V. GEP FOR COHERENT STATES AND SQUEEZED STATES OF ANHARMONIC OSCILLATOR

5.1 Introduction

For the past three decades, developments in the field of coherent states and their applications have been widely discussed. The history of study of coherent states goes back to the early days of quantum mechanics, when around 1926 Schrödinger [43] reported the existence of a certain class of states that display the classical behaviour of the oscillator. Later Glauber [44] called these states coherent states and applied them to the radiation field. In the literature the coherent states have been called the minimum uncertainty product states in the sense that the relation $\text{var}(x)\text{var}(p) = \frac{1}{4}$ holds for the $|0\rangle$ or vacuum state.

The minimum uncertainty method has been applied to general Hamiltonian potential systems, to obtain both generalized coherent states and generalized squeezed states [111-114]. Recently, to produce the wave function of a quantum anharmonic oscillator, a variational procedure has been proposed [115] and found that the wave function looks like a coherent state constructed from the solution of the classical equations of motion with a semiclassical construction. In this chapter we calculate the GEP of an anharmonic oscillator in coherent states and squeezed states.
5.2 Coherent states

We choose the Hamiltonian:

$$H = \frac{1}{2} p^2 + \frac{1}{2} m^2 x^2 + \lambda x^4$$  \hspace{1cm} (5.1)

where

$$x = x_\alpha + \left(\frac{\hbar}{2\omega}\right)^{1/2} (a + a^*_\alpha)$$  \hspace{1cm} (5.2)

$$p = -\frac{i\hbar}{2} (a - a^*_\alpha), x_\alpha = \langle\alpha|x|\alpha\rangle$$  \hspace{1cm} (5.3)

and $\omega$ is the mass parameter. The subscript $\omega$ is a reminder that $a_\alpha$ and $a^*_\alpha$ depend on the frequency of the harmonic oscillator.

The state $|\alpha\rangle$ is the coherent state which can be taken as a Gaussian trial wave function which depends on $\omega$. To evaluate GEP, we determine the minimum of the expectation value of the Hamiltonian in the coherent state:

$$V_G(x_\alpha) = \min_{\omega} V_G(x_\alpha, \omega) = \min_{\alpha} \langle\alpha|H|\alpha\rangle$$  \hspace{1cm} (5.4)

Term by term, we have

$$\langle\alpha|\frac{1}{2} p^2 |\alpha\rangle = -\frac{\hbar^2}{4} (\alpha^2 - 1 - 2|\alpha|^2 \alpha^*)$$  \hspace{1cm} (5.5)

$$\langle\alpha|\frac{1}{2} m^2 x^2 |\alpha\rangle = \frac{1}{2} m^2 (x_\alpha^2 + \left(\frac{\hbar}{2\omega}\right)^{1/2} 2x_\alpha (\alpha + \alpha^*)$$

$$+ \frac{\hbar}{2\omega} (\alpha^2 + \alpha^*^2 + 1 + 2|\alpha|^2))$$  \hspace{1cm} (5.6)

$$\langle\alpha|\lambda x^4 |\alpha\rangle = \lambda x^4_\alpha + 4x_\alpha^3 \lambda \left(\frac{\hbar}{2\omega}\right)^{1/2} (\alpha + \alpha^*)$$

$$+ 6x^2_\alpha \lambda \left(\frac{\hbar}{2\omega}\right) (\alpha^2 + \alpha^*^2 + 1 + 2|\alpha|^2)$$
On summation we get the expectation value of the Hamiltonian as

$$
\langle \xi | H | \alpha \rangle = \sum_{\lambda=0}^{4} C_{\lambda} X_{\alpha} = V_{G}(X_{\alpha}, \xi) \tag{5.8}
$$

where

$$
C_{0} = \frac{-\hbar \xi}{4} (\xi^{2}-1-2|\alpha|^{2}+\alpha^{*}^{2}) + \frac{\hbar m^{2}}{4 \xi \Lambda} (\xi^{2}+\alpha^{*}^{2}+1+2|\alpha|^{2})
$$

