CHAPTER - 6

NON-DARCY CONVECTIVE HEAT TRANSFER
OF A VISCOUS FLUID THROUGH A POROUS
MEDIUM IN A VERTICAL CHANNEL WITH
RADIATION EFFECT AND HEAT GENERATING
HEAT SOURCES
1. INTRODUCTION

Non-Darcy effects on natural convection in porous media have received a great deal of attention in recent years because of the experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters which indicate that the experimental data for systems other than glass at low Rayleigh numbers, do not agree with theoretical predictions based on the darcy flow model. This divergence in the heat transfer results has been reviews in detail in Cheng [3] and Prasad et al.[10] among others. Extensive effects are thus being made to include the inertia and viscous diffusion terms in the flow equations and to examine their effects in order to develop a reasonable accurate mathematical model for connective transport in porous media. The work of Vafai and Tien[15] was one of the early attempts to account for the boundary and inertia effects in the momentum equation for a porous medium. They found that the
momentum boundary layer thickness is of order of $\sqrt{\frac{k}{\varepsilon}}$. Vafai and Thiyagaraja [16] presented analytical solutions for the velocity and temperature fields for the interface region using the Brinkman-Forchheimer extends Darcy equation. Detailed accounts of the recent efforts of non-darcy conception have been recently reported in Tien and Hong [13], Cheng [4], Prasad et. al [10], and Kladas and Prasad [7]. Poulikakkos and Benjon [9] investigated the inertia effects through the inclusion of Forchheimer velocity squared term, and presents the boundary layer analysis for tall cavities. They also obtained numerical results for few cases in ordered to verify the accuracy of their boundary layer solutions. Latter Prasad and Tuntomo [11] reported an extensive numerical work for a wide range of parameters, and demonstrated that effects of Prandtl numbers remain almost unaltered while the dependence on the modified Grashof number, Gr, changes significantly with an increase in the Forchheimer number. This results on reversal of flow regimes form boundary layer to asymptotic to condition as the contribution of the inertia term increases in comparison with that of the boundary term. They also report's a certain for the Darcy flow limit.

The Brinkman-Extended-Darcy model was considered in Tong and Subramamian [14], ands Lauriat and Prasad [8] to examine the boundary effects on free convection in a vertical cavity, while Tong and Subramamian performed a Weber-type boundary layer analysis. Lauriat and Prasad solved the problem numerically for $A=1$ and 5. It was shown that for a fixed modified Rayleigh number, $Ra$, the Nusselt number decreases with an increase in the darcy number, the reduction being larger at higher values of $Ra$. A scale analysis as well as the computational data also showed that the transport term $(v \cdot \nabla)v$ is
of low order of magnitude compared to the diffusions plus buoyancy terms [8]. A numerical study based on the Forchheimer-Brinkman extended Darcy equation of motion has also been reports recently by Beckermann et. al [1]. They demonstrated that the inclusion of both the inertia and boundary effects is important for convection in a rectangular packaged-sphere cavity. Recently Shiva Shankar Reddy (12) has discussed the effect of radiation on Non-Darcy convective flow in a circular duct under different conditions.

The role of thermal radiation of major importance of in some industrial applications such as glass production and furnace design and in space technology applications, such as comical flight aerodynamics rocket, propulsion system, plasma physics and space craft reentry aero thermo dynamics which operate at high temperature. When radiation is taken into account, the governing equation become quite complicated and hence many difficulties arise while such equations Grief et al (5) shown that in the optically thin limit the physical situation can be simplified, and then they derived exact solution to fully developed vertical channel for a radioactive fluid. Hossain and Takhar (6) studied the radiation effect on mixed convection along a vertical plate with uniform surface temperature using Keller box finite difference method. Chamka (5) investigated thermal radiation and buoyancy effects on hydro magnetic flow over an accelerating permeable surface with heat source or sink.

Keeping in view the above maintained facts, in this chapter we discuss free and forced convection flow through a porous medium in a vertical channel where the boundaries are maintained at constant temperatures. The Brinkman-Forchheimer-
Extented Darcy equations which takes into account the boundary and inertia effects are used in the governing linear momentum equations. The effect of density variation is confined to the buoyancy term under Boussinesq approximation. The velocity and temperature in the flow field have been analytically evaluated using perturbation scheme and their behaviour is discussed computationally for variations in the governing parameters viz. \( G \) the Groshoff number, \( D^{-1} \) the inverse Darcy parameter, \( \alpha \) the Forchheimer number, \( N \) dimensionless temperature gradient and \( \alpha \) the non dimensional heat source parameter. The shear stress and the Nusselt number on the walls have been analytically evaluated and their behavior is discussed for variations in the above mentioned parameters.

2. FORMULATION OF THE PROBLEM

We consider a fully developed laminar convective heat transfer flow of a viscous fluid through a porous medium confined in a vertical channel bounded by flat walls. We choose a Cartesian co-ordinate system \( O(x,y,z) \) with \( x \)-axis in the vertical direction and \( y \)-axis normal to the walls. The walls are taken at \( y = \pm 1 \). The walls are maintained at constant temperature and concentration. The temperature gradient in the flow field is sufficient to cause natural convection in the flow field. A constant axial pressure gradient is also imposed so that this resultant flow is a mixed convection flow. The porous medium is assumed to be isotropic and homogeneous with constant porosity and effective thermal diffusivity. The thermo physical properties of porous matrix are also assumed to be constant and Boussinesq approximation is invoked by confining the density variation
to the buoyancy term. In the absence of any extraneous force flow is unidirectional along the x-axis which is assumed to be infinite.

The Brinkman-Forchheimer-extended Darcy equation which account for boundary inertia effects in the momentum equation is used to obtain the velocity field. Based on the above assumptions the governing equations in the vector form are

\[ \nabla \cdot \bar{q} = 0 \quad (\text{Equation of continuity}) \]  

\[ \frac{\rho}{\delta} \frac{\partial \bar{q}}{\partial t} + \frac{\rho}{\delta^2} (\bar{q} \cdot \nabla) \bar{q} = -\nabla p + \rho g - \left( \frac{\mu}{k} \right) \bar{q} - \frac{\rho F}{\sqrt{k}} \bar{q} \bar{q} + \mu \nabla^2 \bar{q} \quad (2.2) \]

\[ \rho C_p \left( \frac{\partial T}{\partial t} + (\bar{q} \cdot \nabla) T \right) = \lambda \nabla^2 T + Q(T_o - T) - \frac{\partial(q_r)}{\partial y} \quad (2.3) \]

\[ \rho - \rho_0 = -\beta \rho_0 (T - T_0) \quad (2.4) \]

