### List of Figures

1.1 Elementary vertices of QCD .......................................................... 5
1.2 Virtual correction to the gluon propagator ........................................ 6
1.3 Schematic diagram of Deep Inelastic Scattering .............................. 8
1.4 Schematic diagram of electron proton scattering in the quark parton model 9
1.5 Deep Inelastic Scattering in the QCD improved parton model .......... 13
1.6 Ladder diagram in DGLAP evolution ............................................. 18
1.7 HERA and fixed target kinematic range ......................................... 22
1.8 Schematic diagram of a characteristic curve ................................. 27

2.1 Gluon distributions given by Eq.(2.36) compared with DLLA and earlier results at $Q^2 = 20 GeV^2$ .................................................. 39
2.2 Slope $dF_2(x, Q^2)/dlnQ^2$ using the gluon Eq.(2.36) at different points of expansion $\alpha$ compared with data from H1 and ZEUS ......................... 40
2.3 Slope $dF_2(x, Q^2)/dlnQ^2$ using Eq.(2.36), (2.38), (2.39) and (2.40) at the point of expansion $\alpha = 0.7$ compared with data from H1 and ZEUS ................. 41
2.4 Slopes $dF_2(x, Q^2)/dlnQ^2$ using different gluons at the point of expansion $\alpha = 0.5, 0.7, 0.9$ and 0.98 respectively ............................................ 42
2.5 Gluon distribution by Eq.(2.36) compared with GRVLO98 exact ........ 44

3.1 Exponent $\lambda(x, Q^2)$ as a function of $x$ at several representative $Q^2$ 49
3.2 Exponent $\lambda(x, Q^2)$ as a function of $Q^2$ at different fixed $x$ ............. 50

4.1 Gluon momentum distribution at NLO compared with GRVNLO98 exact distributions ................................................................. 69
4.2 Same as fig.4.1 but at LO ................................................................. 70
4.3 Copatibility of our LO solution Eq.(4.40) with the DLA solution Eq.(4.44) 73
4.4 Slopes $dF_2(x, Q^2)/dlnQ^2$ using Prytz formula and our NLO solution as a function of $x$ at fixed $Q^2$ ...................................................... 74
4.5 $x$- slopes $dF_2(x, Q^2)/dln(1/x)$ of the structure function at different $Q^2$ ... 75
5.1 Characteristic curves given by Eq.(5.71), Eq.(5.72) and Eq.(5.107) from an initial line \( t = t_0 = \ln \left( \frac{Q^2}{m^2} \right) \) to the point \( \bar{x} = 0.001, \bar{t} = 7.65(Q^2 = 80\text{GeV}^2) \) ......................................................... 96

5.2 Semi-analytical singlet structure function \( F_2^S(x, t) \) (Eq.5.85) compared with the exact MRST01LO solutions at nine different \( Q^2 \) ......................................................... 98

5.3 Semi-analytical gluon momentum distribution given by Eq.(5.84) compared with the exact MRST01LO solutions at nine different \( Q^2 \) ......................................................... 99

5.4 Non-singlet structure function \( F_{2NS} \) (Eq.5.110) compared with the exact solution ................................................................. 102

5.5 Proton structure function \( F_2^p(x, t) \) calculated using Eq.(5.85) and Eq.(5.110) in Eq.(5.111) compared with the exact MRST01LO solution ........................................ 103

5.6 Proton structure function \( F_2^p(x, t) \) calculated using Eq.(5.85) and Eq.(5.110) in Eq.(5.111) compared with the H1 data ................................................................. 104

5.7 \( F_2^p \) (same as in Fig.(5.6)) as a function of \( Q^2 \) at six different fixed \( x \) and compared with the same H1 data ................................................................. 105

5.8 Singlet structure function \( F_2^S(x, Q^2) \) given by the analytic expression Eq.(5.89) for three different values of the parameter \( f_0 \) compared with the exact MRST2001LO solution ................................................................. 107

5.9 Gluon momentum distribution given by the analytic expression Eq.(5.88) compared with the exact solution ................................................................. 108

5.10 \( F_2^p \) calculated using the analytic expression Eq.(5.89) and Eq.(5.110) in Eq.(5.111) and compared with the exact MRST01 solution and the H1 data ................................................................. 109

6.1 Polarized gluon given by the analytic expression Eq.(6.86) compared with the exact (a)AAC00LO, (b)LSS02LO and (c) GRSV01LO distributions ................................................................. 128

6.2 Gluon asymmetry with two different inputs compared with data from SMC, COMPASS and HERMES ................................................................. 130

6.3 \( x - Q^2 \) range where the positivity condition \( \frac{dG}{dx} \leq 1 \) holds good ................................................................. 131

6.4 Polarized singlet distribution given by Eq.(6.85) as a function of \( x \) compared with the exact AAC00LO and LSS02LO distributions at two fixed \( Q^2 = 2\text{GeV}^2 \) and \( Q^2 = 5\text{GeV}^2 \) ................................................................. 133