CHAPTER VI

PERISTALTIC FLOW OF A DUSTY-COUPLE STRESS FLUID WITH HEAT TRANSFER IN A FLEXIBLE CHANNEL
6.1 INTRODUCTION

Saffmen [19] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. He made the following assumptions:

i) The dust particles are uniform in size and shape so that their number density and velocity are given as \( v(x, y, z, t) \), \( N(x, y, z, t) \). Further from particular situations, the number density can also be assumed to be constant \( N_0 \).

ii) The volume concentration of the dust is assumed to be so small that the net effect of the dust on the gas is \( K N (u_p - u) \) per unit volume.

iii) The Reynolds number of the relative motion of dust and gas is supposed to be very small compared to unity so that the force between the dust and gas is proportional to the relative velocity.

iv) If the dust particles are assumed to be spheres of radius ‘a’ the stoke's drag formula holds good, so that \( K=6 \pi \mu a \), \( \mu \) being the viscosity.

Couple stresses are found to appear in noticeable magnitudes in polymer solutions (liquids with large molecules). Stokes [28] gave the equation of motion and constitutive equations for a couple stress fluids on the basis of couple stress in elastic materials. This theory is developed in an effort to examine the simplest generalization of the classical theory which allows polar effects.

In order to develop a mathematical theory of blood flow in arteries, Alihasan Nayfeh [1] considered blood as binary system of plasma (liquid phase) and blood cells a (solid phase). Saffman’s[19]dusty fluid serves as a better model to describe blood as a binary system. Nag,S.K.[17] studied the two-dimensional flow of unbounded dusty fluid induced by the sinusoidal transverse motion of an infinite wall. The two-phase flow in a flexible channel on which a traveling sinusoidal wave is imposed on the boundary resulting in a peristaltic flow has been discussed by
Mallikarjuna Goud [12]. Dust velocity shear driven rotational waves and associated vertices in a non-uniform dusty plasma has been discussed by P.K. Shukla et al. [22].

Hence, in this chapter, we have investigated the interaction of peristalsis with heat transfer for the motion of a Dusty couple stress fluids in a two dimensional flexible channel under long wavelength approximation. The effects of various parameters on temperature and heat transfer have been studied. The computational analysis has been carried out for drawing temperature, heat transfer coefficient, average temperature, average heat transfer, pressure drop and frictional force.
6.2 FORMULATION OF THE PROBLEM

We consider a peristaltic flow of a dusty viscous incompressible couple stress fluid through a two-dimensional channel bounded by flexible walls.

The geometry of the flexible walls are represented by

$$y = \eta(x,t) = d + a \sin \frac{2\pi}{\lambda}(x - ct)$$  \hspace{1cm} (6.2.1)$$

Where ‘d’ is the mean half width of the channel ‘a’ is the amplitude of the peristaltic wave, ‘c’ is the wave velocity, ‘$\lambda$’ is the wave length and ‘c’ is the phase speed of the wave.

The governing equations of motion for dusty viscous incompressible couple stress fluid in vector form are

$$\rho \frac{D}{Dt} \mathbf{q} = -\nabla p + \mu \nabla^2 \mathbf{q} - \eta \nabla \cdot \mathbf{q} + K N(q_t^p - q)$$ \hspace{1cm} (6.2.2)$$

$$q_t^p + [q_t^p, \nabla] q_t^p = \frac{k(q - q_t^p)}{m}$$ \hspace{1cm} (6.2.3)$$

$$\nabla \cdot q = 0$$ \hspace{1cm} (6.2.4)$$

$$N_t + \nabla \cdot [N q_t^p] = 0$$ \hspace{1cm} (6.2.5)$$

Where $q[u,v]$, $q_t^p[u^p,v^p]$ denote the velocities of the fluid and dust particles respectively, ‘p’ is the fluid pressure, ‘$\rho$’ is the density of the fluid, $\mu$ is the coefficient of viscosity, $m$ is the mass of the dust particle, $N$ is the number density of the particles, $k$ is the stokes resistance coefficient. The particles are assumed to be uniform in size and uniformly distributed in the fluid so that $N$ remains a constant.

