Chapter 3

Exponent $\lambda(x, Q^2)$ of the proton structure function $F_2(x, Q^2)$ at low x

One of the most important observations of the HERA experiments measuring the structure functions is the rise of F_2 with decreasing x even down to very small values of $Q^2 \sim$ few GeV^2. The experiments parametrize the structure function as $F_2 \sim x^{-\lambda}$ for each Q^2 bin. The exponent λ determines the soft and hard behaviour of the structure function and understanding its x and Q^2 dependence is very crucial to distinguish the perturbative and nonperturbative behaviour of F_2. There are different theoretical models to explain the behaviour of λ. Using the Regge theory of Pomeron exchange, Donnachie and Landshoff [79] predicts a slow rise of the structure function with c.m. energy in hadron-hadron and real photon-proton interaction with a low value for $\lambda \approx 0.08$. In contrast, the CKMT [80] approach predicts a Q^2 dependent λ leading to a faster growth of the structure function toward small x with increasing Q^2. The ABY [81] approach which introduces a hard singular component ($\sim x^{-\lambda_s}, \lambda_s \approx 0.48$) for $Q^2 < 1GeV^2$ to prevent F_2 decreasing for decreasing x, assumes that perturbative QCD is applicable even down to lowest Q^2.

In this chapter we calculate the exponent λ of the structure function using the solution for the gluon momentum distribution obtained in the previous chapter and compare with

H1 data[33].

3.1 Formalism

The approximate analytical solution of the gluon momentum distribution at low x in LO derived in the previous chapter has the form:

$$G(x,t) = G(\tau)x^{-\left(1-(\frac{t_0}{t})^{\gamma^2}\right)}\left(\frac{t_0}{t}\right)^{\frac{\gamma^2 n_f}{18}} exp\left[-\frac{11}{12}\left\{1-(t_0/t)^{\gamma^2}\right\}\right]. \tag{3.1}$$

In the above equation $\gamma = \sqrt{12/\beta_0}$, $\beta_0 = 11 - \frac{2}{3}n_f$, $t = ln(Q^2/\Lambda^2)$ and $t_0 = ln(Q_0^2/\Lambda^2)$ where n_f is the number of active flavours, Λ is the QCD cut-off parameter and Q_0^2 is the starting scale for evolution. In Eq.(3.1), $G(\tau)$ is the input gluon momentum distribution which is obtained from any specific non-perturbative input available in the literature by the formal replacement $x \to \tau$. The variable τ is given in terms of x and t (see Eq.(2.37)) by

$$\tau = exp\left[\left(-ln\frac{1}{x}+\frac{11}{12}\right)\left(\frac{t_0}{t}\right)^{\gamma^2}-\frac{11}{12}\right]. \tag{3.2}$$

At low x the gluon being the dominant parton, the scaling violation of F_2 arises mainly from the gluon ($g \to q\bar{q}$) and so the contribution from the quark can be neglected. In the DGLAP formalism an approximate relationship can be obtained between the gluon momentum distribution $G(x,Q^2)(\equiv x\,g(x,Q^2))$ and the logarithmic slope of the structure function $F_2(x,Q^2)$. The most general one [75] in the LO reads (Eq.2.43):

$$\frac{\partial F_2(x,Q^2)}{\partial lnQ^2} \approx \frac{10\alpha_s(Q^2)}{27\pi}G\left[\frac{x}{1-\alpha}\left(\frac{3}{2}-\alpha\right)\right], \tag{3.3}$$

where $\alpha < 1$ is a suitable point of expansion of the gluon momentum distribution under the integral in the DGLAP equation for F_2 and $\alpha_s(Q^2)$ is the strong coupling constant in the LO. In chapter 1 we showed that our result was fairly successful in explaining the

HERA data[27, 34] if α is chosen around ≈ 0.7. The exponent $\lambda(x, Q^2)$ of the structure function is defined as the derivative:

$$\lambda(x, Q^2) = \frac{\partial\, lnF_2(x, Q^2)}{\partial\, ln(\frac{1}{x})} |_{Q^2} \ . \tag{3.4}$$

The exponent $\lambda(x, Q^2)$ being directly measurable from the structure function data can give us helpful insight into the behaviour of the structure function specially at low x. Theoretical justification of the use of the exponent for such a study has been reported in the literature [82] on the basis of the j-plane singularity of the Mellin transform of the structure function. To obtain an expression for $\lambda(x, Q^2)$ we first differentiate Eq.(3.3) with respect to $ln(1/x)$ and then integrate it from a low scale Q_0^2 to Q^2. Finally we get

$$\lambda(x, Q^2) = \lambda(x, Q_0^2)\frac{F_2(x, Q_0^2)}{F_2(x, Q^2)} + \frac{1}{F_2(x, Q^2)} \times \frac{10}{27\pi}\int_{Q_0^2}^{Q^2} \alpha_s(Q^2)\frac{\partial G(x', Q^2)}{\partial\, ln(1/x)}\, dlnQ^2, \tag{3.5}$$

where

$$x' = \left(\frac{1.5 - \alpha}{1 - \alpha}\right) x \tag{3.6}$$

and $\alpha_s(Q^2)$ is the strong coupling constant given in the LO as

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)ln(Q^2/\Lambda^2)}\,. \tag{3.7}$$