$$
+ \lambda (\frac{\hbar}{2 \xi \Lambda})^{3/2} (\alpha^{3}+\alpha^{*}^{3}+3\alpha+3\alpha|\alpha|^{2}) + 3|\alpha|^{2} + 6(\alpha^{2}+\alpha^{*}^{2})
$$

$$
+ 4|\alpha|^{2}(\alpha^{2}+\alpha^{*}^{2})+6|\alpha|^{4}+12|\alpha|^{2}+3
$$

$$
C_{1} = m^{2}(\frac{\hbar}{2 \xi \Lambda})^{1/2} (\alpha+\alpha^{*}) + 4\hbar (\frac{\hbar}{2 \xi \Lambda})^{3/2} (\alpha^{3}+\alpha^{*}^{3}+3\alpha+3\alpha^{*}+3|\alpha|^{2}(\alpha+\alpha^{*}))
$$

$$
C_{2} = \frac{1}{2} m^{2} + \frac{3\hbar \Lambda}{\xi \alpha} (\alpha^{2}+\alpha^{*}^{2}+1+2|\alpha|^{2})
$$

$$
C_{3} = 4\hbar (\frac{\hbar}{2 \xi \Lambda})^{1/2} (\alpha+\alpha^{*})
$$

$$
C_{4} = \lambda
$$

Minimizing the quantity $\langle \xi | H | \alpha \rangle$, we get the optimum condition for $\xi$ as

$$
\frac{3}{\xi}(\xi^{2}-1-2|\alpha|^{2}+\alpha^{*}^{2}) + (\frac{\hbar}{2})^{-1/2} \xi^{3/2} (\alpha+\alpha^{*})
$$

$$
(m^{2}X_{\alpha}+4\hbar X_{\alpha}^{3}) + 2\hbar (\alpha^{2}+\alpha^{*}^{2}+1+2|\alpha|^{2}) (\frac{1}{2} m^{2}+6\hbar X_{\alpha}^{2})
$$

$$
+ (\frac{\hbar}{2})^{1/2} \xi^{1/2} 12\hbar X_{\alpha}(\alpha^{3}+\alpha^{*}^{3}+3(\alpha+\alpha^{*}) + 3|\alpha|^{2}(\alpha+\alpha^{*})) \tag{5.9}
$$

$$
+ 2\hbar \lambda (\alpha^{4}+\alpha^{*4}+6(\alpha^{2}+\alpha^{*}^{2})+ 4|\alpha|^{2}(\alpha^{2}+\alpha^{*}^{2})+ 6|\alpha|^{4}+12|\alpha|^{2}+3) = 0
$$

This equation can have six roots and the largest positive root, designated as $\bar{\xi}$, is to be employed by convention [16], when the effective potential is calculated.
If $a$ is real, we have the equation as

$$
\frac{S^3}{2} - 2\left(\frac{\hbar}{2}\right)^{-1/2} a S^{3/2} \left( m^2 + \hbar^2 x^2 \right) - 12 \left(\frac{\hbar}{2}\right)^{1/2} \left( u S^{1/2} \right) \left( 8a^3 + 6a \right) - 2\hbar \left( 16a^4 + 24a^2 + 3 \right) = 0 \tag{5.10}
$$

$$
\frac{\delta V}{\delta x} \bigg|_{x = 0} = \frac{dV}{dx} \tag{5.11}
$$

Since $\frac{\delta V}{\delta x}$ vanishes at $x = 0$,

$$
\frac{dV}{dx} = m^2 \left( \frac{\hbar}{2\hbar} \right)^{1/2} \left( a + a^* \right) + 4 \left( \frac{\hbar}{2\hbar} \right)^{3/2} \left( a^3 + a^* 3 + 3a + 3a^* + 3|a|^2 (a + a^*) \right)
+ 2x \left( \frac{\hbar}{2\hbar} \right)^{1/2} \left( a^2 + a^* 2 + 1 + 2|a|^2 \right)
+ 3x^2 \left( 4 \left( \frac{\hbar}{2\hbar} \right)^{1/2} \left( a + a^* \right) \right) + 4 \lambda \nu^3 \tag{5.12}
$$