The equations governing the two-dimensional flow of a dusty viscous incompressible couple stress fluid in fluid phase are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ \hspace{1cm} (6.2.6)$$

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\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \eta \left( \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \right) - K N(u^p - u) \tag{6.2.7}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \eta \left( \frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4} + 2 \frac{\partial^4 v}{\partial x^2 \partial y^2} \right) - K N(v^p - v) \tag{6.2.8}
\]

Equation of energy

\[
\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + p u \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)^2 + \frac{N m C_s}{\tau_p} (T^p \cdot T) \tag{6.2.9}
\]

The equations governing the two-dimensional flow of a dusty viscous incompressible couple stress fluid in particle phase are

\[
\frac{\partial N}{\partial x} + \mu \left( \frac{\partial u^p}{\partial x} + \frac{\partial v^p}{\partial y} \right) = 0 \tag{6.2.10}
\]

\[
\frac{\partial u^p}{\partial t} + u^p \frac{\partial u^p}{\partial x} + v^p \frac{\partial u^p}{\partial y} = \frac{k}{m} (u - u^p) \tag{6.2.11}
\]

\[
\frac{\partial v^p}{\partial t} + u^p \frac{\partial v^p}{\partial x} + v^p \frac{\partial v^p}{\partial y} = \frac{k}{m} (v - v^p) \tag{6.2.12}
\]

Equation of energy

\[
\frac{\partial T^p}{\partial t} + u^p \frac{\partial T^p}{\partial x} + v^p \frac{\partial T^p}{\partial y} = \left( \frac{T - T^p}{\tau_p} \right) \tag{6.2.13}
\]

The relative boundary conditions are

\[
u = 0 \quad \text{at} \quad y = \pm \eta \tag{6.2.14}
\]

\[
\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \eta \tag{6.2.15}
\]

\[
v = 0 \quad \text{at} \quad y = 0 \tag{6.2.16}
\]

\[
T = T_0 \quad \text{at} \quad y = -\eta, \quad T = T_1 \quad \text{at} \quad y = \eta \tag{6.2.17}
\]
Equation (6.2.14) corresponds to no slip on the boundary, (6.2.15) indicates the boundary condition related to couple stress fluid, (6.2.16) indicates velocity at the centre of the channel and (6.2.17) represents the conditions on temperature.

Introducing a wave frame \((x, y)\) moving with velocity \(c\) away from the fixed frame \((X, Y)\) by the transformation

\[
x = X - ct, \quad y = Y
\]

We introduced the non-dimensional variables as

\[
x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\beta}, \quad u' = \frac{u - c}{c}, \quad u_p' = \frac{u_p - c}{c}, \quad v' = \frac{v}{c \delta}, \quad v_p' = \frac{v_p}{c \delta}, \quad t' = \frac{ct}{\lambda}, \quad \eta' = \frac{\eta}{\beta},
\]

\[
p' = \frac{p}{c \mu \lambda}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \theta^* = \frac{T_p - T_0}{T_1 - T_0}
\]

(6.2.18)

Substituting (6.2.18) in (6.2.6) – (6.2.17), these equations reduces to (after dropping primes)

\[
R \delta \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left( \frac{\partial^2 u}{\partial y^2} + \delta^2 \frac{\partial^2 u}{\partial x^2} \right) - S \left( \frac{\partial^4 u}{\partial y^4} + \delta^4 \frac{\partial^4 u}{\partial x^4} + 2 \delta^2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \right) + Ra \frac{1}{\zeta} (u_p - u)
\]

(6.2.19)

\[
R \delta^3 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left( \frac{\partial^2 v}{\partial y^2} + \delta^2 \frac{\partial^2 v}{\partial x^2} \right) - S \delta^2 \left( \frac{\partial^4 v}{\partial y^4} + \delta^4 \frac{\partial^4 v}{\partial x^4} + 2 \delta^2 \frac{\partial^4 v}{\partial x^2 \partial y^2} \right) - Ra \frac{1}{\zeta} \delta^2 (v_p - v)
\]

(6.2.20)

\[
\delta \left( \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \frac{1}{R \rho} \left( \frac{\partial^2 \theta}{\partial y^2} + \delta^2 \frac{\partial^2 \theta}{\partial x^2} \right) + \frac{E}{R} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 - \alpha \frac{\partial \theta}{\partial x} (\theta_p - \theta)
\]

(6.2.21)

\[
\frac{\partial N}{\partial t} + N \left( \frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} \right) = 0
\]

(6.2.22)
The boundary conditions are

\[ u = -1 \quad \text{at} \quad y = \pm \eta \]  
\[ \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \eta \]  
\[ v = 0 \quad \text{at} \quad y = 0 \]  
\[ \theta = 0 \quad \text{at} \quad y = -\eta, \quad \theta = 1 \quad \text{at} \quad y = \eta \]