In Eq.(3.5) $\lambda(x, Q_0^2)$ is the exponent at the input scale Q_0^2 given by

$$\lambda(x, Q_0^2) = \frac{\partial\, lnF_2(x, Q_0^2)}{\partial\, ln(\frac{1}{x})}, \tag{3.8}$$

where $F_2(x, Q_0^2)$ is the input structure function. $F_2(x, Q^2)$ in Eq.(3.5) is obtained from Eq.(3.3) where we use our gluon Eq.(3.1) on the right hand side with x replaced by $\left(\frac{1.5-\alpha}{1-\alpha}\right) x$. Similarly $G(x', Q^2)$ is our gluon with x replaced by $x' = \left(\frac{1.5-\alpha}{1-\alpha}\right)x$ because the gluon appears here only through the scaling violation relation.

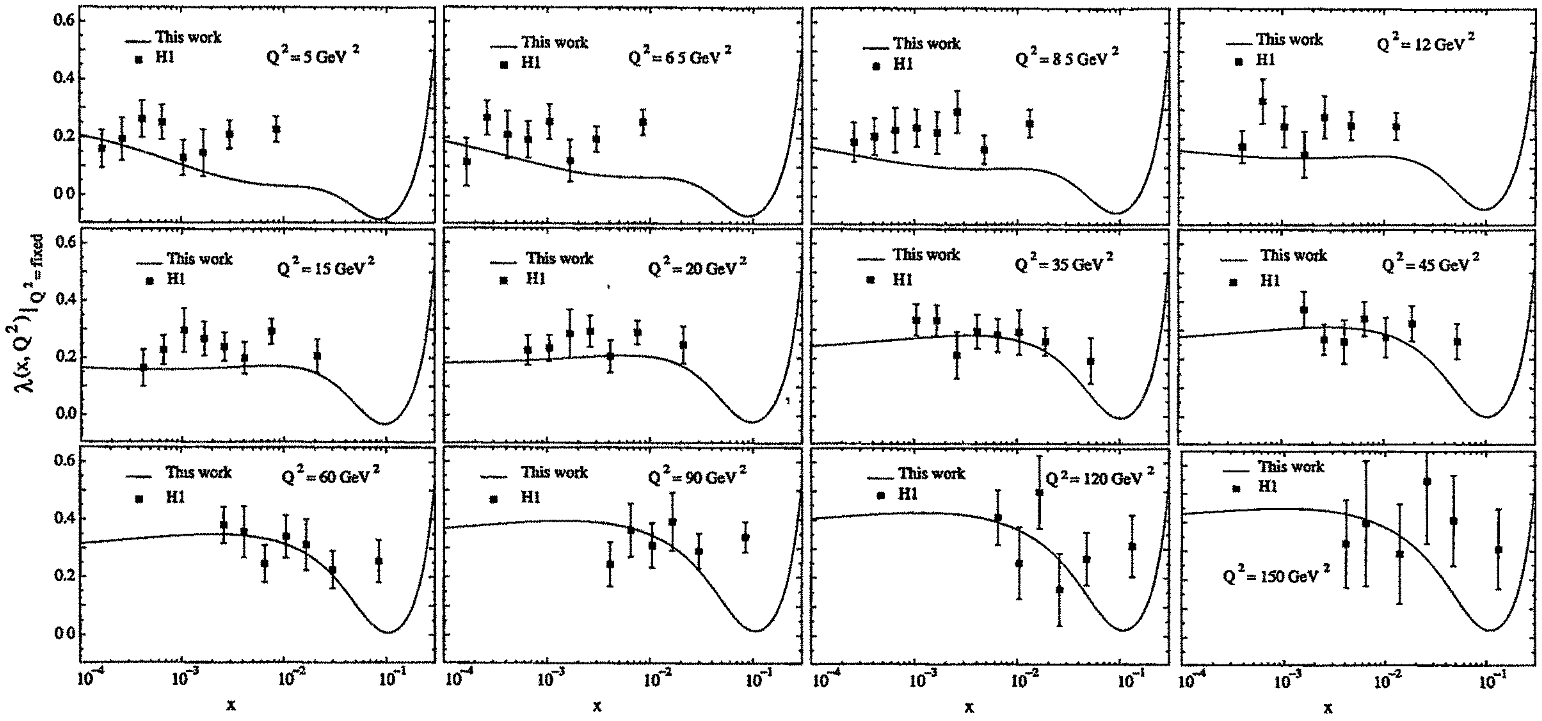

Figure 3.1: Exponent $\lambda(x, Q^2)$ plotted as a function of x at several representative Q^2 values and compared with data from H1[33]. The error bars represent the statistical and systematic uncertainties added in quadrature.