\[ \text{Equation of State} \]

where \( \bar{q} = (u, 0, 0) \) is the velocity, \( T \) is the temperature, \( p \) is the pressure, \( \rho \) is the density of the fluid, \( C_p \) is the specific heat at constant pressure, \( \mu \) is the coefficient of viscosity, \( k \) is the permeability of the porous medium, \( \delta \) is the porosity of the medium, \( \beta \) is the coefficient of thermal expansion, \( \lambda \) is the coefficient of thermal conductivity, \( F \) is a function that depends on the Reynolds number and the microstructure of porous medium, and \( Q \) is the strength of the heat generating source. Here the thermo physical properties of the solid and fluid have been assumed to be constant except for the density variation in the body force term (Boussinesq approximation) and the solid particles and the fluid are considered to be in the thermal equilibrium.
Since the flow is unidirectional, the continuity of equation (2.1) reduces to

\[ \frac{\partial u}{\partial x} = 0 \] where \( u \) is the axial velocity implies \( u = u(y) \)

The momentum and energy equations in the scalar form reduces to

\[
\begin{align*}
-\frac{\partial p}{\partial x} + \left(\frac{\mu}{\delta} \right) \frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu}{k} \right) u - \frac{\rho \delta F}{\sqrt{k}} u^2 - \rho g & = 0 \\
\rho_C \gamma u \frac{\partial T}{\partial x} & = \lambda \frac{\partial^2 T}{\partial y^2} + Q(T_o - T) - \frac{\partial q_r}{\partial y}
\end{align*}
\] (2.5) (2.6)

The boundary conditions are

\[
\begin{align*}
u = 0 & , \quad T = T_1 \quad \text{on} \quad y = -L \\
u = 0 & , \quad T = T_2 \quad \text{on} \quad y = +L
\end{align*}
\] (2.7)

The axial temperature gradients \( \frac{\partial T}{\partial x} \) is assumed to be a constant say, \( A \) Invoking Rosseland approximation for radiation flux we get

\[ q_r = \frac{4 \sigma^*}{\beta_R} \frac{\partial (T'^4)}{\partial y} \] (2.8)

and linearising \( T'^4 \) about \( T_e \) by using Taylor’s expansion and neglecting higher order terms we get

\[ T'^4 \equiv 4T_e^3T - 3T_e^4 \] (2.9)

We define the following non-dimensional variables as

\[
\begin{align*}
u' & = \frac{u}{(\nu/L)} , \quad (x', y') = (x, y)/L \\
p' & = \frac{p \delta}{(\rho \nu^2/L^2)} \\
\theta & = \frac{T - T_e}{T_1 - T_2}
\end{align*}
\] (2.10)
Introducing these non-dimensional variables the governing equations in the
dimensionless form reduce to (on dropping the dashes)

\[
\frac{d^2u}{dy^2} = \pi + \delta(D^{-1})u - \delta G(\theta + NC) \tag{2.11}
\]

\[
\frac{d^2\theta}{dy^2} - \alpha \theta + \frac{4}{3N_1} \frac{d^2\theta}{dy^2} = (PN_1)u \tag{2.12}
\]

where

\[ A = FD^{-1/2} \] (Inertia or Fochhemeir parameter)

\[ G = \frac{\beta g(T_1 - T_e) L^3}{v^2} \] (Grashof Number)

\[ D^+ = \frac{L^2}{k} \] (Darcy parameter)

\[ P = \frac{\mu C_p}{\lambda} \] (Prandtl Number)

\[ \alpha = \frac{QL^2}{\lambda} \] (Heat source parameter)

\[ N_1 = \frac{4\sigma T_e^3}{\beta_k} \] (Radiation parameter)

The corresponding boundary conditions are

\[
u = 0 \ , \ \theta = 1 \ on \ y = -1
\]

\[
u = 0 \ , \ \theta = m \ on \ y = +1 \tag{2.13}
\]
3. SOLUTION OF THE PROBLEM

The governing equations of flow and heat transfer are coupled non-linear differential equations. Assuming the porosity $\delta$ to be small we write

$$u = u_0 + \delta u_1 + \delta^2 u_2 + \ldots$$

$$\theta = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 + \ldots$$  \hspace{1cm} (3.1)

Substituting the above expansions in the equations (2.11)&(2.12) and equating like powers of $\delta$, we obtain equations to the zeroth order as

$$\frac{d^2 u_0}{dy^2} = \pi$$  \hspace{1cm} (3.2)

$$\frac{d^2 \theta_0}{dy^2} - \alpha_1 \theta_0 = (P_i N_T) u_0$$  \hspace{1cm} (3.3)

The equations to the first order are

$$\frac{d^2 u_1}{dy^2} - (D^{-1}) u_1 = -G(\theta_0)$$  \hspace{1cm} (3.4)

$$\frac{d^2 \theta_1}{dy^2} - \alpha_1 \theta_1 = (P_i N_T) u_1$$  \hspace{1cm} (3.5)

The equations to the second order are

$$\frac{d^2 u_2}{dy^2} - (D^{-1}) u_2 = -G(\theta_1) - A u_0^2$$  \hspace{1cm} (3.6)
The corresponding conditions are

\[ u_0(l) = u_0(-1) = 0, \ \theta_0(+1) = 0, \ \theta_0(-1) = 1 \quad (3.11) \]

\[ (u_1(l) = u_1(-1) = 0, \ \theta_1(+1) = 0, \ \theta_1(-1) = 0 \quad (3.12) \]

\[ u_2(l) = u_2(-1) = 0, \ \theta_2(+1) = 0, \ \theta_2(-1) = 0 \quad (3.13) \]

Solving the equations (3.2)-(3.7) subject to the boundary conditions (3.11)-(3.13) we get

\[ u_0(y) = \frac{\pi}{2} (y^2 - 1) \]

\[ \theta_0 = a_1 \left( \frac{Ch(\beta_1, y)}{Ch(\beta_1)} - y^2 \right) + a_2 \left( \frac{Ch(\beta_1, y)}{Ch(\beta_1)} - 1 \right) + 0.5 \left( \frac{Ch(\beta_1, y)}{Ch(\beta_1)} + \frac{Sh(\beta_1, y)}{Sh(\beta_1)} \right) \]

\[ u_1 = a_{14} \left( \frac{Ch(\beta_1, y)}{Ch(\beta_2)} - Ch(\beta_1, y) \right) + a_{16} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_1)} - y^2 \right) - \frac{a_{18} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_2)} - 1 \right) + a_{15} \left( \frac{Sh(\beta_1, y)}{Sh(\beta_1)} - \frac{Sh(\beta_1, y)}{Sh(\beta_2)} \right) + a_{17} (y - \frac{Sh(\beta_2, y)}{Sh(\beta_2)})}{Sh(\beta_2)} \]