Where

\[ \varepsilon = \frac{a}{d}, \quad \delta = \frac{d}{\lambda} \]  
\[ R = \frac{cd}{v} \]  
\[ Pr = \frac{\rho C_p v}{k_1} \]  
\[ S = \frac{\eta}{\mu d^2} \]  
\[ \gamma = \frac{C_s}{C_p} \]  
\[ \zeta = \frac{c m}{k d} \]  
\[ \alpha = \frac{N m}{\rho} \]  
\[ E = \frac{c^2}{C_p(T_1 - T_0)} \]
6.3 METHOD OF SOLUTION

We seek perturbation solution for the following functions in terms of small parameter \( \delta \) as follows:

\[
\begin{align*}
\mathbf{u} & = u_0 + \delta u_1 + \delta^2 u_2 + \ldots \\
\mathbf{u}^p & = u_0^p + \delta u_1^p + \delta^2 u_2^p + \ldots \\
\mathbf{v} & = v_0 + \delta v_1 + \delta^2 v_2 + \ldots \\
\mathbf{v}^p & = v_0^p + \delta v_1^p + \delta^2 v_2^p + \ldots \\
\mathbf{\theta} & = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 + \ldots \\
\mathbf{\theta}^p & = \theta_0^p + \delta \theta_1^p + \delta^2 \theta_2^p + \ldots
\end{align*}
\]

The equations corresponding to zeroth order in \( \delta \) are

\[
\begin{align*}
S \frac{\partial^4 u_0}{\partial y^4} - \frac{\partial^2 u_0}{\partial y^2} - \frac{R\alpha}{\tau} (u_0^p - u_0) & = -\frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial y} & = 0 \\
\frac{1}{\zeta} (u_0 - u_0^p) & = 0 \\
\frac{1}{\zeta} (v_0 - v_0^p) & = 0 \\
\frac{1}{R Pr} \left( \frac{\partial^2 \theta_0}{\partial y^2} \right) + \frac{E}{R} \left( \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right)^2 + \frac{E \gamma}{\zeta} (\theta_0^p - \theta_0) & = 0 \\
\frac{1}{\zeta} (\theta_0^p - \theta_0) & = 0
\end{align*}
\]
The corresponding boundary conditions are

\[ u_0 = -1 \quad \text{at} \quad y = \pm \eta \quad (6.3.13) \]
\[ \frac{\partial^2 u_0}{\partial y^2} = 0 \quad \text{at} \quad y = \pm \eta \quad (6.3.14) \]
\[ v_0 = 0 \quad \text{at} \quad y = 0 \quad (6.3.15) \]
\[ \theta_0 = 0 \quad \text{at} \quad y = -\eta, \quad \theta_0 = 1 \quad \text{at} \quad y = \eta \quad (6.3.16) \]

**Zeroth-order problem**

Solving the equations (6.3.7-6.3.12) subject to the conditions (6.3.13-6.3.16), we get

\[ u_0 = u_0^P = G_1 + G_2 \cosh \beta y + G_3 y^2 \quad (6.3.17) \]
\[ v_0 = v_0^P = -\left( a_1 y + \frac{a_2 \sinh \beta y}{\beta} \right) \quad (6.3.18) \]
\[ \theta_0 = \theta_0^P = \frac{L_6}{12} y^4 + \frac{L_2}{\beta^2} y \sinh [\beta y] - \frac{2 L_2}{\beta^3} \cosh [\beta y] + \frac{L_8}{4 \beta} \sinh [2 \beta y] \]
\[ - \frac{L_8}{2} y + C_9 y + C_{10} \quad (6.3.19) \]

The equations corresponding to the order \( \delta \) are

\[ S \frac{\partial^4 u_1}{\partial y^4} - \frac{\partial^2 u_1}{\partial y^2} - \frac{R \alpha}{\tau} (u_1^P - u_1) = -R \left( \frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) \quad (6.3.20) \]
\[ \frac{\partial u_0^P}{\partial t} + u_0 \frac{\partial u_0^P}{\partial x} + v_0 \frac{\partial u_0^P}{\partial y} = \frac{1}{\tau} (u_1 - u_1^P) \quad (6.3.21) \]
\[ \frac{\partial v_0^P}{\partial t} + u_0 \frac{\partial v_0^P}{\partial x} + v_0 \frac{\partial v_0^P}{\partial y} = \frac{1}{\tau} (v_1 - v_1^P) \quad (6.3.22) \]
\[ \frac{1}{R Pr} \left( \frac{\partial^2 \theta_1}{\partial y^2} \right) + \frac{E}{R} \left( \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial y} \right)^2 + \frac{\alpha \gamma}{\tau} (\theta_1^P - \theta_1) = \left( \frac{\partial \theta_0}{\partial t} + u_0 \frac{\partial \theta_0}{\partial x} + v_0 \frac{\partial \theta_0}{\partial y} \right) \quad (6.3.23) \]
\[ \frac{\partial \theta_0^P}{\partial t} + u_0 \frac{\partial \theta_0^P}{\partial x} + v_0 \frac{\partial \theta_0^P}{\partial y} = \frac{1}{\tau} (\theta_1 - \theta_1^P) \quad (6.3.24) \]
The corresponding boundary conditions are

