Chapter 1

New Astronomy

1.1 Introduction

The beginning of the 21st century is blessed with the bright prospect of a new window of study of the universe - the gravitational wave window. This is a unique window in the sense that information derived from it will be complementary to that either from electromagnetic wave or neutrino. With the impressive progress in the development of gravitational wave detectors over last couple of years, astronomical researches will soon be tuned to observe various types of astrophysical events through this window. At the same time theoretical research towards generation as well as detection of gravitational waves have also been upsurged by the advent of this new method of observation. Though gravitational waves are yet to put its evidential signatures to the detectors developed around the globe, the limits to the astrophysical parameters are progressively narrowed down with the help of the data acquired through these detectors.

Gravitational waves are peculiar kind of waves in the sense that the perturbation associated with it is not a directly measurable physical quantity like density, pressure, temperature or chemical composition. It is the perturbation in the space-time geometry itself on which various physical parameters are defined. Propagation of these perturbations also takes place in the same space-time waltz. Thus these waves are non-linear by nature. These waves originate from the most energetic events in the Universe such as fast rotating neutron stars, colliding neutron star binaries, supernova explosion, gravitational collapse in black holes and others. The effect of the gravita-
tional waves when impinged on matter is to cause a strain to develop to periodically stretch and compress the matter. The amplitude of these strain are amazingly tiny that the detection of gravitational waves as of today is more of a technological challenge rather than a theoretical one. To get a feeling of the order-of-magnitude of the gravitational wave strength, let us consider a gravitational wave strain produced by a binary in-spiral of two stars of mass and size typical of a neutron star at a distance of say, 10 kiloparsecs from the detector of arm length 1 km., on Earth. Then the change in the detector arm length induced by the impinge of gravitational waves from this system will be of the order of $10^{-22}$ km, that is about ten thousandth of nuclear diameter!

Gravitational waves were first predicted by Albert Einstein in 1916 as part of his theory of general relativity through the perturbative analysis of his famous equations—the Einstein field equations. It was first considered a mere theoretical exercise. It was not until the late 1950s that H. Bondi [11], rigorously proved that gravitational radiation is in fact a physically observable entity, that gravitational waves carry energy, and as a result, a system that emits gravitational waves should lose energy. The idea to detect gravitational waves could then be conceived of as a measure of the energy deposition on to the detector. The first ever detector to detect gravitational waves was designed and developed by Joseph Weber in the late 1960s. His arrangement was basically a cylindrical Aluminium bar about 2 metres long, 50 cm in diameter, weighing 1 tonne and attached to it a piezoelectric crystal oriented broadside to incoming gravitational waves. This crystal can convert the internal pressure due to the strain developed in the bar on impinge of gravitational waves onto the bar, to an electrical signal which could then be transported through high fidelity cables and fed to electronic devices for analysis. The detector development journey starting from Joseph’s bar type detector has now reached a stage with the design and development of Michelson-type and laser beam based with much improved efficiency of detection since then.

At the present time the gravitational wave community is focusing its efforts on narrow band resonant bar as well as wide band interferometric methods of detecting gravitational waves. ALLEGRO in US, AURIGA in Italy, EXPLORER in CERN, NAUTILUS in Italy, NIOBE in Australia and GRAIL in Netherlands form the resonant bar category detectors that are developed. LIGO in the United States [115, 116];
TAMA in Japan [119]; VIRGO in Italy [117]; GEO600 in Germany [118] and AIGO in Western Australia form the interferometric category detectors on Earth. A space-based sub-frequency range interferometric detector, LISA [120, 121], could also be launched in the later part of the next decade (possibly around 2018). Detection of gravitational waves in the near future not only prove the existence of gravitational waves thereby re-confirming the Einstein’s general theory of relativity, but also carry a lot of information for us by probing a broad range of gravitational wave frequencies that will arise from a variety of astrophysical sources.

First indirect confirmation of the existence of gravitational waves came when the binary pulsar PSR 1913+16 discovered in 1974 by Hulse and Taylor [12], showed a systematic variations of the arrival times of radio pulses. This systematic variation could be attributed to the emission of energy as gravitational waves as the orbit shrinks and the latter is a good match with the theoretically calculated value using general theory of relativity. And for this, the 1993 Nobel prize was awarded to Hulse and Taylor.

