Chapter 8

Summary and Outlook

The focus of the thesis has been on the phenomenological study of a class of neutrino models involving three active neutrinos in the framework of see-saw mechanism. These models are capable of describing the atmospheric neutrino oscillation data, the LMA MSW solar solution which is now uniquely specified by recent data, and the constraint from reactor angle.

In the introductory chapter we have presented some survey work on neutrino physics. We have briefly reviewed the historical background of neutrinos and their implications in other branches of physics. Then we have given the theory of neutrino oscillations in vacuum and in matter with constant and varying densities. In a semi-quantitative way, we have discussed the MSW neutrino flavor conversion in the adiabatic approximation. Then we have presented brief notes on some of the neutrino experiments which are either completed or presently taking data or to start taking data in near future. In this chapter we have introduced see-saw mechanism as an extension of the Standard Model (SM) of particle physics. In this mechanism, SM is extended to include the right-handed Majorana neutrinos, one per family of the fermions. These right-handed Majorana neutrinos are singlets under the transformation of the standard model gauge group. So they are not restricted to have very high mass also. As a result, the see-saw mechanism can generate tiny mass of the left-handed Majorana neutrinos required for
the phenomenon of the neutrino oscillation. Two of the different possible realisations of see-saw mechanism, viz., Type-I and Type-II see-saw formulae are presented through a cursory glance on the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Then we have presented the scope of the thesis outlining some of the problems in neutrino physics and our attempt for resolving some of them.

In our work, we have been motivated to explore the light left-handed Majorana neutrino mass matrices with the non-diagonal texture of the right-handed Majorana neutrino mass matrix $M_R$ and the diagonal Dirac neutrino mass matrix $m_{LR}$ in see-saw formulae, which can describe the current data. In the present analysis we have assumed $m_{LR}$ to be either diagonal charged lepton type, or up-quark type mass matrix. In this case, one can expect that the left-handed rotations diagonalising $m_{LR}$ will not exceed the Cabbibo mixing in quark sector which is very small. Thus the large leptonic mixing observed in low-energy sector is a consequence of the see-saw enhancement of leptonic mixing. Such an enhancement requires a strong (quadratic) mass hierarchy of RH neutrinos and/or off-diagonal structure of $M_R$.

So far, the neutrino oscillation experiments can provide us the values of the solar and the atmospheric neutrino mass squared differences, not the absolute scale of the neutrino mass. Based on this information, three types of neutrino mass patterns can be realised, which are, namely, normal hierarchical, inverted hierarchical, and nearly degenerate. Chapters 2 & 3 deal with construction of normal hierarchical (Type [III]), inverted hierarchical (Type [IIA] & [IIB]) and nearly degenerate neutrino mass models (Type [IA], [IB] & [IC]) [77, 89, 96]. These models can predict the experimental best fit values of neutrino oscillation parameters available during the preparation of our original published papers: $\Delta m^2_{21} = 4.2 \times 10^{-5}eV^2$, $\Delta m^2_{23} = 3 \times 10^{-3}eV^2$, $\sin^2 2\theta_{12} = 0.819$, $\sin^2 2\theta_{23} = 1$, CHOOZ angle $\sin \theta_{13} \leq 0.16$ and mass splitting parameter $\xi = \frac{\Delta m^2_{21}}{\Delta m^2_{23}} = 0.014$, except the degenerate model of the Type[IA] and inverted hierarchical model of
the Type[IIB] predicting nearly maximal solar mixing $\sin^2 2\theta_{12} = 0.999$. Any change of the parameters of the inverted hierarchical mass matrix can hardly affect the almost maximal solar mixing.