We define the effective mass $m_c^2(a)$ of the anharmonic oscillator by the relation

$$
m_c^2(a, a^*) = \left. \frac{d^2 \mathcal{V}}{dx^2} \right|_{x = 0} = \left. \frac{m^2 \left( \frac{\hbar}{2\hbar} \right)^{1/2} \mathcal{V}_0 - 3/2 (a + a^*) \frac{d \mathcal{V}}{dx} \right|_{x = 0}
- 6\lambda \left( \frac{\hbar}{2\hbar} \right)^{3/2} \mathcal{V}_0 - 5/2 (a^3 + a^* 3 + 3(a + a^*)
+ 3|a|^2 (a + a^*) \right) \frac{d \mathcal{V}}{dx} \bigg|_{x = 0}
+ \frac{m^2 + 6\lambda \hbar}{\mathcal{V}_0} \left( a^2 + a^* 2 + 1 + 2|a|^2 \right) \tag{5.13}
$$

where $\frac{d \mathcal{V}}{dx} \bigg|_{x = 0}$ can be obtained from (5.9) as

$$
\frac{d \mathcal{V}}{dx} \bigg|_{x = 0} = -\left. \frac{1}{2} \lambda \left( \frac{\hbar}{2\hbar} \right)^{-1/2} \mathcal{V}_0 \left( m + a + a^* \right) + 12 \lambda \left( \frac{\hbar}{2\hbar} \right)^{1/2} \mathcal{V}_0 \left( a^3 + a^* 3 + 3a \right)
$$
Here, $\overline{\omega}_0 = \overline{\omega}$ at $X = 0$

For the ground state expectation value of the Hamiltonian [16],
\[ \frac{d\overline{\omega}}{dX_0}|_{X=0} = 0, \] and hence (5.14) gives marked difference in the
behaviour of $\overline{\omega}$ from the ordinary ground state and excited
state results.

Substituting (5.14) into (5.13), we have
\[
m^2_c(\alpha) = m^2 + \frac{6\lambda \hbar}{\overline{\omega}_0} (\alpha^2 + \alpha^{*2} + 1 + 2|\alpha|^2) + \left[ \frac{m^4}{4}(\alpha + \alpha^*)^2 \right.
\]
\[ + 9m^2\lambda \left( \frac{\hbar}{2} \right) \overline{\omega}_0^{-1}(\alpha + \alpha^*) + 18\lambda^2 \hbar^2 \overline{\omega}_0^{-2}
\]
\[ \left. (\alpha^3 + \alpha^{*3} + 3(\alpha + \alpha^*) + 3|\alpha|^2(\alpha + \alpha^*))^2 \right] / [3 \overline{\omega}_0^2(a^2 - 1 - 2|\alpha|^2 + \alpha^{*2}) +
\]
\[ m^2(\alpha^2 + \alpha^{*2} + 1 + 2|\alpha|^2)] (5.15) \]

We may also introduce an effective coupling constant $\lambda_C(\alpha, \alpha^*)$ for the coherent state $|\alpha\rangle$ through the relation
\[
\lambda_C(\alpha, \alpha^*) = \frac{1}{4!} \left( \frac{d^4 V_G}{d\alpha^4} \right)|_{\alpha = 0} (5.16)
\]