Gravitational waves expected from anticipated astrophysical sources can be divided into four classes: the chirp signal from the spiral of compact binaries; periodic waves from pulsars; impulsive waveforms like the one from supernova bursts and stochastic background from the Big Bang or from distant faint sources. Their detection would provide us with information about strong-field gravity, about the structure and dynamics of astrophysical sources such as binary inspirals, pulsars and supernovae as well as relic gravitational waves from the Big Bang. Various groups of gravitational wave researchers have been focussing on these sources with the hope of extracting vital information from these sources.

Astrophysicists predict that inspiraling neutron star binary systems in the final seconds before merger are likely to be the first sources to be seen by the LIGO detector, as they fall within the LIGO frequency band. Gravitational waves emitted by such systems will have a chirp waveform. Gravitational waves carry with them information about their origins and fundamental properties such as speed and polarisation, or equivalently, the mass and spin of the associated quantum mechanical particle, the graviton.

One of the most interesting astrophysical scenarios to search for the gravitational waves is the gravitational collapse of the inner iron cores of massive stars with masses.
of the order of 8 solar masses. The resulting object, a hot proto-neutron star or a black hole surrounded by accreting material, lies at the heart of some of the most energetic observable events in the universe: type Ib/Ic/II supernovae, gamma-ray bursts and jet formation. These are all promising sources of gravitational waves, and their theoretical investigation is of paramount importance in order to understand their observational implications.

Astrophysicists have been long puzzled by the origins of the gamma-ray bursts since their incidental discovery in the late sixties. Some recent observations imply that the long-duration gamma-ray bursts are associated with core-collapse supernovae. From the theoretical point of view, the gamma-ray bursts are considered to be accompanied by the failed core-collapse supernovae, in which not the neutron star but the black hole is left behind. It is one of the most exciting issues to understand how the failed core-collapse supernovae can produce the observed properties of the gamma-ray bursts. In order to understand these astrophysical phenomena related to core-collapse supernovae and the properties of neutrino and gravitational-wave emissions, it is indispensable to understand the explosion mechanism of core-collapse supernovae. However, one still cannot express it exactly even with the elaborate efforts made during last 40 years or so. At present, detections of gravitational waves from nearby core-collapse supernovae are becoming a reality. Since gravitational wave can offer an alternative window that enables us to directly see the innermost part of core-collapse supernovae, information from gravitational wave is expected to help us to understand the explosion mechanism itself.

Present day understanding about core collapse supernova reveals that at the core bounce the core is supposed to oscillate in various modes - both radial and non-radial. These afterbounce oscillations are also expected to give rise to a quadrupole change in the structure and thus emit a low amplitude gravitational wave. Out of these modes, the $r$-mode oscillation caught the attention of the scientists in the later part of the last decade because such oscillation in a very rapidly rotating neutron star resulting from core collapse, can go unstable at all states of rotation and thus emit copious amount of gravitational radiation. This is a kind of run-away process that keep the star converting its rotational kinetic energy into gravitational wave energy and radiate out into space till the angular velocity of the star falls to a significantly low value. The gravitational waves so emitted would carry some vital information.
about the internal structure and dynamics of the star. The detection of such waves can lead to estimation of some vital parameters such as the nature of the Equation of State (EoS), Differential rotation, Magnetic field pattern etc.

1.2 Core Collapse dynamics

1.2.1 Collapse and bounce

About seventy five years ago W. Baade and F. Zwicky [23, 24] were the first to coin the term *Supernova*. They first pointed out that one of the brightest astronomical phenomenon, supernovae (SNe), can be due to explosion of massive stars at the end of their evolution. The formation of a dense neutron core (neutron star) results in a sudden energy release of order of the gravitational binding energy of the neutron star which amounts to $E_g \sim GM_{NS}^2/R_{NS} \approx 10^{53}$ ergs for the standard values of the Neutron star mass $M_{NS} \approx 1M_\odot$ and radius $R_{NS} \approx 10$ km.