\[
\begin{align*}
u_j &= 0 \quad \text{at} \quad y = \pm \eta \quad (6.3.25) \\
\frac{\partial^2 u_j}{\partial y^2} &= 0 \quad \text{at} \quad y = \pm \eta \quad (6.3.26) \\
v_j &= 0 \quad \text{at} \quad y = 0 \quad (6.3.27) \quad \theta_1 = 0 \quad \text{at} \quad y = -\eta, \quad \theta_1 = 1 \quad \text{at} \quad y = \eta \quad (6.3.28)
\end{align*}
\]

**First-order problem**

Solving the equations (6.3.20-6.3.24) subject to the conditions (6.3.25-6.3.28), we get

\[
\begin{align*}
u_j &= C_5 + C_7 \cosh \beta y + h_1 y \sinh \beta y + h_2 y^3 \sinh \beta y + h_3 y^2 \cosh \beta y - h_4 y^2 - h_5 y^4 \
\theta_1 &= R_1 \cosh \beta y + R_9 \sinh \beta y + R_8 \cosh 2\beta y + R_3 \sinh 2\beta y + R_10 \sinh 3\beta y \\
&\quad + R_{11} y \cosh \beta y + R_2 y \sinh \beta y - R_{16} y \cosh 2\beta y + R_7 y \sinh 2\beta y + R_6 y^2 \cosh \beta y + R_{19} y^2 \sinh \beta y + R_{25} y^2 \cosh 2\beta y + R_{15} y^2 \sinh 2\beta y \\
&\quad + R_{21} y^3 \cosh \beta y + R_5 y^3 \sinh \beta y + R_{27} y^3 \sinh 2\beta y + R_4 y^4 \cosh \beta y - R_{23} y^4 \sinh \beta y + R_{26} y^4 \cosh 2\beta y + R_{22} y^4 \cosh \beta y + R_{20} y^5 \sinh \beta y \\
&\quad + R_{28} + R_{24} y^6 \cosh \beta y + R_{28} y^6 \sinh 2\beta y + R_{29} y^6 \cosh 2\beta y - R_{12} y^6 + R_{17} y^5 + R_{18} y^4 + R_{13} y^3 + R_{14} y^2 + R_{30} y + R_{31}
\end{align*}
\]
\[ \theta^p_1 = R_{32} \cosh \beta y + R_{33} \sinh \beta y + R_{34} \cosh 2\beta y + R_{35} \sinh 2\beta y + R_{36} \beta y + R_{37} \beta y + R_{38} \cosh \beta y + R_{39} \sinh \beta y + R_{40} \cosh 2\beta y + R_{40} \cosh 2\beta y + R_{41} \cosh 2\beta y + R_{42} \beta y + R_{43} \cosh \beta y + R_{44} \cosh \beta y + R_{45} \sinh \beta y + R_{46} \cosh \beta y + R_{47} \sinh \beta y + R_{48} \cosh \beta y + R_{49} \sinh \beta y + R_{50} \beta y + R_{51} \beta y \]

\[ \text{(6.3.34)} \]

The heat transfer coefficient in terms of wall slope parameter 'δ' is

\[ z = z_0 + \delta z_1 + \ldots \]

\[ \text{(6.3.35)} \]

\[ z_0 = \left( \frac{\partial \eta}{\partial x} \right) \left( \frac{\partial \theta_0}{\partial y} \right) \]

\[ \text{(6.3.36)} \]

\[ z_1 = \left( \frac{\partial \theta_0}{\partial x} \right) + \left( \frac{\partial \eta}{\partial x} \right) \left( \frac{\partial \theta_1}{\partial y} \right) \]

\[ \text{(6.3.37)} \]

The average temperature, \( \bar{\theta} \) is given by

\[ \bar{\theta} = \frac{1}{\theta} \int_0^1 \theta \, dt \]

\[ \text{(6.3.38)} \]

The average heat transfer, \( \bar{z} \) is given by

\[ \bar{z} = \frac{1}{z} \int_0^z z \, dt \]

\[ \text{(6.3.39)} \]