Then on calculation, we find
\[
\frac{1}{4!} \left. \frac{d^4 V_G}{d\alpha^4} \right|_{\alpha = 0} = \frac{1}{4!} \left[ - \left( \frac{\hbar}{2} \right)^{1/2} \overline{\omega}_0^{-3/2} (\alpha + \alpha^*) \right.
\]
\[ + 6\lambda \left( \frac{\hbar}{2} \right)^{3/2} \overline{\omega}_0^{-5/2} (\alpha^3 + \alpha^{*3} + 3\alpha + 3\alpha^*)
\]
\[ + 3|\alpha|^2(\alpha + \alpha^*) \] \[ \left. \frac{d^2 \overline{\omega}}{d\alpha^4} \right|_{\alpha = 0} \]
\[-\frac{18\lambda}{\bar{\nu}_o^2}(a^2 + a^* + 2 + 2|a|^2) \left. \frac{d^2 \bar{\nu}}{d\alpha^2} \right|_{\alpha=0} \]
\[+ 3 \left[ \frac{3m^2}{2} \right]^{1/2} \bar{\nu}_o^{5/2} (a + a^*) \]
\[+ 15\lambda (\bar{\nu}_o)^{3/2} \bar{\nu}_o^{-7/2} \]
\[+ 3 |a|^2 (a + a^*) \right] \left. \frac{d^2 \bar{\nu}}{d\alpha^2} \right|_{\alpha=0} \]
\[+ 3 \left[ \frac{36\lambda}{\bar{\nu}_o^3} \right]^{1/2} \bar{\nu}_o^{-3/2} (a + a^*) \]
\[- 36\lambda (\bar{\nu}_o)^{1/2} \bar{\nu}_o^{-7/2} (a + a^*) \]
\[- \left[ m^2 (\bar{\nu}_o)^{1/2} \right] \frac{15}{8} \bar{\nu}_o^{-7/2} (a + a^*) \]
\[+ \frac{105}{2} \lambda (\bar{\nu}_o)^{3/2} \bar{\nu}_o^{-9/2} (a + a^* + 3a + 3a^*) \]
\[+ 3 |a|^2 (a + a^*) \right] \left. \frac{d^3 \bar{\nu}}{d\alpha^3} \right|_{\alpha=0} + 24\lambda \]

(5.17)

where

\[\left. \frac{d^2 \bar{\nu}}{d\alpha^2} \right|_{\alpha=0} = \left[ - \left[ \frac{3m^2}{2} \right]^{1/2} \bar{\nu}_o^{1/2} (a + a^*) \right. \]
\[+ (\bar{\nu}_o)^{1/2} \bar{\nu}_o^{-1/2} 12\lambda (a + a^* + 3a + 3a^* + 3|a|^2 (a + a^*) \right] \left. \frac{d \bar{\nu}}{d\alpha} \right|_{\alpha=0} \]
\[+ 24\lambda \bar{\nu}_o (a^2 + a^* + 2 + 2|a|^2) \left. \right] \]

(5.18)

and

\[\left. \frac{d^3 \bar{\nu}}{d\alpha^3} \right|_{\alpha=0} = \left[ - \left[ \frac{3m^2}{2} \right]^{1/2} \bar{\nu}_o^{1/2} (a + a^*) \right. \]
\[+ 24\lambda \bar{\nu}_o (a^2 + a^* + 2 + 2|a|^2) \left. \right] \]
+ $18\left(\frac{\hbar}{2}\right)\frac{1}{2\pi\hbar^2} - \frac{1}{2} \quad (\alpha^3 + \alpha^*^3 + 3(\alpha^2 \alpha^*))$

+ $3|\alpha|^2(\alpha + \alpha^*) \left[ \frac{\delta}{dx_{\alpha}} \bigg| x_{\alpha} = 0 \right]$

- $18\tilde{\alpha}_0 (\alpha^2 - 1 - 2|\alpha|^2 + \alpha^2 \alpha^*) \left[ \frac{\delta}{dx_{\alpha}} \bigg| x_{\alpha} = 0 \right]$

- $\left[ \frac{9m^2}{4} \left( \frac{\hbar}{2}\right)^{-1/2} \frac{\hbar}{\pi} \right]^{1/2} (\alpha + \alpha^*)$

- $\left( \frac{\hbar}{2}\right)^{1/2} \frac{\pi}{2} \frac{\hbar}{\pi}^{3/2} \alpha^3 + \alpha^*^3 + 3(\alpha^2 \alpha^*)$

+ $3|\alpha|^2(\alpha + \alpha^*) \left[ \frac{\delta}{dx_{\alpha}} \bigg| x_{\alpha} = 0 \right]$

- $72(\alpha^2 + \alpha^*^2 + 1 + 2|\alpha|^2) \left[ \frac{\delta}{dx_{\alpha}} \bigg| x_{\alpha} = 0 \right]$

- $6(\alpha^2 - 1 - 2|\alpha|^2 + \alpha^2 \alpha^*) \left[ \frac{\delta}{dx_{\alpha}} \bigg| x_{\alpha} = 0 \right]^3$