\[ \theta_1 = (a_{24} Sh(\beta_1) - a_{23} - a_{19} Ch(\beta_1)) \frac{Ch(\beta_2, y)}{Ch(\beta_1)} + (a_{22} - a_{23} Ch(\beta_1)) \frac{Sh(\beta_2, y)}{Sh(\beta_1)} + \frac{a_{19} Ch(\beta_2, y) + a_{20} Sh(\beta_2, y) + a_{21} y^2 - a_{22} y + a_{23} - a_{24} y Sh(\beta_1, y) + a_{25} y Ch(\beta_1, y)}{Sh(\beta_2)} \]

\[ u_2 = a_{63} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_1)} - Ch(\beta_2, y) \right) + a_{62} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_2)} - y Sh(\beta_1, y) \right) + \frac{a_{64} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_1)} - Sh(\beta_2, y) \right) + a_{66} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_1)} - y^3 \right) - a_{68} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_1)} - y^2 \right)}{Sh(\beta_2)} \]

\[ + a_{70} \left( \frac{Ch(\beta_2, y)}{Ch(\beta_2)} - 1 \right) + a_{61} \left( \frac{Sh(\beta_2, y)}{Sh(\beta_1)} - y Sh(\beta_2, y) \right) - a_{69} \left( \frac{Sh(\beta_2, y)}{Sh(\beta_2)} - y^3 \right) \]

\[ - y Ch(\beta_2, y) - a_{65} \left( \frac{Sh(\beta_2, y)}{Sh(\beta_2)} - Ch(\beta_1, y) \right) - a_{67} \left( \frac{Sh(\beta_2, y)}{Sh(\beta_2)} - y^3 \right) - a_{69} \left( \frac{Sh(\beta_2, y)}{Sh(\beta_2)} \right) \]
\[ \theta_2 = a_{17} (Ch(\beta_2, y) - Ch(\beta_2)) + a_{24} (ySh(\beta_2, y) - Sh(\beta_2) \frac{Ch(\beta_2, y)}{Ch(\beta_2)}) + \\
+ a_{75} (ySh(\beta_1, y) - Sh(\beta_1) \frac{Ch(\beta_2, y)}{Ch(\beta_2)}) - a_{75} (y^2 - 1)Ch(\beta_1, y) + \\
+ a_{79} (y^4 - 1) + a_{81} (y^2 - 1) + a_{83} (1 - \frac{Ch(\beta_1, y)}{Ch(\beta_1)}) + a_{72} (Sh(\beta_2, y) - \\
\frac{Sh(\beta_2, y)}{Sh(\beta_1)} Sh(\beta_2)) + a_{75} (yCh(\beta_1, y) - \frac{Sh(\beta_2, y)}{Sh(\beta_1)} Ch(\beta_1)) + \\
+ a_{76} (yCh(\beta_1, y) - \frac{Sh(\beta_2, y)}{Sh(\beta_1)} Ch(\beta_1)) + a_{78} (y^2 - 1)Sh(\beta_1, y) + \\
+ a_{96} (y^3 - \frac{Sh(\beta_1, y)}{Sh(\beta_1)}) + a_{92} (y - \frac{Sh(\beta_2, y)}{Sh(\beta_1)}) \\
\]

4. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the boundaries \( y = \pm 1 \) is given by

\[ \tau_{y=\pm 1} = \mu \left( \frac{du}{dy} \right)_{y=\pm 1} \]

which in the non-dimensional form is

\[ \tau_{y=1} = \left( \frac{du}{dy} \right)_{y=1} \]

and the corresponding expressions are

\[ \tau_{y=1} = \pi + \delta b_{31} + \delta^3 b_{37} \]
\[ \tau_{y=-1} = \pi + \delta b_{32} + \delta^3 b_{38} \]

The rate of heat transfer (Nusselt Number) is given by

\[ Nu_{y=\pm 1} = \left( \frac{d\theta}{dy} \right)_{y=\pm 1} \]
and corresponding expressions are:

\[ Nu_{y=+1} = b_{27} + \delta b_{33} + \delta^2 b_{39} \]
\[ Nu_{y=-1} = b_{28} + \delta b_{34} + \delta^2 b_{40} \]

5. DISCUSSION

The velocity, temperature, shear stress and the rate of heat transfer have been analyzed for different variations of the governing parameter \( G, D^{-1}, \alpha, \) and \( N1. \) The velocity \( u \) is presented in Figs 1 to 4 in view of the chosen pressure gradient. The actual flow is axially downwards. We find that for \( G > 0, \) the velocity \( u \) changes from negative to positive near the right boundary \( y=1 \) which indicates the occurrence of reversal flow in the vicinity of \( y=1 \) and for \( G < 0 \) the reversal flow occurs in the vicinity of the left boundary \( y=-1. \) The region of reversal flow enhances with increase in \( |G| (< > 0). \) The magnitude of \( u \) enhances with increase in \( |G| \) (fig. 1). The variation of \( u \) with porous parameter \( D^{-1} \) is shown in Fig. 2 we notice a reversal flow in the vicinity of \( y=1 \) and the region increases with increase in \( D^{-1} \) and this region decreases with increase in \( D^{-1} \leq 2 \times 10^2 \) and enhances for further increase in \( D^{-1} \geq 3 \times 10^2. \) We notice that the fluid moves with higher velocity in the left region for an increase in \( D^{-1} \) whereas in the central core region (0.2 to 0.4), \( |u| \) depreciates with \( D^{-1} \leq 2 \times 10^2 \) and enhances with \( D^{-1} \geq 10^3 \) and a narrow region (0.6, 0.8) abutting \( y=1, \) the velocity depreciates with \( D^{-1} \leq 2 \times 10^2 \) and enhances for higher \( D^{-1} \geq 3 \times 10^2. \) Thus lesser the permeability of porous medium greater the fluid velocity in the left boundary and it exhibits an oscillating nature in the right region.
Fig(1) Variation of $u$ with $G$
$D^{-1}=2\times10^2$ $N=1$ $\alpha=2$ $N_1=0.5$

Fig(2) Variation of $u$ with $D^{-1}$
$N=1$ $\alpha=2$ $N_1=0.5$
Fig(3) Variation of u with $\alpha$
$N=1$ $N_i=0.5$ $D'=10^2$

Fig(4) Variation of u with $N_i$
$N=1$ $\alpha=2$ $D'=10^2$
Fig(5) Variation of $\theta$ with $G$
$D'_{1}=2\times10^{2}$ $N=1$ $\alpha=2$ $N_{I}=0.5$