Stars that can explode as supernovae are quite massive, with masses of more than about $8M_\odot$. At their centres, nuclear burning has processed all the way to iron, and they have an onion-like structure, with a core of iron, and lighter elements in shells around that core [10]. The outer shell consists mainly of hydrogen. When the core is mainly iron, there is no energy to gain from further fusion and the core is kept from collapsing only by the pressure of degenerate relativistic electrons, as in a white dwarf. The size of this *iron white dwarf* core is $\sim 1.5R_\oplus$ ($R_\oplus$ being the radius of the Earth), and its mass is slightly larger than the Chandrasekhar limit. The structure of the progenitor star depends on the stellar evolution models and therefore the size of the core is model-dependent, which introduces an uncertainty for the supernova models.

When the temperature of the core becomes large enough, two processes induce collapse to this core. First the electron captures by nuclei reduce the electron pressure while neutrinos produced carry away energy from the core. Second, for densities above $10^{10}$ g cm$^{-3}$, break-up of iron nuclei into helium ($^{56}Fe \rightarrow 13\alpha + 4n$), an endothermic process, is responsible for stealing a part of the photon pressure, thus cooling the core and leading to core collapse. At the time the collapse sets in, the core has a radius of about a few thousand km, a central density of $\rho \sim 10^{10}$ g cm$^{-3}$ and a
temperature of $T \sim 10^{10}$ K. Part of the gravitational binding energy of the core is released into kinetic energy which accelerates the infalling material to a significant fraction of the speed of light. Beyond densities of $\sim 10^{12}$ g cm$^{-3}$, neutrinos become trapped and the composition quickly approaches beta-equilibrium when the density exceeds $\sim 10^{13}$ g cm$^{-3}$. As the trapped neutrinos can escape only on their diffusion timescale which is larger than the collapse timescale, no further energy losses by neutrinos take place and the collapse proceeds almost adiabatically. While the density increases, beta-equilibrium shifts towards more neutron-rich matter. As the density approaches nuclear matter density, $\rho_n \sim 2 \times 10^{14}$ g cm$^{-3}$, the core gets divided into an inner homologous core and an outer core. The inner core collapses homologously (infall velocity proportional to the radius) and the outer core collapses supersonically. As the density reaches nuclear matter density, the nuclear forces between nucleons begin to play a prevailing role as the primary source of pressure. If the mass of the collapsing core is low enough not to undergo black hole formation, the collapse of the inner core is stopped at about 10 km, bouncing back and forming a strong shock wave [6, 25]. This shock plays a key role in most proposed explosion mechanisms [26, 27, 28, 29, 30, 31, 32]. For sufficiently high enough initial shock strength, the shock wave propagates outwards through the infalling outer core material, photodissociate iron nuclei and transform the kinetic energy of the shock into thermal energy. As the shock exits the neutrino-sphere, neutrinos leave freely carrying energy from the shock. As a consequence the shock stalls at about 100 km after a few 10 ms. At this time a hot proto-neutron star (PNS) is formed in the inner $\sim 30$ km, with a huge amount of neutrinos trapped inside. Neutrino diffusion timescales become important, and copious amount of thermal neutrino-antineutrino pairs are emitted, which carry away energy of the PNS and cool it to form a compact neutron star (NS) of $\sim 10$ km radius. A small fraction of the immense energy that leaves with the neutrinos is deposited behind the stalled shock which is revived, and a delayed supernova explosion is produced. As the explosion reaches lower density regions, the shock accelerates and disrupts the whole star carrying away most of the mass. Only a fraction of about $1 \ M_\odot$ remains at the centre forming the neutron star. The electromagnetic radiation emitted by the outer layers forms the observable supernova.

The collapse scenario described above is generally in the absence of angular momentum of the collapse progenitor star only. However, for a collapse of the progenitor endowed with an initial amount of angular momentum, i.e., rotating progenitor stars,
the collapse proceeds in a different way. Matter in the equatorial plane does not fall towards the center as fast as matter at the pole leading to a progressive flattening of the core. In addition, the equation of state and the collapse time scale is affected by rotation when compared to non-rotating collapse. Propagation of shock waves after core bounce may also be influenced by rotation as a consequence of the fact that the centrifugal forces in rotating cores may stop the collapse before nuclear matter densities are reached, which implies that the kinetic infall energy of the inner core and hence the initial shock energy of rotating cores is reduced [35, 36, 37].