- $\left( \frac{\hbar}{2}\right)^{-1/2} \frac{\pi}{2} (\alpha + \alpha^*) 24 \lambda^2$

$(3\tilde{\alpha}_0^2 (\alpha^2 - 1 - 2|\alpha|^2 + \alpha^2 \alpha^*) + m^2(\alpha^2 + \alpha^*^2 + 1 + 2|\alpha|^2))$ \hspace{1cm} (5.19)

To obtain the effective potential, we integrate the equation (5.12) with respect to $x_{\alpha}$ and get

$$\bar{V}_G = \frac{1}{2} m^2 x_{\alpha}^2 + m^2 x_{\alpha}^2 \left( \frac{\hbar}{2\pi} \right)^{1/2} (\alpha + \alpha^*)$$

+ $\lambda x_{\alpha}^4 + 4 \lambda x_{\alpha}^3 \left( \frac{\hbar}{2\pi} \right)^{1/2} (\alpha + \alpha^*)$

+ $\lambda x_{\alpha}^2 \left( \frac{\hbar}{2\pi} \right) (\alpha^2 + \alpha^*^2 + 1 + 2|\alpha|^2) + 4 \lambda x_{\alpha} \left( \frac{\hbar}{2\pi} \right)^{3/2} (\alpha^3 + \alpha^*^3 + 3(\alpha + \alpha^*)$

+ $3|\alpha|^2(\alpha + \alpha^*)$ \hspace{1cm} (5.20)

Here the effective potential $\bar{V}_G$ is real even though the coherent state parameter $\alpha$ may be complex. On inspection of the expression
for the effective coupling constant $\lambda_c(\alpha, \alpha^*)$ it is seen that it contains some non-analytic terms in $\hbar$, namely those in negative powers of $\hbar$. As a result $\lambda_c$ diverges in the limit $\hbar \to 0$. However $\lambda_c$ remains finite in this limit if the 'bare mass parameter' $m$ is taken to zero. It is also noted that there is no similar problem in the ground state $|\phi\rangle$.

5.3 Squeezed states

The generalizations of coherent states namely, squeezed states, have become of more and more interest in recent times [48,116-122]. A wide range of applications have been suggested, ranging from gravitational wave detection and polariton theory [123] to low-noise optical communications, to the inhibition of atomic phase decay [124]. Recently, there have been several attempts at generalizing the notion of squeezing. Very recently, Nieto [125] generalized the notion of squeezed states to arbitrary symmetry systems and discussed its relationship to squeezed states obtained for general potentials. It is shown that the coherent light interacting with a nonlinear nonabsorbing medium modelled as anharmonic oscillator can also give rise to the amplitude-squared squeezing effect [126]. This model system has previously been shown to give rise to usual second order squeezing in terms of the field amplitude [127]. In this section we extend the GEP method to define the effective potential of anharmonic oscillator for squeezed states $|\beta\rangle$ defined by (1.87) and (1.88).
Expressing $X$ and $P$ in the form

$$
X = X_\beta + \left(\frac{\hbar}{2\pi}\right)^{1/2} (a_\beta + a_\beta^+) \\
P = -\frac{i}{2}(2\hbar\pi)^{1/2} (a_\beta - a_\beta^+),
$$

where $X_\beta = \langle\beta|X|\beta\rangle$,

we calculate the expectation value for each term of the Hamiltonian given by (5.1) for a squeezed state $|\beta\rangle$ with the operator

$$b = \mu a + \nu a^+;$$

$$
\langle\beta|\frac{1}{2}P^2|\beta\rangle = -\frac{1}{4}\hbar\pi[\mu^*2\beta^2 + \nu^*2\beta^2 - \mu^*(2|\beta|^2+1) + \mu^2\beta^2 \\
- \nu^*(2|\beta|^2+1) + \nu^2\beta^2 - 2(|\mu|^2 + |\nu|^2)|\beta|^2 - |\nu|^2 \\
- 2\nu^*\beta^2 + 2\nu^2\beta^2 - |\mu|^2 ]
$$