Fig(6) Variation of $\theta$ with $D'_{1}$
$N=1$ $\alpha=2$ $N_{I}=0.5$
Fig(7) Variation of $\theta$ with $\alpha$
$N=1$ $N_i=0.5$ $G=10^3$ $D'=10^2$

Fig(8) Variation of $\theta$ with $N_i$
$N=1$ $G=10^3$ $D'=10^2$
The variation of $u$ with heat source parameter $\alpha$ exhibits that the reversal flow which occurs in the vicinity of $y=1$ shrinks in its sides with increase in $\alpha$. Also the magnitude of $u$ depreciates in the region adjacent to the boundaries $y=\pm 1$ and enhances in the central core $-0.2 \leq y \leq 0.4$ (Fig. 3). The influence of the radiation parameter $N_1$ on $u$ is shown in Fig. 4. It is found that the reversal flow which occurs in the region adjacent to $y=1$ decreases in its sides with an increase in $N_1$. For an increase in $N_1 \leq 0.3$, $|u|$ decreases in the fluid region except in the region $0 \leq y \leq 0.2$, where it enhances. For higher $N_1 \geq 0.5$, $|u|$ experiences an enhancement in the region $-0.8 \leq y \leq 0.4$ and depreciates in the remaining region.

The non-dimensional temperature ($\theta$) is exhibited from Figs. (5-8) for different variations of $G$, $D'^1$, $\alpha$ and $N_1$. The temperature $\theta$ is positive and negative according as the actual temperature in the fluid region is greater than or less than the equilibrium temperature. We notice that $\theta$ is positive for all variations. From Fig. 5 it is found that the temperature decreases with increase in $G > 0$ and enhances with $|G| (<0)$. The variation of $\theta$ with $D'^1$ is exhibited in Fig. 6. It is found that $\theta$ enhances with $D'^1 \leq 5 \times 10^2$ and for higher $D'^1 \geq 10^3$, $\theta$ experiences depreciation in the left region and enhances in the right region. Thus lesser the permeability of porous medium larger the temperature and for further lowering of permeability smaller $\theta$ in the left region and higher $\theta$ in the right region. From Fig 7 it is found that $\theta$ experiences depreciation with increase in the heat source parameter $\alpha$. The variation of $\theta$ with $N_1$ is shown in Fig 8. It is found that an increase in the radiation parameter $N_1$ leads to a depreciation in the temperature $\theta$ in the entire flow region.
Table-1
Shear Stress ($\tau$) at $y=1$
P=0.71, $N_1=0.5$

<table>
<thead>
<tr>
<th>$G/\tau$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>1.17403</td>
<td>1.17067</td>
<td>1.20183</td>
<td>1.04596</td>
<td>1.00418</td>
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<tr>
<td>$2\times10^3$</td>
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<td>1.07547</td>
<td>1.00516</td>
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<tr>
<td>$3\times10^3$</td>
<td>1.25198</td>
<td>1.40955</td>
<td>1.54744</td>
<td>1.0883</td>
<td>1.00275</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.73543</td>
<td>0.79511</td>
<td>0.77872</td>
<td>0.93698</td>
<td>0.99204</td>
</tr>
<tr>
<td>$2\times10^{-3}$</td>
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<td>0.53811</td>
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<td>0.98088</td>
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<tr>
<td>$3\times10^{-3}$</td>
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<td>$3\times10^2$</td>
<td>$10^2$</td>
<td>$10^2$</td>
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<tr>
<td>$\alpha$</td>
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<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
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Table-2
Shear Stress ($\tau$) at $y=1$
P=0.71, $\alpha=2$

<table>
<thead>
<tr>
<th>$G/\tau$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
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<td>1.1535</td>
<td>1.13861</td>
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<td>$2\times10^3$</td>
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<td>1.30681</td>
<td>1.3271</td>
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<td>$3\times10^3$</td>
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<td>1.25198</td>
<td>1.45974</td>
<td>1.56527</td>
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<tr>
<td>$-10^3$</td>
<td>0.47878</td>
<td>0.73543</td>
<td>0.84574</td>
<td>0.91068</td>
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<tr>
<td>$2\times10^{-3}$</td>
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<tr>
<td>$3\times10^{-3}$</td>
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<td>0.53644</td>
<td>0.88148</td>
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<tr>
<td>$N_1$</td>
<td>0.1</td>
<td>0.5</td>
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<td>5</td>
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</table>
Table 3
Shear Stress ($\tau$) at $y=-1$
P=0.71, $N_1=0.5$

<table>
<thead>
<tr>
<th>$\frac{G}{\tau}$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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<tbody>
<tr>
<td>$10^3$</td>
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<td>-1.10413</td>
<td>-1.03192</td>
<td>-1.07212</td>
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<td>$-10^3$</td>
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<td>-0.92596</td>
<td>-0.96894</td>
</tr>
<tr>
<td>$2\times-10^3$</td>
<td>-0.46785</td>
<td>-0.71943</td>
<td>-0.89393</td>
<td>-0.85058</td>
<td>-0.94334</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>-0.11808</td>
<td>-0.54312</td>
<td>-0.81986</td>
<td>-0.77367</td>
<td>-0.92301</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D^{-1}$</th>
<th>$10^2$</th>
<th>$2\times10^2$</th>
<th>$3\times10^2$</th>
<th>$10^2$</th>
<th>$10^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4
Shear Stress ($\tau$) at $y=-1$
P=0.71, $\alpha=2$

<table>
<thead>
<tr>
<th>$\frac{G}{\tau}$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.09955</td>
<td>-1.18201</td>
<td>-1.15736</td>
<td>-1.14016</td>
</tr>
<tr>
<td>$2\times10^3$</td>
<td>-0.80302</td>
<td>-1.30834</td>
<td>-1.34197</td>
<td>-1.35425</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>-0.11022</td>
<td>-1.37882</td>
<td>-1.55364</td>
<td>-1.64207</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>-0.50381</td>
<td>-0.76176</td>
<td>-0.86932</td>
<td>-0.93319</td>
</tr>
<tr>
<td>$2\times-10^3$</td>
<td>0.38846</td>
<td>-0.46785</td>
<td>-0.76589</td>
<td>-0.94032</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>1.677</td>
<td>-0.11808</td>
<td>-0.68952</td>
<td>-1.02118</td>
</tr>
</tbody>
</table>

