussed in the chapter 6, average tip velocity at a particular point can be calculated from the experimental observation. On the other hand, for any direction, computer model assumes the velocity of the last submerged node in that direction as the tip velocity of wave front. Therefore, there exist a basic difference between the computed and observed tip velocities. In spite of this fundamental difference, computed and experimental tip velocities have been compared. The trend of variation of both the tip velocities with distance, for different combinations of bed slopes and bed roughness, will definitely help towards assessment of the proposed model.

For the purpose of comparison, tip velocity in the direction normal to the gate has been used. A comparative plot of computed and experimental tip velocities in case of horizontal and adverse (1:100)P.C.C bed has been presented in the Fig.7.7 and Fig.7.8 respectively. Fig.7.9 and Fig.7.10 have shown the similar plot in case of horizontal and adverse stone chips bed. Variation of experimental
FIG 7.7: TIP VELOCITY Vs DISTANCE FOR HORIZONTAL PCC BED
FIG 7.8: TIP VELOCITY VS DISTANCE FOR ADVERSE PCC BED
FIG 7.9: TIP VELOCITY Vs DISTANCE FOR HORIZONTAL JUTE BED
FIG 7.10: TIP VELOCITY vs DISTANCE FOR ADVERSE JUTE BED

- - - - EXPERIMENTAL
COMPUTED

$u_t$ (m/s)

0.0 0.5 1.0 1.5

DISTANCE (m)

0.0 0.5 1.0 1.5
FIG 7.11: TIP VELOCITY vs DISTANCE for Horizontal Stone Chips Bed
FIG 7.12: TIP VELOCITY Vs DISTANCE FOR ADVERSE STONE CHIPS BED
and computed tip velocities with distance, in case of horizontal and adverse (1:100) jute bed has been presented in Fig. 7.11 and Fig. 7.12 respectively.

### 7.3.2.1 Model Assessment on The Basis of Tip Velocity:

From the graphical representation of tip velocity Vs distance (Fig. 6.12 to 6.17) it has already been observed in chapter-6 that bed slope and bed roughness influence the tip velocity significantly. Identical influence of bed slopes and bed roughness has been observed from the comparative plot of experimental and computed tip velocities.

It has been observed that although trend of variation is same, computed tip velocities are less than the experimental tip velocities in all the cases. But this cannot be regarded as an error of computed tip velocity; rather this difference is due to the limitation in the calculation of experimental tip velocity. This can be explained as given below.

Tip velocity drops from a very high value to a considerably small value within a very short distance. The rate of change of tip velocity with distance is also not uniform within that distance. This rate decreases with increase of distance. But in the experimental observations tip velocity midway between two successive wave tips has been calculated as

\[ (V_{tip})_r = \frac{r_2 - r_1}{t_2 - t_1} \]

Where \( r_1 \) = Distance up to the first (preceding) wave tip

\( r_2 \) = Distance up to the second (succeeding) wave tip.
\[ r' = \frac{(r_1 + r_2)}{2} \]

\[ t_1 = \text{Time required to reach the first wave tip.} \]

\[ t_2 = \text{Time required to reach the second wave tip.} \]

The above equation is based on the assumption that variation of tip velocity is linear between two successive wave tips under consideration, i.e., assumption of constant \( \frac{dV_{tip}}{dr} \) between the two points has been made. But in reality \( \frac{dV_{tip}}{dr} \) is not constant but decreases with the increase of distance. Fig 7.13 illustrates how the experimental tip velocity curve deviates from the actual tip velocity curve towards upward direction because of this assumption.

Fig. 7.13: DEVIATION OF EXPERIMENTAL TIP VELOCITY CURVE FROM ACTUAL TIP VELOCITY CURVE.

In Fig. 7.13, \( r_1, r_2, r_3 \) and \( r_4 \) represent the successive wave tip positions. The curve AB represent the variation of actual tip velocities \( (V_{t_1}', V_{t_2}', V_{t_3}' \text{ and } V_{t_4}') \) with distance. Following the assumption made in the calculation of tip velocity, \( V_{t_1}', V_{t_2}', \text{ and } V_{t_3}' \) will represent the calculated tip velocities corresponding to the points \( r_1', r_2' \text{ and } r_3' \).
midway between $r_1-r_2$, $r_2-r_3$, and $r_3-r_4$ respectively. Thus the experimental tip velocity curve $A'B'$ will deviates in the upward direction from the actual tip velocity curve $AB$. Further, because of the same reason the deviation is more at the beginning and decreases gradually with the increase of distance.

Similar deviation of the experimental tip velocity curve from the computed tip velocity curve has been observed in the comparative plots of computed and experimental tip velocities (Fig. 7.7 - 7.12).

On the basis of above analysis the computed tip velocities can be regarded as more accurate than the experimental tip velocities. Thus accuracy of the proposed model may be claimed to be quite reliable.

7.4 CONCLUSION:

Validity of the proposed model has been assessed on the basis of two types of comparison. Comparison between computed and experimental wave tip position has been made first and then the variation of tip velocities with distances has been compared. Comparisons of computed and experimental results have shown very good agreement between the experimental results and model solution.

A maximum error of one grid distance has been observed in the comparative plot of wave front against time. This error is of course, expected primarily because of discretization of the flow field, which is essential for any Finite Difference scheme. However, from practical point of view, this error is not very much significant. The comparative
plots of variation of tip velocity with distance have revealed that the
trend of variation for different combinations of bed slopes and bed
roughness is identical in the experimental and computed plots.
Because of an inherent limitation in the calculation of experimental
tip velocities, the experimental tip velocity curve deviates slightly from
the actual tip velocity curve in the upward direction. The comparative
plots of experimental and computed curves have shown similar
deviation of the experimental curve from the computed one. Thus
computed curve can be regarded as more accurate than the
experimental curve.

On the basis of these comparisons the accuracy of the proposed
model is justified and thus the model is claimed to be satisfactory for
computing two-dimensional flow in the proposed situation.