(5.21)

$$
\langle\beta|\frac{1}{2}m^2x^2|\beta\rangle = \frac{1}{2}m^2x_\beta^2 + m^2x_\beta + \left(\frac{\hbar}{2\pi}\right)^{1/2} (\mu^* - \nu^* + \mu^* - \nu^*) \\
+ \frac{\hbar}{2\pi}(\mu^*2\beta^2 + \nu^*2\beta^2 - \mu^*(2|\beta|^2+1) + \mu^2\beta^2 \\
+ \nu^*2\beta^2 - \mu^*(2|\beta|^2+1) + 2(|\mu|^2 + |\nu|^2)|\beta|^2 - |\nu|^2 \\
- 2\nu^*\beta^2 - 2\nu^2\beta^2 )
$$

(5.22)

$$
\langle\beta|\lambda x^4|\beta\rangle = \lambda x_\beta^4 + 4\lambda x_\beta^3 \left(\frac{\hbar}{2\pi}\right)^{1/2} A + 6\lambda x_\beta^2 \left(\frac{\hbar}{2\pi}\right)^{1/2} B \\
+ 4\lambda x_\beta \left(\frac{\hbar}{2\pi}\right)^{3/2} C + \left(\frac{\hbar}{2\pi}\right)^2 D
$$

(5.23)

where

$$A = \mu^* \beta - \nu^* \beta + \mu^* \beta - \nu^* \beta$$

$$B = \mu^*2\beta^2 + \nu^*2\beta^2 - \mu^*\nu^*(2|\beta|^2+1) + \mu^2\beta^2 + \nu^2\beta^2 - \mu\nu^*(2|\beta|^2+1) + 2(|\mu|^2 + |\nu|^2)|\beta|^2 + |\mu|^2 + |\nu|^2 \\
- 2\mu^*\nu^*\beta^2 - 2\mu\nu\beta^2$$
\[ C = \beta^3 (\mu^* \gamma^* - \gamma \mu^* \gamma^* + 3 \gamma^* \mu^* \gamma^* ) \\
+ \beta^3 (\mu^* \gamma^* - \gamma \mu^* \gamma^* + 3 \gamma^* \mu^* \gamma^* ) \\
+ ( |\beta|^2 \beta \gamma^* \beta \gamma^* ) (\mu^* \mu^* - \gamma^* \gamma^* ) \\
+ \mu^* \mu^* - \gamma^* \gamma^* + 2 \mu^* \gamma^* \mu^* - 2 \gamma^* \mu^* \gamma^* ) \\
+ [ |\beta|^2 (3 \beta^* ) + 3 \beta^* ] [ \gamma^* \mu^* - \mu^* \gamma^* + \mu^* \gamma^* - \gamma^* \mu^* + 2 \mu^* \gamma^* ] \\
- 2 \gamma^* \mu^* \gamma^* ] \\
and \;
D = \beta^4 (\mu^* \gamma^* - 4 \mu^* \gamma^* - 4 \mu^* \gamma^* + 6 \mu^* \gamma^* ) \\
+ \beta^4 (\mu^* \gamma^* - 4 \mu^* \gamma^* + 6 \mu^* \gamma^* ) \\
+ 6 \beta^2 |\beta|^2 \mu^* \gamma^* \mu^* - \gamma^* \mu^* \gamma^* + 3 \gamma^* \mu^* \mu^* - 3 \mu^* \gamma^* \mu^* - 3 \gamma^* \mu^* \gamma^* ] \\
+ 6 \beta^2 + 4 \beta^2 |\beta|^2 [ - \mu^* \gamma^* \mu^* + 3 \gamma^* \mu^* + \mu^* + 3 \gamma^* \mu^* ] \\
+ 3 \mu^* \gamma^* \mu^* - 3 \gamma^* \mu^* \gamma^* - 3 \mu^* \gamma^* \gamma^* + 12 |\beta|^2 + 6 |\beta|^4 + 3 |\beta|^2 \\
+ \mu^* \gamma^* \mu^* + \gamma^* \mu^* \gamma^* - \gamma^* \mu^* \gamma^* + 2 \mu^* \gamma^* \mu^* - 2 \gamma^* \mu^* \gamma^* - 2 \mu^* \gamma^* \gamma^* ] \\
- 2 \gamma^* \mu^* \gamma^* ] \\
\]