The volumetric flow rate in the wave frame is defined by

\[ q = \int_0^\eta u \, dy = (A_1 + A_2 + A_3) P^2 + (A_4 + A_5) P + A_6 \]

\[ \text{(6.3.40)} \]
where

\[ A_1 = \delta((M_1 \eta + m_{10} \eta^3) \cosh[\beta \eta] + M_2 \eta^2 \sinh[\beta \eta] + M_1 \eta^5 + M_3 \eta^3 + M_4 \eta) \]

\[ A_2 = \frac{\delta \sinh[\beta \eta]}{\beta} (((M_5 + m_{16}) + m_{15} \eta^2) \sech[\beta \eta] + ((M_7 + m_{17} + m_{20} + m_{21} + M_9) \eta + m_{14} \eta^3) \tanh[\beta \eta] + (m_{11} + m_{14} + M_{10}) \eta^2 + (M_6 + m_9 + m_{10} + M_8 + M_{11} + M_{12})) \]

\[ A_3 = \delta\left(\frac{1}{\beta^3} (m_5 \eta^3 \beta^2 + 6 m_5 \eta - 2 M_{15} \eta \beta + (m_8 + M_{14} + M_{16}) \eta \beta^2) \cosh[\beta \eta] \right. \\
\left. + \frac{1}{\beta^4} (2 M_{15} \beta - (m_8 - M_{14} + M_{16} + 3 m_5 \eta^2) \beta^2 + M_{15} \eta^2 \beta^3 - 6 m_5) \sinh[\beta \eta] \right) + \\
\frac{m_{10}}{5} \eta^5 + \frac{M_3}{3} \eta^3 \]

\[ A_4 = \delta\left(\frac{1}{\beta} (m_3 \eta^2 + m_{19} \eta) \cosh[\beta \eta] + \left(\frac{1}{\beta} (m_1 \tanh[\beta \eta] + m_7 \eta \sinh[\beta \eta] \tanh[\beta \eta] + m_6) + \frac{1}{3} (3 m_{14} \eta^3 + m_{16} \eta^3) \right) \right) \]

\[ A_5 = C_1 \eta + \frac{1}{\beta} C_3 \sinh[\beta \eta] + \eta^3 \quad \text{and} \quad A_6 = \frac{\eta^2}{4} \frac{\partial p_i}{\partial x} - \eta \]

The instantaneous flux \( Q(x, t) \) in the laboratory frame is

\[ Q = \int_0^\eta (u + 1) \, dy = q + \eta \quad (6.3.41) \]

The average flux \( \overline{Q} \) over one period of peristaltic wave is

\[ \overline{Q} = \frac{1}{T} \int_0^T Q \, dt = q + 1 \quad (6.3.42) \]

From equations (5.3.30) and (5.3.32), the pressure gradient is obtained as

\[ \frac{dp}{dx} = \frac{-M \pm \sqrt{M^2 - 4L(N - \overline{Q} + 1)}}{2L} \quad (6.3.43) \]

Where \( L = A_1 + A_2 + A_3 \), \( M = A_4 + A_5 \), \( N = A_6 \)

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The pressure rise (drop) over one cycle of the wave can be obtained as

\[ \Delta p = \frac{1}{0} \left( \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + \ldots \right) \, dx \tag{6.3.44} \]

The dimensionless frictional force \( F \) at the wall across one wavelength is given by

\[ F = \int_0^1 \eta^2 \left( \frac{\partial p}{\partial x} \right) \, dx \tag{6.3.45} \]

In the absence of heat transfer, this problem reduces to S.Ravi Kumar [18].

The constants

\[ a_1, a_2, \ldots, a_{43}, C_1, C_3, \ldots, C_{10}, g_1, g_2, \ldots, g_{25}, G_1, G_2, \ldots, G_7, h_1, h_2, \ldots, h_{22}, L_1, L_2, \ldots, L_8, \]
\[ m_1, m_2, \ldots, m_{21}, M_1, M_2, \ldots, M_{16}, R_1, R_2, \ldots, R_{51} \]

are given in appendix.
6.4. RESULTS AND DISCUSSION

In this analysis we investigate the effect of various governing parameters are R, δ, e, α, γ, P_r and S on Heat transfer in the fluid phase and the particle phase.

The non-dimensional temperature distribution ‘θ’ is depicted in figs.(6.1-6.7) for a different parametric values R, α ,γ, P_r, ε, δ, and S. Figs.(6.1 - 6.3) it is noticed that the non-dimensional temperature distribution ‘θ’ depreciates with increase in R, α and γ and attains the prescribed value at y = 1. We find that temperature ‘θ’ experiences an appreciable enhancement with increasing δ, P_r and S in the entire flow region and it attains prescribed value at y =1 figs(6.4-6.6).From fig.(6.7) we find that ‘θ’ experiences depreciation with increase in the amplitude ratio ε.