Despite the significant recent progress in our understanding of core collapse so far, a number of questions need yet to be answered. The initial angular momentum distribution, its evolution with time, the role of shear viscosity and other factors such as magnetic field which can cause a redistribution of angular momentum that manifests itself in terms of differential rotation at the time of the collapse, are unknown. Further role of convective motion needs to be understood.

Study of gravitational waves from core collapse ought to be able to ease the situation by putting constraints on rotation states of iron core and outcoming neutron star, supernuclear EoS, degree of convection, the differential rotation, possible role of magnetic fields and others.

1.2.2 Gravitational Wave from Core Collapse

Not only electromagnetic radiation and neutrinos are emitted in the supernova explosion, but the asymmetric bulk motion of the collapsing star, that bounces at densities around nuclear matter density, produces a gravitational wave burst. Depending on how the progenitor star rotates and how the collapse proceeds, different types of waveforms are emitted [62]. Although the estimated amplitudes for realistic progenitors are small to be detectable in present-day gravitational wave detectors outside the Galaxy, other processes can lead to stronger gravitational waves after the collapse. In particular, convective motions behind the shock driven by neutrinos can lead to high amplitude gravitational waves even for slowly rotating cores [73].

Thus it is apparent that the modelling of progenitor star along with its rotational state has a key role to play in the emission of gravitational waves in core collapse supernova. For high enough rotation rates or for high degrees of differential rotation
nonaxisymmetric instabilities develop in dynamical timescales, namely the so-called bar-mode instability [80, 82], which produces strong gravitational wave signals. When the neutron star has cooled to about $10^{10}$ K after its formation, it can be subject to the Chandrasekhar-Friedman-Schutz instability [84, 85] and it becomes an important source of gravitational waves [86]. Therefore, a detailed modelling of the progenitor star is essential in making predictions about the emission of gravitational waves from neutron stars. Additional attention has also to be paid to the rotation rates, the distribution of angular momentum and the equation of state, as these parameters too, would play significant role in the evolution of rotating core collapse.

1.3 r-mode instability in neutron stars

Stars are not perfectly rigid systems; they naturally oscillate and the nonradial pulsations generate gravitational radiation that removes energy and angular momentum from the star. In nonrotating stars this process is always dissipative and inevitably, the oscillation of the star dies off. However, if the star rotates this is not necessarily the case: under certain conditions the amplitude of the pulsation mode grows and a Gravitational radiation (GR) instability sets in.

The oscillation patterns of a star are classified according to the dominant restoring force that tries to push a displaced fluid element back to its equilibrium position. The two main families of modes, which exists for any type of star irrespective of whether they are rotating or not, are those with pressure as restoring forces, called the p-modes and buoyancy to yield the so called g-modes. Studies of these different oscillation modes of a star can yield different pieces of physics. For a rotating star a new type of restoring force exists - the coriolis force which associate a new family of modes called the rotational modes or the r-modes and are familiar in geophysics as Rossby waves.

r-modes are primarily horizontal velocity perturbations and cause very small disturbances in the star’s density. More precisely, the velocity perturbations are of order $\Omega$, while the density perturbations are of order $\Omega^2$, where $\Omega$ is the star’s angular velocity. The unique feature that clearly distinguishes the r-modes from other stellar
modes, turn them potentially interesting for astrophysics is the fact that they are driven unstable by GR in all perfect-fluid rotating stars no matter how slowly they rotate. This property was discovered by Andersson [93] and confirmed theoretically by Friedman and Morsink [94]. So, unlike other modes, \( r \)-modes are always retrograde in the star's corotating frame and prograde in the inertial frame, i.e., the sign of the \( r \)-mode frequencies is always opposite in the two frames. In other words, the gravitational radiation driven instability known as Chandrasekhar-Friedman-Schutz (CFS) instability always occurs for the \( r \)-modes because the angular pattern speed \( \omega \) of a \( r \)-mode with angular quantum number \( l \) in an inertial frame satisfies the CFS condition \( 0 < \omega < \Omega \), for any value of the stellar rotation and for \( l \geq 2 \).