Putting together the expressions given by (5.21), (5.22) and (5.23) the expectation value of the Hamiltonian in a squeezed state \(|\beta\rangle\) is

\[ \langle \beta | H | \beta \rangle = -\frac{\hbar }{4} B^* + \frac{1}{2} m^* X^* \beta + m^* X^* \beta (\frac{\hbar}{2 \gamma^*})^{1/2} A + \frac{1}{2} m^* \gamma^* (\frac{\hbar}{2 \gamma^*})^{1/2} B \\
+ \lambda X^* \beta + 4 \lambda X^* (\frac{\hbar}{2 \gamma^*})^{1/2} A + 6 X^* \lambda (\frac{\hbar}{2 \gamma^*}) B + 4 \lambda X^* (\frac{\hbar}{2 \gamma^*})^{3/2} C \\
+ \lambda (\frac{\hbar}{2 \gamma^*}) D \tag{5.24} \]

where \( B^* = \mu^* \gamma^* + \nu^* \beta^* - \mu^* \gamma^* (2 |\beta|^2 + 1) \\
- \mu^* \gamma^* (2 |\beta|^2 + 1) + \mu^* \gamma^* + \nu^* \beta^* - 2 (|\mu|^2 + |\nu|^2) |\beta|^2 \\
- |\mu|^2 - |\nu|^2 + 2 \mu^* \gamma^* + 2 \beta^* \mu^* \gamma^* \)].
Minimizing the expectation value of $H$ with respect to $\varphi$, we have

$$\frac{d\langle H \rangle}{d\varphi} = \frac{\hbar}{4} B' + \frac{m^2}{2} X_\beta^2 \left( \frac{\hbar}{2} \right)^{1/2} A \frac{1}{2} \varphi - \frac{3}{2}$$

$$+ \frac{1}{2} m^2 \frac{\hbar}{2\gamma^2} B + 2 \lambda X_\beta^2 \left( \frac{\hbar}{2} \right)^{1/2} \varphi - \frac{3}{2} A$$

$$+ 6 \lambda^2 \left( \frac{\hbar}{2} \right)^{3/2} C \varphi - \frac{5}{2}$$

$$+ \lambda \left( \frac{\hbar}{2} \right)^{2} \frac{2}{\gamma^3} D = 0 \quad (5.25)$$

The effective mass is calculated for the squeezed state $|\beta\rangle$ as in the case of coherent states:

$$m_s^2 = \frac{d^2\varphi}{dx_\beta^2} \bigg|_{x_\beta=0} = m^2 + \frac{6\hbar \lambda B}{\gamma^2} + \left[ \frac{m^4}{4} \right] A^2 + 9 m^2 \left( \frac{\hbar}{2} \right) \varphi - \frac{3}{2} A$$

$$+ 72 \lambda^2 C^2 \left( \frac{\hbar}{2} \right)^2 \varphi - \frac{5}{2} \bigg[ 3 \gamma^2 \frac{B}{3} + m^2 B \bigg]$$

To find the effective coupling constant we have to evaluate the fourth derivative of the GEP:

$$\lambda_s = \frac{1}{4!} \frac{d^4 \varphi}{dx_\beta^4} \bigg|_{x_\beta=0} = \frac{1}{4!} \left[ \frac{m^2}{2} \left( \frac{\hbar}{2} \right)^{1/2} \varphi - \frac{3}{2} A \right]$$

$$+ 6 \lambda \left( \frac{\hbar}{2} \right)^{3/2} \varphi - \frac{5}{2} \bigg[ \frac{9 m^2}{4} \left( \frac{\hbar}{2} \right)^{1/2} \varphi - \frac{5}{2} A + 45 \lambda \left( \frac{\hbar}{2} \right)^{3/2} \varphi - \frac{7}{2} \bigg]$$