Figs(6.8-6.14) represents temperature distribution ‘θ^p’ in the particle phase with different parametric values R, α ,γ, P_r, ε, δ, and S. It is noticed that the non-dimensional temperature distribution ‘θ^p’ depreciates with increase in R, α and γ and attains the prescribed value at y = 1 figs.(6.8 - 6.10). We find that temperature ‘θ^p’ experiences an appreciable enhancement with increasing δ, P_r and S in the entire flow region and it attains prescribed value at y =1 figs(6.11-6.13).From fig.(6.14) we find that ‘θ^p’ experiences depreciation with increase in the amplitude ratio ε.

Figs.(6.15-6.21) represent heat transfer coefficient z characteristics in fluid phase. fig.(6.15) represent the variation of heat transfer z with Reynolds number R, we find that an increase in the Reynolds number R leads to a depreciation in the heat transfer coefficient z. The depreciation in z with R increases with increase in R. The variation of z with Ratio of specific heats γ, reveals that the depreciation in z is very marginal with the increase in γ in the flow region fig.(6.16). In fig.(6.17) we find that an increase in Dust concentration parameter α reduces z in entire flow region, the depreciation in z with α is almost negligible on the central line y = 0 and it is
appreciable as we move towards the boundary $y = 1$. The variation of $z$ with Prandtl number $p_r$, shows that the heat transfer coefficient experiences a depreciation with $p_r$, the depreciation on the boundary at $y = 1$ is appreciably large in fig(6.18). The effect of amplitude ratio $\varepsilon$ on the coefficient of Heat transfer is shown in fig.(6.19). The variation in $z$ with different $\varepsilon$ is almost linear as we move from the central line $y = 0$ to the boundary $y = 1$, the heat transfer coefficient $z$ experiences with $\varepsilon$ in the entire flow region. The effect of slope parameter $\delta$ on $z$ is exhibited in fig.(6.20). The profiles of $z$ with $\delta$ shows that the values of $z$ gradually reduces as we move from the central line $y = 0$ to $y = 1.0$. An increase in $\delta$ leads to an enhancement in $z$. Fig.(6.21) represent the variation of $z$ with $S$, we find that the values $S \leq 2$ the heat transfer coefficient $z$ reduces as we move from the central line to the boundary, while for $S \geq 3$, it enhances enhancement with Couple stress parameter $S$.

From figs.(6.22-6.28) represent the variation of heat transfer coefficient $z^*$ with $R, \delta, \varepsilon, \alpha, \gamma, p_r$ and $S$ in the particle phase. From figs(6.22 - 6.25) we find that the heat transfer coefficient $z^*$ in the particle phase reduces gradually as we move from the central line $y = 0$ to boundary $y = 1$. An increase in the Reynolds number $R$ results a depreciation in $z^*$. Fig.(6.22) The depreciation in $z^*$ with $R$ is marginal at the central line $y = 0$ and is appreciably large at $y = 1$. Fig.(6.23) The variation of transfer coefficient $z^*$ with Ratio of specific heats $\gamma$ reveals that $z^*$ experiences marginal enhancement with increase in $\gamma$. From fig.(6.24) the variation of $z^*$ with Dust concentration parameter $\alpha$ shows that the depreciation in $z^*$ is marginal in the central line and appreciable at $y = 0$. From fig.(6.25) the variation of heat transfer coefficient $z^*$ with Prandtl number $P_r$, we find that as we move from central line $y = 0$ to $y = 1$ heat transfer coefficient $z^*$ gradually reduces and it depreciates with increase in $P_r$. Fig.(6.27) the variation of $z^*$ with amplitude ratio $\varepsilon$, shows that $z^*$ is almost linear
with any value of $\epsilon$, as we move from $y = 0$ to $y = 1$, also it experiences an enhancement with increase in $\epsilon$. Fig. (6.28) represent the variation of $z^*$ with slope parameter $\delta$, it is found that higher the slope $\delta$ of the boundary layer the heat transfer coefficient $z^*$. The variation of heat transfer coefficient $z^*$ with couple stress parameter $S$ shows that for $S \leq 2$, $z^*$ decreases gradually from $y = 0$ to $y = 1$ and for higher $S \geq 3$, it enhances as we move the central line to the boundary also higher the couple stress parameter $S$ larger the heat transfer coefficient $z^*$.