The gravitational wave amplitude, the emitted energy, and amount of angular momentum radiated depend sensitively on the \( r \)-mode dimensionless saturation amplitude \( \alpha_{\text{max}} \). If not limited by viscosity or magnetic fields, early optimistic estimates by Lindblom et al. [95, 88, 100, 101] saw the \( r \)-mode amplitude grow to \( \alpha_{\text{max}} \sim 1 \) within about 10 minutes of core bounce for a millisecond neutron star. Similar results were found by Stergioulas and Font [99].

Recent studies of non-linear mode couplings between the unstable \( r \)-mode and higher-order inertial modes by Arras et al. [108, 109] and non-linear 3D Newtonian hydrodynamics simulations by Gressman et al. [104] suggest that \( \alpha_{\text{max}} \ll 1 \). If \( \alpha_{\text{max}} \) is indeed \( \ll 1 \), then the gravitational wave emission from \( r \)-modes in compact stars is unlikely to be detectable.

### 1.3.1 Gravitational waves from \( r \)-mode instability of neutron stars

Studies so far reveal that \( r \)-mode instability is capable of emitting, though not as promising as a burst source, a reasonable amount of gravitational wave lasting for about a year. Besides, emission of gravitational wave from \( r \)-mode instability can throw some lights on the millisecond pulsars.
1.4 Objectives of the present study

Our interest in this work will be to look into the problem of rotating core collapse supernova and \( r \)-mode instability of the hot young rapidly rotating neutron stars from the viewpoint of gravitational wave emission. Since a hot young and rapidly rotating neutron star is the outcome of a core collapse supernova, it is felt that a basis for future studies in gravitational waves is to be constructed for treating these two separate issues as a single integrated issue so that the outcome of the gravitational waves from core collapse supernova could be fed straight as the starting point from the \( r \)-mode instability from a hot young rapidly rotating neutron star. To this end, it is desired that a fresh look into the behaviours of various parameters of both the above issues would provide a better and better understanding the kinematics or the dynamics of the entire issue. It is always best to assume very simple models to start and investigate the parameters rigorously, their roles in the emission of gravitational waves and so on. This thesis encompasses our work in this line, i.e., understanding the parameters governing the core collapse supernova and the \( r \)-modes with the help of simplest possible starting models.

The main objective of this thesis is the numerical simulation of the gravitational radiation pattern produced in an idealised gravitational core collapse and study the functional dependence of collapse parameters, i.e., the influence of equation of state, angular velocity and differential rotation in the gravitational wave emission. Further, we numerically study the growth of \( r \)-mode instability in a hot newborn model neutron star and evaluate the role of differential rotation and magnetic field in the emission of gravitational waves along with their detectibility in the currently operating earth-bound detectors, LIGO, advanced LIGO and VIRGO.

1.5 Organisation of the thesis

This thesis is organised as the following:

In Chapter 2, we describe the theoretical framework of gravitational waves, their basic characteristics, generation and propagation properties. We also present the gravitational wave extraction formula suitable for implementation in numerical core collapse simulations. The last section of this chapter describes the numerical hy-
dynamical code, the ZEUS-2D, that has been applied to simulate the collapse of various models.

In Chapter 3, we describe our preparation for parameter and model selections to be fed into the numerical simulation code for core collapse simulations. We describe the simulation results in the form of Tables and graphs. The inferences drawn from the investigation is incorporated towards the last part of this chapter.

In Chapter 4, we derive our equations to deal with $r$-mode instability in a typical neutron star by incorporating differential rotation as well as a simple dipole magnetic field. Previous studies included either the differential rotation or the magnetic field only. Our evolution equations are numerically solved for different values of differential rotation parameters and magnetic field strengths, the gravitational wave output are evaluated along with the detectibility studies for the interferometric detectors LIGO, Advanced LIGO and VIRGO.

In Chapter 5, we summarise the results of the investigations carried out. Also we present the future scopes and possible outlook in core collapse and $r$-mode instability studies.

Detailed Reference and Appendices relevant to the thesis are included at the end.