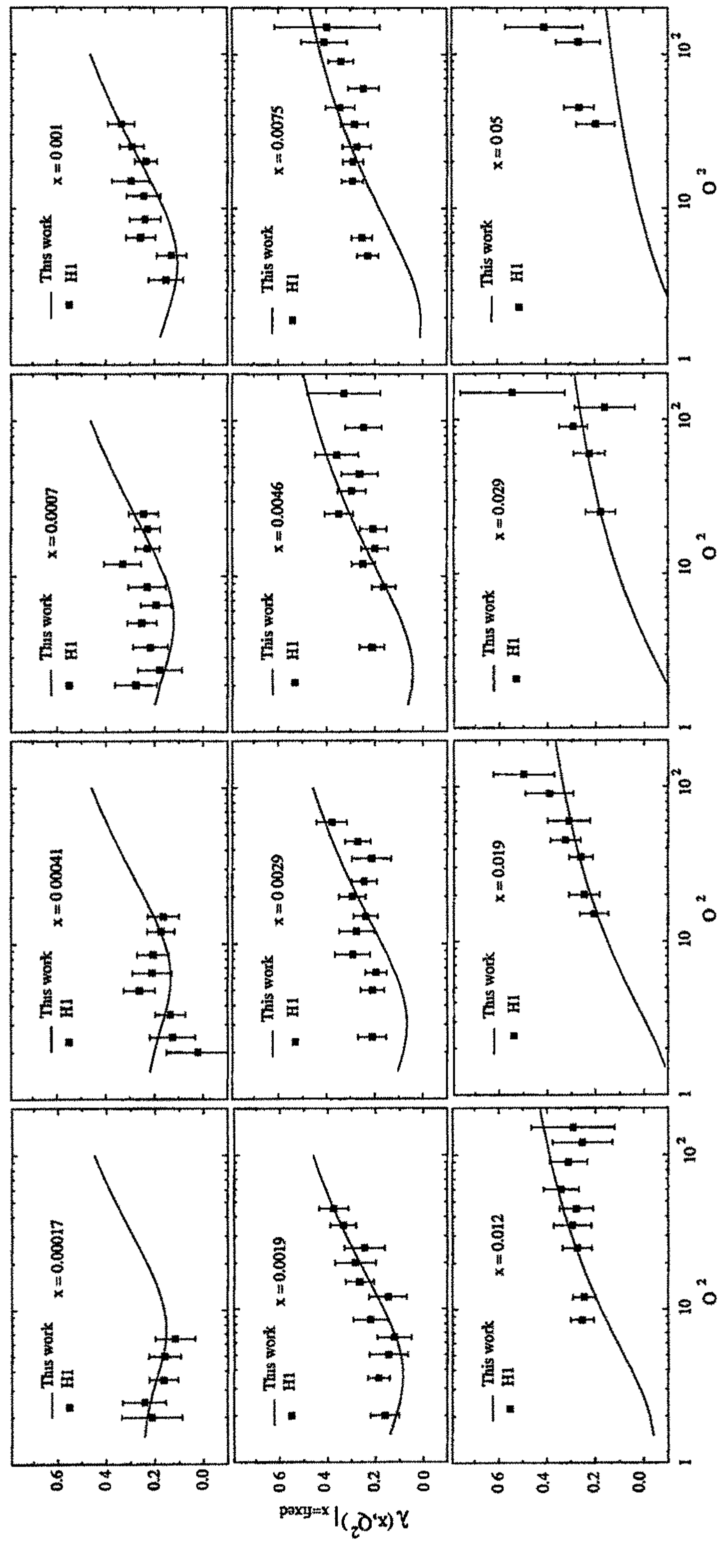

Figure 3.2: Exponent $\lambda(x, Q^2)$ plotted as a function of Q^2 at different fixed x and compared with data from H1[33]. The error bars represent the statistical and systematic uncertainties added in quadrature