| $N_1$ | 0.1 | 0.5 | 1.5 | 5    |

### Table-5
Nusselt Number (Nu) at y=1
\( P=0.71, N_1=0.5 \)

<table>
<thead>
<tr>
<th>G/Nu</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>-4.6552</td>
<td>-4.72818</td>
<td>-4.74933</td>
<td>-2.95354</td>
<td>-2.31323</td>
</tr>
<tr>
<td>(2\times10^3)</td>
<td>-4.40113</td>
<td>-4.61948</td>
<td>-4.68426</td>
<td>-2.86282</td>
<td>-2.25974</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>-4.16832</td>
<td>-4.52818</td>
<td>-4.62717</td>
<td>-2.78264</td>
<td>-2.21257</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>-5.22708</td>
<td>-4.98318</td>
<td>-4.90339</td>
<td>-3.16663</td>
<td>-2.43918</td>
</tr>
<tr>
<td>(2\times-10^3)</td>
<td>-5.54489</td>
<td>-5.12947</td>
<td>-4.9924</td>
<td>-3.28898</td>
<td>-2.51164</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>-5.88396</td>
<td>-5.2883</td>
<td>-5.08938</td>
<td>-3.42188</td>
<td>-2.59042</td>
</tr>
</tbody>
</table>

### Table-6
Nusselt Number (Nu) at y=1
\( P=0.71, \alpha=2 \)

<table>
<thead>
<tr>
<th>G/Nu</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>-2.45949</td>
<td>-4.6552</td>
<td>-6.09042</td>
<td>-7.19892</td>
</tr>
<tr>
<td>(2\times10^3)</td>
<td>-0.63575</td>
<td>-4.40113</td>
<td>-5.90794</td>
<td>-6.96754</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>1.91418</td>
<td>-4.16832</td>
<td>-5.74959</td>
<td>-6.71346</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>-3.92836</td>
<td>-5.22708</td>
<td>-6.52773</td>
<td>-7.59358</td>
</tr>
<tr>
<td>(2\times-10^3)</td>
<td>-3.5735</td>
<td>-5.54489</td>
<td>-6.78257</td>
<td>-7.75687</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>-2.49244</td>
<td>-5.88396</td>
<td>-7.06153</td>
<td>-7.89746</td>
</tr>
<tr>
<td>(N_1)</td>
<td>0.1</td>
<td>0.5</td>
<td>1.5</td>
<td>5</td>
</tr>
</tbody>
</table>
### Table-7
Nusselt number (Nu) at y = -1
P = 0.71, N1 = 0.5

<table>
<thead>
<tr>
<th>G/Nu</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>4.669</td>
<td>4.72</td>
<td>4.732</td>
<td>2.951</td>
<td>2.307</td>
</tr>
<tr>
<td>2x10^3</td>
<td>4.477</td>
<td>4.614</td>
<td>4.655</td>
<td>2.871</td>
<td>2.253</td>
</tr>
<tr>
<td>3x10^3</td>
<td>4.356</td>
<td>4.532</td>
<td>4.59</td>
<td>2.815</td>
<td>2.21</td>
</tr>
<tr>
<td>-10^3</td>
<td>5.262</td>
<td>5.003</td>
<td>4.926</td>
<td>3.182</td>
<td>2.45</td>
</tr>
<tr>
<td>2x-10^3</td>
<td>5.665</td>
<td>5.18</td>
<td>5.042</td>
<td>3.332</td>
<td>2.538</td>
</tr>
<tr>
<td>3x10^3</td>
<td>6.137</td>
<td>5.381</td>
<td>5.17</td>
<td>3.506</td>
<td>2.638</td>
</tr>
<tr>
<td>D^1</td>
<td>10^2</td>
<td>2x10^2</td>
<td>3x10^2</td>
<td>10^2</td>
<td>10^2</td>
</tr>
<tr>
<td>a</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

### Table-8
Nusselt Number (Nu) at y = -1
P = 0.71, α = 2

<table>
<thead>
<tr>
<th>G/Nu</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>2.535</td>
<td>4.669</td>
<td>6.075</td>
<td>7.155</td>
</tr>
<tr>
<td>2x10^3</td>
<td>0.82</td>
<td>4.477</td>
<td>5.934</td>
<td>6.936</td>
</tr>
<tr>
<td>3x10^3</td>
<td>-1.59</td>
<td>4.356</td>
<td>5.872</td>
<td>6.75</td>
</tr>
<tr>
<td>-10^3</td>
<td>3.884</td>
<td>5.262</td>
<td>6.599</td>
<td>7.693</td>
</tr>
<tr>
<td>2x-10^3</td>
<td>3.581</td>
<td>5.665</td>
<td>6.981</td>
<td>8.011</td>
</tr>
<tr>
<td>3x10^3</td>
<td>2.456</td>
<td>6.137</td>
<td>7.443</td>
<td>8.362</td>
</tr>
<tr>
<td>α</td>
<td>0.1</td>
<td>0.5</td>
<td>1.5</td>
<td>5</td>
</tr>
</tbody>
</table>
The shear stress and the rate of heat transfer at the boundaries have been shown in tables 1-8 for different variations of \( G, D^{-1}, \alpha \) and \( N_1 \). From tables 1-4 we find that the shear stress at the left boundary \( y=-1 \) is negative and that at the right boundary \( y=1 \) is positive for all variations. The shear stress \( \tau \) at \( y=-1 \) enhances with increase in \( G > 0 \) and decreases with increase in \( |G| (<0) \) where as at \( y=1 \) the shear stress depreciates for all \( |G| (<>0) \). Also in the heating case lesser the permeability of porous medium larger \( |\tau| \) at \( y=1 \) and smaller \( |\tau| \) at \( y=-1 \) and in the cooling case larger \( |\tau| \) at \( y=1 \) and for further lowering of the permeability it depreciates while at \( y=-1 \) larger \( |\tau| \). The variation of \( \tau \) with \( \alpha \) indicates that an increase in the heat source parameter \( \alpha \) depreciates with \( |\tau| \) at \( y=\pm 1 \) in the heating case and enhances it in the cooling case (tables 1 and 3). The variation of \( \tau \) with the radiation parameter \( N_1 \) is shown in tables 2 and 4. It is found that the stress at \( y=1 \) is negative for higher \( |G| (<0) \) for \( N_1=0.1 \) and is positive for higher values of \( N_1 \). The magnitude of stress \( \tau \) at \( y=-1 \) enhances with increase in \( N_1 \) for all \( |G| (<0) \) while at \( y=1 \) it enhances with \( N_1 \), in heating case and for \( |G|=10^3 \). For higher \( |G| \geq 2 \times 10^3 \) it depreciates with \( N_1 \geq 0.5 \) and enhances with \( N_1 \geq 1.5 \).