The variation of Average temperature $\bar{\theta}$ is depicted in figs. (6.29-6.35). Fig. (6.29) show that the effect of Reynolds number $R$ on the average temperature $\bar{\theta}$, the effect is almost uniform. The effect of $R$ on $\bar{\theta}$ gradually enhances as we move from the centre from $y = 0$ to $y = 1$. From figs. 6.30 & 6.31, it is observed that the average temperature $\bar{\theta}$ increases with $\gamma$ and $\alpha$, is more significant at the central of the channel than at the boundary. It is observed that the average temperature $\bar{\theta}$ increases with $P_r$ and $\epsilon$ and is more significant at the central of the channel than at the boundary figs 6.32 & 6.33. It is observed from fig. (6.34) that the average temperature $\bar{\theta}$ enhances with increases the amplitude ratio $\epsilon$. The enhancement of $\bar{\theta}$ in the central region is large in comparative that near the boundary, also, an increase in couple stress parameter $S$ increases $\bar{\theta}$ in fig. (6.35).

Figs. (6.36 - 6.42) represent the variation of average heat transfer $\bar{z}$ with different parametric values $R, \delta, \epsilon, \alpha, \gamma, P_r$ and $S$. From the above figures we notice that the average heat transfer $\bar{z}$ gradually reduces from the maximum value at $y = 0$ and attains prescribed value on the boundary $y = 1$. From fig. (6.36), we find that an increase in the Reynolds number $R$ enhances the average heat transfer $\bar{z}$ in the entire flow region. The variation of average heat transfer $\bar{z}$ with $\gamma$ shows that average heat
transfer $\bar{z}$ experiences the depreciation in magnitude with $y$ in fig.(6.37). From fig.(6.38) we notice that the variation of average heat transfer $\bar{z}$ with $\alpha$ is marginal in the region $0 \leq y \leq 0.4$ and it is noticeable in the remaining region. From fig.(6.39) we observe that an amplitude ratio $\varepsilon$ increases, the average heat transfer $\bar{z}$ experiences a depreciation in magnitude in the flow region. From fig.(6.40) the variation of average heat transfer $\bar{z}$ with $S$ shows that average heat transfer $\bar{z}$ gradually reduces from $y = 0$ to $y = 1$ for $S \leq 2$, and for higher $S \geq 3$ it enhances as we move the central line to the boundary also average heat transfer $\bar{z}$ experiences an enhancement with $S$. Fig.(6.41) we find that lesser the slope $\delta$ of the wall smaller the magnitude of average heat transfer $\bar{z}$, also average heat transfer $\bar{z}$ depreciates with increase in the Prandtl number $P_r$ in the fig.(6.42).

Figs.(6.43 to 6.50) represent the variation of average heat transfer $\bar{z}$ with different parametric values $R$, $\delta$, $\varepsilon$, $\alpha$, $\gamma$, $P_r$, $\zeta$ and $S$. The variation of average heat transfer $\bar{z}$ with Reynolds number $R$ shows that, it depreciates in magnitude with increase in $R$ fig.(6.43). Figs.(6.44-6.46) we find that the average heat transfer $\bar{z}$ in particle phase gradually enhances, attains the maximum value at $y = 0.8$ and then falls to prescribed value at $y = 1$. It is found that average heat transfer $\bar{z}$ experiences an enhancement with $\gamma$, $\alpha$, $P_r$. Fig.(6.47) represent the variation of average heat transfer $\bar{z}$ with the amplitude ratio $\varepsilon$ for $\varepsilon \leq 0.02$, the variation of average heat transfer $\bar{z}$ is linear as we central line $y = 0$ to the boundary $y = 1$, and for higher value of $\varepsilon$ average heat transfer $\bar{z}$ gradually enhances, attain the maximum at $y = 0.8$ and then reduces on $y = 1$. An increase in $\varepsilon$ leads to enhancement in magnitude in average heat transfer $\bar{z}$, also from fig.(6.48) we find that higher the slope $\delta$ larger $|\bar{z}|$ in the entire
flow region. The variation of average heat transfer $\vec{z}$ with the Relaxation time $\zeta$ shows that $|\vec{z}|$ is maximum at $y = 0$ and gradually enhances from $y = 0$ to $y = 1$. $|\vec{z}|$ enhances in the region $0 \leq y \leq 0.9$ and in the region adjacent to $y = 1$, it reduces in magnitude in Fig.(6.49). The variation of average heat transfer $\vec{z}$ with the Couple stress parameter $S$, is shown in Fig (6.50). It is found that for $S \leq 0.2$, enhances in the region $y = 0$ to $y = 0.8$, and reduces in the remaining region and for higher $S \geq 3$, $|\vec{z}|$ depreciates in the entire flow region.