3.1.1 Phenomenological study of the exponent

We use Eq.(3.5) to calculate the exponent. In our analysis we take the point of expansion α of the gluon to be 0.7 because it gave good phenomenological test and it has an important interpretation as discussed in the last chapter. We take all our inputs from MRS98LO[83]. In Fig.3.1 we show $\lambda(x, Q^2)$ calculated from Eq.(3.5) as a function of x at twelve different fixed Q^2 values from $5GeV^2$ to $150GeV^2$ along with the data from H1[33] where the measurement of the exponent in a large kinematic domain at low x, $3.10^{-5} \leq x \leq 0.2$ and $1.5 \leq Q^2 \leq 150\,GeV^2$ has been reported. From the figure we observe that at low $x \lesssim 10^{-2}$, and for $8.5GeV^2 < Q^2 \leq 150\,GeV^2$ the derivative $\lambda(x, Q^2)$ is almost independent of x consistent with the H1 [33] data explored in this range. As x increases above 10^{-2}, $\lambda(x, Q^2)$ falls sharply with increasing x reaching a minimum at $x \approx 0.1$ and then increases rapidly. It presumably indicates the breakdown of the low x assumption and transition to the valence quark region as noted in ref.[33]. In Fig.3.2 we compare our prediction with the H1 data for $\lambda(x, Q^2)$ as a function of Q^2 at twelve different fixed $x\,(0.00017 \leq x \leq 0.05)$ values. For $x \lesssim 0.0011$ we notice that there is a slow fall of λ logarithmically with Q^2 upto about $Q^2 \approx 8GeV^2$, but above this value the exponent rises almost linearly with lnQ^2 consistent with the H1 observation. However, the maximum value reached by λ which is about ~ 0.4 for $x \lesssim 0.019$ gradually falls as the value of the fixed x is increased so that when $x \approx 0.029 - 0.05$ it rises only upto a maximum value of ≈ 0.2. Thus in this explored kinematic range it might suggest a form $\lambda(x, Q^2) \sim a(x)\, ln(Q^2/\Lambda^2)$ rather than a constant $a(= 0.0481)$ as suggested in ref.[33]. However, we must be cautious. This deviation may be due to the approximate nature of the calculational technique done in LO only which might have large corrections at small x and large Q^2. But it is also to be noted that Desgrolard *et al* [84] have also indicated such x-dependence of the derivative $\lambda(x, Q^2)$ based on various Regge-type models[85, 86]. To see quantitatively the agreement of our prediction with experiment, we quote in Table(3.1) some χ^2 [87, 88] in different ranges of the kinematic variables. As is evident, the agreement of our prediction with

Table 3.1: Comparison of the prediction for $\lambda(x, Q^2)$ with H1 data

For $\lambda(x, Q^2)$ vs x with Q^2 fixed		
$Q^2(GeV^2)$	x	$\chi^2/d.o.f.$
$3.5 \leq Q^2 \leq 150$	$0.000105 \leq x \leq 0.132$	1.858
$3.5 \leq Q^2 \leq 150$	$0.000105 \leq x \leq 0.05$	1.327
$5.0 \leq Q^2 \leq 150$	$0.000165 \leq x \leq 0.132$	1.768
$5.0 \leq Q^2 \leq 150$	$0.000165 \leq x \leq 0.05$	1.195
$8.5 \leq Q^2 \leq 90$	$0.00026 \leq x \leq 0.05$	1.039
For $\lambda(x, Q^2)$ vs Q^2 with x fixed		
x	$Q^2(GeV^2)$	$\chi^2/d.o.f.$
$0.00017 \leq x \leq 0.05$	$2.0 \leq Q^2 \leq 150$	1.678
$0.00017 \leq x \leq 0.05$	$5.0 \leq Q^2 \leq 150$	1.410
$0.00041 \leq x \leq 0.05$	$8.5 \leq Q^2 \leq 150$	0.985
$0.00041 \leq x \leq 0.05$	$8.5 \leq Q^2 \leq 90$	1.009

the H1 data is not very satisfactory if we consider the entire range of $x - Q^2$ explored in ref.[33]. This high value of χ^2/d.o.f is mainly due to large deviations of our predictions in the lower $Q^2(\lesssim 5 - 8.5GeV^2)$ region. It is also to be noted that there are some large fluctuations of the data also. But if we squeeze the domain in both x and Q^2, then in a limited kinematic range our prediction is comfortable with the experimental data.

3.2 Conclusion

The exponent $\lambda(x, Q^2)$ of the structure function computed from the LO gluon distribution proposed in the previous chapter conforms to the qualitative features of the H1[33] data for low $x(x \lesssim 10^{-2})$ and high $Q^2(Q^2 \gtrsim 8.5GeV^2)$ region. However, a simple parametrization like $F_2 = c\,x^{-\lambda(Q^2)}$ in the entire $x - Q^2$ range explored in ref.[33] seems to be not possible in our formalism which we have carried out only at the leading order.