The Nusselt No which measures the rate of heat transfer at \( y=\pm 1 \) is shown in tables 5-8 for different \( G, D^{-1}, \alpha \) and \( N_1 \). It is noticed that the rate of heat transfer at \( y=-1 \) is positive and is negative at \( y=1 \) for all variations. The magnitude of \( Nu \) reduces with \( G > 0 \) and enhances with \( |G| (<0) \). The variation of \( Nu \) with \( D^{-1} \) exhibits that lesser the permeability of the porous medium greater \( |Nu| \) in the heating case and smaller \( |Nu| \) in the cooling case. Also \( |Nu| \) at \( y=\pm 1 \) experiences a depreciation with increase in \( \alpha \) for all \( |G| (<>0) \) (figs.5 and 7). The variation of \( Nu \) with radiation parameter \( N_1 \) shows that the
magnitudes of Nu experiences an enhancement with increase in N1, (tables 7-8). We notice that |Nu| at y=-l is greater than that at y=1. Thus the inclusion of the radiation effect in the energy equation enhances the rate of heat transfer at both the boundaries.
6. REFERENCES


\[ a_1 = \frac{p N_T}{2 \alpha_1} \quad a_2 = \frac{2}{\alpha_1} + 1 \]

\[ a_3 = 0 \quad a_4 = 0 \]

\[ a_5 = 0 \quad a_6 = -\frac{a_4}{\beta_1^2} \text{Sh}(\beta_1) - 0.5 \]

\[ a_8 = G\left(\frac{a_1 + Na_3}{\beta_1^2 \text{Ch}(\beta_1)} + a_4 a_3\right) \quad a_9 = G(a_1 + 0.5 a_3) \]

\[ a_{10} = G(a_1 a_2 + \frac{Na_3}{\beta_1^2 \text{Ch}(\beta_1)}) \quad a_{11} = G\left(\frac{Na_4}{\beta_1^2 \text{Sh}(\beta_1)} - 0.5\right) \]

\[ a_{12} = G\left(\frac{a_4}{\beta_1^2 \text{Sh}(\beta_1)} + 0.5\right) \quad a_{13} = GN(1 + a_4)/2 \]

\[ a_{14} = \frac{a_8}{\text{Ch}(\beta_1)(\beta_1^2 - \beta_2^2)} \quad a_{15} = \frac{a_9}{\beta_2^2} \]

\[ a_{16} = \frac{a_{11}}{\text{Sh}(\beta_1)(\beta_1^2 - \beta_2^2)} \quad a_{17} = \frac{a_{12}}{\beta_2^2} \]

\[ D_3 = \frac{(a_{14} \text{Ch}(\beta_1) + a_{16} - a_{18})}{\text{Ch}(\beta_2)} \quad a_{19} = \frac{P N_T D_4}{\beta_1^2 - \beta_2^2} \]

\[ a_{20} = \frac{P N_T D_4}{\beta_1^2 - \beta_2^2} \quad a_{21} = \frac{a_{16}}{\beta_2^2} \]

\[ a_{22} = \frac{a_{17}}{\beta_1^2} \quad a_{23} = \frac{2a_{16} - a_{18}}{\beta_2^2 - \beta_1^2} \]

\[ a_{24} = \frac{a_{14}}{2 \beta_1} \quad a_{25} = \frac{a_{18}}{2 \beta_1} \]
\[ D_3 = \frac{a_{26} \text{Sh}(\beta_1) - a_{23} - a_{21} - a_{19} \text{Ch}(\beta_1)}{\text{Ch}(\beta_1)} \]

\[ D_6 = -\frac{(a_{20} \text{Sh}(\beta_1) + a_{22} - a_{25} \text{Ch}(\beta_1))}{\text{Sh}(\beta_1)} \]

\[ a_{26} = 0 \quad a_{27} = 0 \]

\[ a_{28} = 0 \]

\[ a_{29} = 0 \]

\[ a_{30} = 0 \quad a_{31} = 0 \]

\[ a_{32} = 0 \quad a_{33} = 0 \]

\[ a_{34} = 0 \quad a_{37} = \frac{a_{26}}{\beta_1^2} \]

\[ a_{38} = \frac{a_{27}}{\beta_2^2} \quad a_{39} = \frac{a_{28}}{\beta_1^2} \]

\[ a_{40} = \frac{a_{29}}{\beta_1^2} \quad a_{41} = \beta_1^2 a_{30} \]

\[ a_{42} = \beta_1^2 a_{31} \quad a_{43} = \frac{a_{42}}{12} \]

\[ a_{44} = \frac{a_{33}}{6} \quad a_{45} = \frac{a_{34}}{2} \]

\[ a_{46} = D_3 + Na_{39} \]

\[ a_{47} = D_3 + Na_{39} \quad a_{48} = a_{19} + Na_{37} \]

\[ a_{49} = a_{20} + Na_{38} \quad a_{50} = a_{25} - Na_{41} \]
\[
\begin{align*}
a_{51} &= a_{24} + Na_{42}, \quad a_{52} = GNa_{43} \\
a_{53} &= GNa_{44}, \quad a_{54} = a_{21} + Na_{45} \\
a_{55} &= a_{22} - Na_{35}, \quad a_{56} = a_{23} + Na_{36} \\
a_{57} &= a_{52} + \frac{\beta_4^2}{4}, \quad a_{58} = a_{54} + \frac{\beta_4^2}{4} \\
a_{59} &= \frac{\beta_4^2}{4} - a_{56}, \quad a_{60} = -\frac{a_{46}}{\beta_1^2 - \beta_2^2} - \frac{2P_1a_{59}}{(\beta_1^2 - \beta_2^2)^2} \\
a_{61} &= \frac{a_{47}}{\beta_1^2 - \beta_2^2} - \frac{2P_1a_{51}}{(\beta_1^2 - \beta_2^2)^2} \\
a_{62} &= \frac{a_{48}}{2\beta_2}, \quad a_{63} = \frac{a_{49}}{2\beta_2} \\
a_{64} &= \frac{a_{50}}{\beta_1^2 - \beta_2^2}, \quad a_{65} = \frac{a_{51}}{\beta_1^2 - \beta_2^2} \\
a_{66} &= \frac{a_{57}}{\beta_1^2 - \beta_2^2}, \quad a_{67} = \frac{a_{53}}{\beta_2^2} \\
a_{68} &= \frac{12a_{51}}{\beta_2^2} - \frac{a_{58}}{\beta_2^2}, \quad a_{69} = \frac{6a_{43}}{\beta_2^4} + \frac{a_{35}}{\beta_2^2} \\
a_{70} &= \frac{a_{49}}{\beta_2^2}, \quad a_{71} = \frac{(P_1N_T)D_8}{\beta_2^2 - \beta_1^2} \\
a_{72} &= \frac{(P_1N_T)D_7}{\beta_2^2 - \beta_1^2}, \quad a_{73} = \frac{2\beta_2}{\beta_2^2 - \beta_1^2} \\
a_{74} &= \frac{a_{62} - P_1N_T}{\beta_2^2 - \beta_1^2}, \quad a_{75} = -\frac{a_{60}P_1N_T}{2\beta_1} + \frac{(P_1N_T)D_8}{\beta_2^2 - \beta_1^2} + \frac{P_1N_Ta_{65}}{4\beta_1^2}
\end{align*}
\]
$$
a_{76} = \frac{P_1N_T a_{60}}{2\beta_1}, \quad a_{77} = \frac{P_1N_T a_{64}}{4\beta_2^2}
$$