$$\frac{d\varphi}{dx_\beta} \bigg|_{x_\beta=0} \quad \frac{d^2\varphi}{dx_\beta^2} \bigg|_{x_\beta=0}$$
\[- \frac{18 \Lambda \hbar}{\hbar^2} \beta \frac{d^2 \bar{\sigma}}{dx^2} \bigg|_{x_0} = 0 \]

\[- 36 \Lambda (\frac{\hbar}{2})^{1/2} \frac{1}{2} \frac{3}{4} \beta \frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = 0 \]

\[+ 36 \Lambda \hbar \frac{\hbar}{\hbar_0^3} \beta \frac{(d \bar{\sigma})^2}{x} \bigg|_{x_0} = 0 \]

\[- \left[ m^2 (\frac{\hbar}{2})^{1/2} \frac{15}{8} \frac{\hbar_0}{2} \right] \frac{7}{2} \Lambda \]

\[+ \frac{105}{2} \Lambda \hbar \frac{3}{2} \frac{1}{2} \frac{9}{2} \beta \frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = 0 \]

\[+ 24 \Lambda \beta \]  

\[(5.26)\]

where

\[
\frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = - \left[ (\frac{\hbar}{2})^{-1/2} \frac{3}{4} \frac{1}{2} \frac{2}{2} \right] \Lambda
\]

\[+ (\frac{\hbar}{2})^{1/2} \frac{1}{2} \frac{12 \Lambda C}{3 \hbar_0^2} \bigg[ B' + m^2 B \bigg] \]  

\[(5.27)\]

\[
\frac{d^2 \bar{\sigma}}{dx^2} \bigg|_{x_0} = \left[ -3 (\frac{\hbar}{2})^{-1/2} \frac{3}{4} \frac{1}{2} \frac{2}{2} \right] \Lambda
\]

\[+ (\frac{\hbar}{2})^{1/2} \frac{1}{2} \frac{12 \Lambda C}{3 \hbar_0^2} \bigg[ \frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = 0 \bigg]
\]

\[- [6 \hbar_0 \beta' \frac{(d \bar{\sigma})^2}{x} \bigg|_{x_0} = 0 \bigg]
\]

\[- 24 \lambda \frac{\hbar_0}{2} \beta \bigg] \bigg[ 3 \hbar_0^2 \frac{B'}{m^2 B} \bigg] \]  

\[(5.28)\]

and

\[
\frac{d^3 \bar{\sigma}}{dx^3} = \left[ -2 (\frac{\hbar}{2})^{-1/2} \frac{3}{4} \frac{1}{2} \frac{2}{2} \right] \Lambda
\]

\[+ 18 (\frac{\hbar}{2})^{1/2} \frac{1}{2} \frac{12 \Lambda C}{3 \hbar_0^2} \bigg[ \frac{d^2 \bar{\sigma}}{dx^2} \bigg|_{x_0} = 0 \bigg]
\]

\[- 18 \hbar_0 \beta' \frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = 0 \bigg] \bigg[ 18 \hbar_0 \beta' \frac{d \bar{\sigma}}{dx} \bigg|_{x_0} = 0 \bigg] \]
The effective potential in the squeezed state is obtained by integrating the \( \frac{\delta V_G}{\delta x_\beta} \) with respect to \( x_\beta \).

\[
\tilde{V}_G = \frac{1}{2}m^2 x_\beta^2 + m^2 x_\beta \left( \frac{\hbar}{2\Lambda} \right)^{1/2} + \lambda x_\beta^4 + 4\lambda x_\beta^3 \left( \frac{\hbar}{2\Lambda} \right)^{1/2} + 6x_\beta^2 \lambda \left( \frac{\hbar}{2\Lambda} \right)^{3/2} + 4\lambda x_\beta \left( \frac{\hbar}{2\Lambda} \right)^{3/2}
\]

where \( \Lambda \) is given by (5.25).

The effective coupling constant \( \lambda_s \) for a squeezed state has a singularity at \( \hbar = 0 \), which disappears for zero bare mass \( m \), an effect already encountered with coherent states. Since the bare mass \( m \) can be arbitrary, it might as well be set equal to zero. The singular behaviour in \( \lambda_c \) and \( \lambda_s \) for \( m \neq 0 \) is perhaps an artifact of the GEP method.