The above outstanding problem of the prediction of nearly maximal solar mixing of the inverted hierarchical model (Type [IIB]) is addressed in chapter 4. Several attempts [69, 70, 97, 99] were made to tone down the solar mixing angle within the observational bound. We present a novel solution [101] to the problem by considering charged lepton mixing matrix $V_{eL}$ having a special structure in 1-2 block. This leads to the corresponding charged lepton mass matrix having special structure in 1-2 block also. The new leptonic mixing matrix $V_{MNS}$ taking the contribution from $V_{eL}$ predicts the following mixing angles: $\sin^2 2\theta_{12} = 0.8517, \sin^2 2\theta_{23} = 0.9494$ and $|V_{e3}| = 0.159$ which are consistent with LMA solar solution. The neutrino mass eigenvalues which are already in agreement with the experimental predictions without taking the contribution from the charged lepton, remain unchanged. We have checked the radiative stability of the model under the running of renormalisation group equations in MSSM. In the process, the neutrino mass matrix has been subject to evolution under RGEs in MSSM, on running from the unification scale $2 \times 10^{16}$ GeV down to top-quark mass scale (174 GeV) for large $\tan \beta \sim (50 - 60)$. We observed that the neutrino mixing angles have undergone a very mild deviation, which indicates the model being stable against radiative correction.

In the second part of this chapter, we have also revisited the inverted hierarchical model (Type[IIB]) [146] in the light of new parametrisation on parameter space $(\tan^2 \theta_{12} - \Delta m^2_{31})$ where $m_2 > m_1$. In this new parametrisation the global analysis of data [32, 125] favours the "light side" of the data, $\tan^2 \theta_{12} < 1$ and disfavours the "dark side" $\tan^2 \theta_{12} > 1$. The model can predict solar mixing within the "light side" of parameter space. We have shown the correlations among the neutrino mixing angles
and expectation value $< m_{ee} >$ of the Majorana neutrino measured in $\beta\beta0\nu$, through the CHOOZ mixing angle $| V_{e3} |$. The important observation is that the correction of solar mixing from the charged lepton, always leads to non-zero value of CHOOZ angle. The radiative stability of the inverted hierarchical model still persists with $\tan^2 \theta_{12} < 1$.

The see-saw predictions of neutrino oscillation parameters are at high-energy scale, say, nearly at unification scale, while the experimental predictions are at low-energy scale. Therefore, the radiative stability of a see-saw model on the running of RGEs in MSSM which can keep on its predictions of the neutrino oscillation parameters with at best, the mild deviation from their values at see-saw scale to those at the top-quark mass scale, is a preferred aspect of a promising neutrino model. In chapter 5, two popular approaches for checking the radiative stability of a neutrino mass matrix in MSSM have been discussed [92]. In one approach [82, 106] the whole neutrino mass matrix is subject to the running of RGEs in MSSM while in the other approach [102], only the mass eigenvalues and mixing angles are subject to renormalisation group evolution, we referred to the first approach as ‘method A’ and the second one as ‘method B’. We have outlined them and studied their numerical consistency for estimation of radiative correction to neutrino masses and mixings. Our numerical analysis revealed that they can predict the same values of mixing angles for the same input values whereas there arises 4.2% difference in predicting mass eigenvalues on running vacuum expectation value (VEV) and 13% for scale-independent VEV. The hierarchical model (Type[III]) and inverted hierarchical model (Type IIB) having opposite CP phase are stable in both approaches. The inverted hierarchical model (Type[IIB]) having same CP phase and degenerate models (Type[IA,IB,IC]) are unstable in both approaches. The radiative unstability of these models remains as an outstanding problem.

We have also carried out numerical analysis on some important conjectures related to radiative corrections in MSSM, viz., radiative magnification of solar and atmospheric
mixings in case of nearly degenerate model having same CP parity (MPR conjecture [139]) and radiative generation of solar mass scale in exactly two-fold degenerate model having opposite CP parity and non-zero \( |U_{e3}| \) (JM conjecture) [138]. We have observed in the MPR conjecture that the running of masses and mixings with or without running of the gauge and the Yukawa couplings, leads to reasonably different magnifications. However, such radiative amplification requires a delicate fine tuning of initial conditions which are not natural [144]. We have investigated JM conjecture [138] and in contrary to the conjecture, we have been able to show radiative generation of solar mass scale even with \( U_{e3} = 0.0 \) and non-zero value of \( m_3 \) at higher scale. In our analysis, we have noticed that the MST [143] conjecture which states that starting with the bimaximal mixings at high scale, the radiative correction leads the solar angle to the dark side of the data at low energy, is not always true. Another observation is that the running VEV gives rise to the magnification factors of 1.68 and 1.91 in neutrino masses at low-energy in methods A & B respectively, compared to the values calculated without running VEV. There is a word of caution that oversimplification of analytical solution and a departure from the simultaneous running of Yukawa and gauge couplings along with neutrino masses and mixings may lead to some misleading results.