$$
a_{78} = \frac{P_1N_T a_{65}}{4\beta_2^2}, \quad a_{79} = \frac{P_1N_T a_{66}}{4\beta_2^2}
$$

$$
a_{80} = \frac{P_1N_T a_{67}}{\beta_1^2}, \quad a_{81} = \frac{12P_1N_T a_{67}}{\beta_1^4} - \frac{P_1N_T a_{68}}{\beta_1^2}
$$

$$
a_{82} = \frac{6P_1N_T a_{68}}{\beta_1^4} + \frac{P_1N_T a_{69}}{\beta_1^4}, a_{83} = \frac{P_1N_T a_{70}}{\beta_1^4}
$$

$$
b_1 = \beta_1^2 D_6 + 2\beta_1 a_{75} - 2a_{77}, \quad b_2 = \beta_1^2 D_{10} + 2\beta_1 a_{76} - 2a_{78}
$$

$$
b_3 = \beta_1^2 a_{76} - 4\beta_1 a_{78}, \quad b_4 = \beta_1^2 a_{75}
$$

$$
b_5 = \beta_1^2 a_{71} + 2\beta_1 a_{74}, \quad b_6 = \beta_1^2 a_{72} + 2\beta_1 a_{73}
$$

$$
b_7 = 0, \quad b_8 = 0
$$

$$
b_9 = 0, \quad b_{10} = 0
$$

$$
b_{11} = 0, \quad b_{12} = 0
$$

$$
b_{16} = \beta_1^2 b_1 + 2\beta_1 b_4, \quad b_{17} = \beta_1^2 b_2 + 2\beta_1 b_3
$$

$$
b_{18} = \beta_1^2 b_3, \quad b_{19} = \beta_1^2 b_4
$$

$$
b_{20} = \beta_1^2 b_4 + 2\beta_1 b_8, \quad b_{21} = \beta_1^2 b_7
$$

$$
b_{22} = \beta_1^2 b_8, \quad b_{24} = 112b_{11}
$$

$$
b_{27} = -a_1(\beta_1 Th(\beta_1) + 2) - a_1 a_2 \beta_1 Th(\beta_1) - \frac{\beta_1}{2} (TH(\beta_1) + Cth(\beta_1))
$$

$$
b_{28} = -a_1(\beta_1 Th(\beta_1) + 2) + a_1 a_2 \beta_1 Th(\beta_1) + \frac{\beta_1}{2} (TH(\beta_1) - Cth(\beta_1))
$$

$$
b_{29} = -a_1 \beta_1 Th(\beta_1) + \frac{a_4}{\beta_3^2 Sh(\beta_1)} (\beta_1 Ch(\beta_1) - Sh(\beta_1)) - 0.2 - \frac{2a_3}{3}
$$
\[
b_{30} = -\frac{a_3}{\beta_1} Th(\beta_1) + \frac{a_3}{\beta_1^2} (pCh(\beta_1) - Sh(\beta_1)) - 0.2 + \frac{2a_5}{3}
\]

\[
b_{31} = -a_{14} \beta_2 Th(\beta_1) Ch(\beta_1) - \beta_1 Sh(\beta_1) - a_6 (\beta_2 Th(\beta_2) + 2) + a_{18} \beta_2 Th(\beta_2) + a_{15} (\beta_1 Ch(\beta_1) - \beta_2 Cth(\beta_2) - Sh(\beta_1)) + a_{17} (1 - \beta_2 Cth(\beta_2))
\]

\[
b_{32} = a_{14} \beta_2 Th(\beta_2) Ch(\beta_1) - \beta_1 Sh(\beta_1) + a_{16} (\beta_2 Th(\beta_2) + 2) - a_{18} \beta_2 Th(\beta_2) + a_{15} (\beta_1 Ch(\beta_1) - \beta_2 Cth(\beta_2) Sh(\beta_1)) + a_{17} (1 - \beta_2 Cth(\beta_2))
\]

\[
b_{33} = -a_{24} \beta_1 Th(\beta_1) Sh(\beta_1) + \beta_1 Ch(\beta_1) + Sh(\beta_1) + a_{23} Th(\beta_1) + a_{21} (2 + \beta_1 Th(\beta_1)) - a_{19} (\beta_2 Sh(\beta_2) - \beta_1 Th(\beta_1) Ch(\beta_1)) + a_{22} (\beta_2 Cth(\beta_1) - 1) - a_{23} (\beta_2 Sh(\beta_1) - Ch(\beta_1) + \beta_1 Cth(\beta_1) Ch(\beta_1))
\]

\[
b_{34} = a_{24} \beta_1 Th(\beta_1) Sh(\beta_1) + \beta_1 Ch(\beta_1) + Sh(\beta_1) - a_{23} Th(\beta_1) - a_{21} (2 + \beta_1 Th(\beta_1)) + a_{19} (\beta_2 Sh(\beta_2) - \beta_1 Th(\beta_1) Ch(\beta_1)) + a_{22} (\beta_2 Cth(\beta_1) - 1) - a_{25} (\beta_1 Sh(\beta_1) - Ch(\beta_1) + \beta_2 Cth(\beta_1) Ch(\beta_1))
\]

\[
b_{35} = -a_{37} \beta_2 Th(\beta_2) + a_{38} (\beta_2 Ch(\beta_1) - Sh(\beta_2) - a_{39} \beta_2 Sh(\beta_1) + a_{40} (\beta_2 Ch(\beta_1) - Sh(\beta_1) + a_{41} \beta_1 Sh(\beta_1) + a_{42} (\beta_1 Ch(\beta_1) - Sh(\beta_1)) - 4a_{43} + 2a_{44} + 2a_{45}
\]

