Chapter 5

Summary and Conclusion

This chapter summarises the results presented in this thesis for two different astrophysical scenarios that have been studied: the gravitational wave emission from a core collapse situation and $\tau$-mode instabilities in neutron stars. The objective of our studies on these two sources is to understand the associated parameters and their influences on the overall dynamics and on the emission of gravitational waves. We summarise our investigations on each of the two sources below.

5.1 Summary of the investigations

5.1.1 Core Collapse

We have performed a numerical simulation of the core collapse with the help of publicly available two-dimensional hydrodynamic code ZEUS-2D by suitably modifying some of its subroutines, e.g., the equation of state is modified to take care of the hybrid equation of state [78]. Moreover, we have written additional subroutines to (1) incorporate the data describing a non-rotating hydrostatic equilibrium structure by solving the Lane-Emden equation for $\Gamma = 4/3$, for use as input progenitor to core collapse simulation (2) calculate the spin harmonic formula for the quadrupole gravitational wave which would result as output data set.

We have chosen to study the emission of gravitational wave from rotation induced axisymmetric core collapse supernova in a parametric way. The parameters are

- The angular velocity $\Omega$ put by hand at the central region of the progenitor.
• The polytropic index $\Gamma$ of the hybrid equation-of-state of the configuration.

• Rotation law parameter or the differential rotation parameter $A$.

We have constructed as many as 100 precollapse model by incorporating the above input parameters in different combinations. The output of the simulation is summarised in tabular form in table 3.2.

Through the core collapse simulations of the models constructed from the Newtonian hydrostatic models with polytropic EoS $4/3$, parametrised by the polytropic index of infall $\Gamma$, the imposed rotation in terms of angular velocity and the nature of angular momentum distribution through a cylindrical rotation law, we see that,

1. As the collapse progresses, the emission of gravitational wave decreases for a brief period due to the oblateness developed in all the models considered in this study. Thereafter progressive infall of the core material leads to a positive value in the gravitational wave strain amplitude till the bounce time. At bounce the strain amplitude drastically drops to a significantly high negative values and rises up again.

2. The precollapse models with larger values of pressure deficit factor $(4/3 - \Gamma)$ would yield smaller amount of gravitational wave in comparison to the models with low pressure deficit, when other parameters of the collapse kept unchanged.

3. Amount of gravitational radiation emitted by models first rises with the increase of angular velocity. After a certain moderate values of initial angular velocity, towards a higher values of initial angular velocities, the amount of radiation is found to decrease progressively.

4. The peak of the gravitational wave strain amplitude at bounce, in general, has a weak dependence on the initial cylindrical differential rotation imposed in the models.

5. As the core collapse progresses, the central density rises monotonically till the bounce. At bounce the central density maximised; subsequently fluctuates and settles down to a lower value.
6. This maximum density at bounce is lowered by rotation and enhanced by the pressure deficit factor.

7. The Central density maxima are enhanced in the case of differentially rotating models compared to the rigidly rotating ones with the same initial rotation or pressure deficit.

8. Time of bounce recorded show a predominance of the pressure deficit factor. More is the pressure deficit, rapid is the collapse and hence less is the time required to the bounce point. The effect of initial rotation on the time of bounce is minimal. It further becomes increasingly insensitive to the pressure deficit factor in case of more and more differentially rotating models.

9. The time-width of the gravitational wave burst roughly decreases with the pressure deficit and increases with initial angular velocity. It has a very weak dependence on the degree of differential rotation.

10. The ratio of rotational to potential energy value $\beta_b$ at bounce is more or less has linear relationship with the its initial ratio $\beta_i$. The proportionality constant is does increase in case of more and more differentially rotating models.

11. In general, it is found that the centrifugal force induced due to the rotation and the differential rotation play a key role in governing the overall dynamics and gravitational wave emission during core collapse.

5.1.2 $\tau$-mode Instability

We have evaluated the behaviour of $\tau$-mode instability including its gravitational wave detectability scenario when a typical neutron star goes unstable through $\tau$-mode, particularly in the presence of differential rotation and magnetic field. Our inclusion of the differential rotation is in the line of Sá et al., [112] and the magnetic field considered is a standard dipole magnetic field. The initial amplitude of the mode is taken to be $\sim 10^{-8}$. Our results are summarised in the following.