In chapter 6 we have tried to discriminate three types of neutrino models (obtained from Type-I see-saw formula) in presence of the Type-II see-saw contribution [186], and this may represent a unique choice of Nature. We have constructed neutrino models of three mass patterns, viz., normal, inverted and degenerate from Type-I seesaw formula, which can predict the values of neutrino parameters in agreement with their experimental values (see chapter 2 & 3). We have considered the term (non-canonical) arising from the left-handed Higgs triplet of Type-II seesaw formula as a perturbation to the other term (canonical) representing Type-I seesaw formula. The Type-I seesaw model which is less perturbed in retaining its original good predictions in presence of the non-canonical term, is more favourable. In other words, the perturbations arising from non-
canonical term are rather small to distort the main canonical predictions. Using this hypothesis we have found that the inverted hierarchical model having even CP phase (Type[IIA]) is the most favourable one. The next to it is the normal hierarchical model (Type[III]). The degenerate models are highly perturbed. This analysis is independent of the textures of $m_{LR}$ and $M_{RR}$. From the viewpoint of radiative stability, Type[IIIB] and [III] are found to be stable under radiative correction in MSSM but Type[IIA] and [IA,B & C] are found to be unstable. Our work is just to get some clues for understanding the model of definite neutrino mass pattern which is an important issue of neutrino physics.

Lacking of the specific method for estimating error/uncertainty propagation in any mathematical calculation, is an outstanding problem in particle physics. Therefore, we have explored for a reliable method and found the quadrature method consistent and reliable for the purpose. In chapter 7 we have dealt with the quadrature method [201] outlined with some useful examples. This method has been employed in estimation of uncertainties in the QCD-QED rescaling factors, $\eta_f$ acquired from the experimental values of the gauge coupling constants $\alpha_3(M_Z)$ and $\alpha(M_Z)$ at electro-weak scale $M_Z$. These $\eta_f$'s govern the running of the fermion mass renormalisation from the top-quark mass scale down to the physical fermion mass scale. There involves several intermediate steps in calculation of the rescaling factors which may induce unbelievably large uncertainties if estimated from the usual minimum-central-maximum value method. But the quadrature method regulates the propagation of uncertainties involving multi-step calculation and using this method we have found new and low symmetric uncertainties in QCD-QED rescaling factors in comparison with earlier attempts [115, 190, 191, 192].

A careful and reliable estimation of uncertainties in rescaling factors is desirable as they are used for low energy predictions of fermion masses, such as low energy prediction of neutrino masses obtained in seesaw mechanism, otherwise uncertainties of very low neutrino masses overshoot the central values which is discouraging. We have found the
quadrature method a reliable and useful technique for controlling the propagation of uncertainties.

We conclude the thesis with the following important observations and remarks.

(a) The non-zero texture of the right-handed Majorana mass matrices $M_R$ presented in the thesis, are not only suitable for left-handed Majorana neutrino mass matrix accounting for the current experimental data, but also useful for study of baryogenesis through leptogenesis [203]. There is a scope for the extension of the present work to this interesting field of research.

(b) The degenerate and inverted hierarchical models having the mass eigenvalues with the same CP phase, are unstable under running of renormalisation group equations (RGEs) in minimal super symmetric model (MSSM), with or without the running of phases, and it remains as an outstanding problem.

(c) The left-handed Majorana neutrino mass matrices of inverted hierarchy (Type[IIA,B]) having 2-3 symmetry can be parametrised with two parameters [204] and their ratio is identified as “flavor twister” [205]. Hence they are able to predict tri-bimaximal mixing or its deviation. But the radiative stability of the model Type[IIB] in MSSM is apparently lost. This is another outstanding problem to be studied in details.

(d) Since all the neutrino mass matrices presented in this thesis, are constructed in a model independent way, so the structures of the matrices may also represent a useful guidance for model building from the general structure of gauge symmetry.

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