\[
b_{36} = a_{36} \beta_2 Sh(\beta_1) + a_{38} (\beta_2 Ch(\beta_2) - Sh(\beta_2) + a_{39} \beta_1 Sh(\beta_1) + a_{40} (\beta_2 Ch(\beta_1) - Sh(\beta_1) + a_{42} (\beta_1 Ch(\beta_1) - Sh(\beta_1)) + 4a_{43} + 2a_{44} - 2a_{45}
\]

\[
b_{37} = -a_{60} (\beta_2 Th(\beta_2) Ch(\beta_1) - \beta_2 Sh(\beta_1) - a_{62} (\beta_2 Th(\beta_2) Sh(\beta_2) + \beta_2 Ch(\beta_2) + Sh(\beta_2) - a_{64} (\beta_2 Th(\beta_2) Sh(\beta_2) + \beta_1 Ch(\beta_1) + Sh(\beta_1)) - a_{66} (\beta_2 Th(\beta_2) + 4) + a_{68} (\beta_2 Th(\beta_2) + 2) - a_{70} \beta_2 Th(\beta_2) + a_{61} (\beta_2 Ch(\beta_2) Sh(\beta_1) - \beta_1 Ch(\beta_1) - a_{63} (\beta_2 Cth(\beta_2) Ch(\beta_1) + \beta_1 Sh(\beta_2) - Ch(\beta_2)) - a_{65} (\beta_2 Cth(\beta_2) Ch(\beta_1) + \beta_1 Sh(\beta_1) - Ch(\beta_1)) - a_{67} (\beta_2 Cth(\beta_2) - 3) - a_{69} (\beta_2 Cth(\beta_2) - 1)
\]
\[ b_{37} = a_{60}(\beta_2 Th(\beta_2) Ch(\beta_1) - \beta_1 Sh(\beta_1)) + a_{62}(\beta_2 Th(\beta_2) Sh(\beta_2)) + \beta_2 Ch(\beta_2) + Sh(\beta_2) + a_{64}(\beta_2 Th(\beta_2) Sh(\beta_1) + \beta_1 Ch(\beta_1) + Sh(\beta_1)) - a_{66}(\beta_2 Th(\beta_2) + 4) - a_{68}(\beta_2 Th(\beta_2) + 2) + a_{70}\beta_2 Th(\beta_2) + a_{61}(\beta_2 Ch(\beta_1) Sh(\beta_1) - \beta_1 Ch(\beta_1) - a_{63}(\beta_2 Ch(\beta_2) Ch(\beta_1)) + \beta_1 Sh(\beta_2) - Ch(\beta_2)) - a_{65}(\beta_2 Ch(\beta_2) Ch(\beta_1) + \beta_1 Sh(\beta_1) - Ch(\beta_1)) - a_{67}(\beta_2 Ch(\beta_2) - 3) - a_{69}(\beta_2 Ch(\beta_2) - 1) \]

\[ b_{39} = -a_{71}(\beta_2 Sh(\beta_2) - \beta_1 Th(\beta_1)) + a_{74}(\beta_2 Ch(\beta_2) + Sh(\beta_2) + \beta_2 Ch(\beta_2) Sh(\beta_2)) + a_{73}(\beta_1 Ch(\beta_1) + Sh(\beta_1) + \beta_2 Th(\beta_2) Sh(\beta_1) - 2a_{77} Ch(\beta_1) + 4a_{78} + 2a_{81} + a_{83} \beta_1 Th(\beta_1) + a_{72}(\beta_2 Ch(\beta_2) - \beta_1 Ch(\beta_1) Sh(\beta_2)) - a_{73}(\beta_2 Sh(\beta_2) + \beta_1 Ch(\beta_1) Ch(\beta_1)) + a_{78} Sh(\beta_1) + a_{80}(3 - \beta_1 Ch(\beta_1)) + a_{82}(1 - \beta_1 Ch(\beta_1)) \]

\[ b_{40} = a_{71}(\beta_2 Sh(\beta_2) - \beta_1 Th(\beta_1)) - a_{74}(\beta_2 Ch(\beta_2) + \beta_2 Th(\beta_2) Sh(\beta_2)) - 4a_{78} - 2a_{81} - a_{83} \beta_1 Th(\beta_1) + a_{72}(\beta_2 Ch(\beta_2) - \beta_1 Ch(\beta_1) Sh(\beta_2)) - a_{73}(\beta_2 Sh(\beta_2) - \beta_1 Sh(\beta_1) + \beta_1 Ch(\beta_1) Ch(\beta_1)) + 2a_{78} Sh(\beta_1) + a_{80}(3 - \beta_1 Ch(\beta_1)) + a_{82}(1 - \beta_1 Ch(\beta_1)) \]

\[ b_{41} = -b_{15} \beta_1 Sh(\beta_1) + b_{19}(\beta_1 Ch(\beta_1) + Sh(\beta_1)) - b_{20} \beta_2 Sh(\beta_2) + b_{23}(\beta_2) Ch(\beta_2) + Sh(\beta_2) + 2b_{24} + b_{17}(\beta_1 Ch(\beta_1) - Sh(\beta_1)) - b_{18} \beta_1 Sh(\beta_1) + b_{21}(\beta_2 Th(\beta_2) - Sh(\beta_2) - b_{22} Ch(\beta_2) \]

\[ b_{42} = b_{16} \beta_1 Sh(\beta_1) - b_{19}(\beta_1 Ch(\beta_1) + Sh(\beta_1)) + b_{20} \beta_2 Sh(\beta_2) - b_{23}(\beta_2 Ch(\beta_2) + Sh(\beta_2) - 2b_{24} + b_{17}(\beta_1 Ch(\beta_1) - Sh(\beta_1)) - b_{18} \beta_1 Sh(\beta_1) + b_{21}(\beta_2 Ch(\beta_2) - Sh(\beta_2) - b_{22} Ch(\beta_2) \]
Dear Prof. Prasada Rao

I am happy to inform you that your papers entitled,
**Non - Darcy Hydromagnetic Convective Heat and Mass Transfer through a Porous medium in Cylindrical Annulus with Soret effect Radiation and Dissipation**

M. SURESH, M. BHARATHI, U. RAJESWARA RAO & D. R. V. PRASADA RAO
Department of Mathematics, S. K. University, Anantapur – 515 055. India.

has been accepted for publication in Journal of Pure & Applied Physics, Vol. 21, No.2, 2009.

With Best Wishes

Prof. ADEEL AHMAD
Associate Chief Editor, JPAP

Ref: 0624/2/JPAP/2009
Date: 24-06-2009
Dear Prof. Prasada Rao,


With Best Wishes

Prof. ADEEL AHMAD
Associate Chief Editor, JPAP