1. The amplitude of the $\tau$-mode first rises exponentially and saturates in a natural way a few hundred seconds after the mode instability sets in.
2. Small initial differential rotation causes the $r$-mode to saturate around unity. Larger and larger differential rotation suppresses the growth by substantial order of magnitude and also comparatively in shorter time scales.

3. Magnetic field plays a dominant role in dramatically suppressing the growth of $r$-mode amplitude

4. In absence of either differential rotation or magnetic field, the angular velocity of a typical neutron star stays constant upto a time scale of few hundred seconds. Subsequently it starts abruptly shedding in another few thousand seconds. Presence of differential rotation causes the star to shed angular velocity early. On the other hand, presence of magnetic field causes the abrupt fall of angular velocity gradually smoother.

5. Initial differential rotation hardly influences the critical curve for $r$-mode instability, while the magnetic field offers to shrink the instability window available for a typical neutron star.

6. Presence of magnetic field tends to suppress the effects of the differential rotation parameters. Further, higher strengths of magnetic field often erase distinctive features induced by different values of differential rotation parameters.

7. The gravitational wave strain amplitude emitted due to the $r$-mode instability in a typical neutron star grows exponentially during first few hundred seconds after which it slowly falls approximately in a linear fashion.

8. Increased values of differential rotation cause the gravitational wave strain amplitude to peak at smaller values as well as early. Further, the magnetic field smoothens this amplitude change-over at the peak.

9. the Fourier transformed gravitational wave amplitude does not register any spike during the evolution of the instability, unlike the case in Owen et al.,[88].

10. The Signal-to-Noise ratio of the gravitational waves due to the models investigated in this study reveal that the Advanced LIGO detector with low values of $K$ and magnetic field upto an order of $10^{15}$ Gauss, will be able to detect $r$-mode unstable neutron stars upto a distance of 20 Megaparsecs.
5.2 Epilogue

Gravitational Wave Astronomy is taking its strides to become the *New Millenium Astronomy* with the coming up of different detectors - both in the ground based as well as space borne categories. The outputs in these detectors are a sequence of virtually *unreadable* data streams. The filtering of these data and decipher the meanings would be the main challenge for the gravitational wave researchers of the day.

In view of incomplete pictures through electromagnetic wave studies about the sources that could emit gravitational waves lead researchers to concentrate on parametric studies of these sources to estimate the gravitational waves.

Through the investigations incorporated in this thesis, we have also attempted to have a parametric study on two prospective gravitational wave sources. We have mainly focussed our studies in a small set of parameters. Initial models taken for studies are made as simple as possible. We believe, it helps clarifying the basic physics operating through the interplay of these parameters. The results even from our narrow range of central angular velocity seem encouraging enough to comply with the general core collapse scenario and to draw simple conclusions. Incorporating correct microphysics and a realistic equation of states, consideration of relativistic collapse and inclusion of magnetic field could only lead to a final picture in core collapse scenario. There have been works by various research groups by taking one or more aspects. We believe, a simpler fresh look into the same problem emboldens one's confidence, makes way for new insights - sometimes even fundamental!

In the study of r-mode instability in neutron stars, we have considered the second order effect giving the differential rotation and a simple dipole magnetic field to see the evolution pattern of the instability and emission of gravitational waves and their detectibility. The initial amplitude factor at the inception of instability could also affect the r-mode evolution. This remains as a future work. Further, the evolution period can be extended to several years once the factors like bulk viscosity, shear viscosity, effects of superfluidity etc. are concretely evaluated. The dipole magnetic field considered here is assumed to be aligned with the rotational axis. But observations on pulsars suggest that magnetic fields make a tilt with the rotation axis. Inclusion of a tilted magnetic field, or finding out any mechanism by which the magnetic axis makes an angle with the rotation axis on neutron stars, would be a future work of
A complete understanding of Gravitational Wave Astronomy, as of the present, may be far from being a simple view-and-understand astronomy. Our attempt towards a simplified understanding of the two very important Gravitational wave production processes, may be helpful to view the two issues into a single perspective. The long data stream from the detector outputs will then be more meaningful to the data analysts to bring correspondences with the physics that is operating behind the processes.