Figs.(6.51-6.55) represent the variation of $\Delta p$ against $\bar{Q}$ in the fluid phase. It is found that the above all figures $\Delta p$ gradually decreases in magnitude to attain the minimum at $\bar{Q} = 1.0$. From Fig.(6.51) we find that $R \leq 2$, the profiles are of straight lines and for higher $R \geq 3$ the profiles are parabolic in nature and the $\Delta p$ experiences an enhancement with $R$. Fig (6.52) $|\Delta p|$ reduces with Dust concentration parameter $\alpha$. From Figs.(6.53-6.55) we find that the variation of $\Delta p$ with for different parameters $\varepsilon$, $\delta$ and $S$, shows that $\Delta p$ experiences an enhancement with $\varepsilon$, $\delta$ and $S$ in the entire flow region.

From Figs.(6.56-6.60) represent the variation of Frictional force $F$ against $\bar{Q}$ for a different parametric values. Fig.(6.56) represents the Frictional force $F$ with $R$, it is found that $R \leq 2$, the profiles of $|F|$ are straight lines and for higher $R \geq 3$ the profiles are almost parabolic in nature. It is found that $F$ experiences an enhancement with increase in $R$. Fig.(6.57) the Frictional force $F$ enhances with increase in Dust concentration parameter $\alpha$. From Figs(6.58-6.60) we notice that the Frictional force $F$ against $\bar{Q}$ decreases from $\bar{Q} = 0$ to $\bar{Q} = 1.0$, also we notice that $F$ experiences an enhancement with increase in $\varepsilon$, $\delta$ and $S$ in the entire flow region.
REFERENCES


13) Mekheimer, Kh.S., "Peristaltic transport of a Couple-stress fluid in a uniform and
14) Memone, A.V. and Masumdar, J., "Mathematical modeling of peristaltic transport
of a non-newtonian fluid", J. Australian Physical and Engineering Sciences in
16) Nadeem, S. and Noreen Sher Akbar, "Influence of Heat Transfer on a peristaltic
transport of Herschel-Bulkley fluid in a non-uniform inclined tube", Non-linear
17) Nag, S.K. "The two-dimensional flow of unbounded dusty fluid in an infinite
19) Sagayamary, S. and Devanathan, R., "Steady flow of couple stress fluid through
tubes of slowly varying cross-sections-Application to blood flows",
20) Shapiro, A.H., Jaffrin, M.Y., Weinberg, S.L.: Peristaltic pumping with long


APPENDIX:

\[ \eta = 1 + \varepsilon \sin 2 \pi x \quad b = \frac{R(1+\alpha)}{\varepsilon} \quad \beta = \frac{1}{\sqrt{s}} \]

\[ C_1 = \frac{1}{\beta^2} - \frac{\eta^2}{2} \quad C_3 = \frac{1}{\beta^2 \text{Sech} [\beta \eta]} \]

\[ G_1 = PC_1 - 1 \quad G_2 = PC_3 \quad G_3 = \frac{P}{2} \]

\[ G_4 = \left( \frac{L_2}{\beta^2} + \frac{6L_4}{\beta^6} - \frac{2L_5}{\beta^5} \right) \text{Cosh} [\beta \eta] + \left( \frac{L_5}{\beta^4} - \frac{4L_4}{\beta^5} \right) \eta \text{Sinh} [\beta \eta] + \frac{L_4}{\beta^4} \eta^2 \text{Cosh} [\beta \eta] + \frac{L_3}{12\beta^2} \eta^4 \]

\[ + \left( \frac{L_1}{2\beta^2} + \frac{3L_3}{\beta^4} \right) \eta^2 - \left( \frac{L_1}{\beta^4} + \frac{4L_3}{\beta^6} + 1 \right) \]

\[ G_5 = \left( \frac{L_1}{\beta^4} + \frac{4L_3}{\beta^6} \right) \text{Sech} [\beta \eta] + \frac{L_3}{\beta^4} \eta^2 \text{Sech} [\beta \eta] + \left( \frac{L_5}{4\beta^4} - \frac{4L_4}{4\beta^5} - \frac{L_2}{2\beta^3} \right) \eta \text{Tanh} [\beta \eta] + \frac{L_4}{6\beta^3} \eta^3 \text{Tanh} [\beta \eta] + \left( \frac{L_4}{4\beta^4} - \frac{L_5}{4\beta^5} \right) \eta^2 + \left( \frac{2L_5}{\beta^5} - \frac{6L_4}{\beta^6} - \frac{L_2}{\beta^4} \right) \eta \]

\[ G_6 = \frac{1}{2\eta} - \frac{L_8}{2} \left( \frac{\text{Sinh} [2\beta \eta]}{2\beta \eta} - 1 \right) \]

\[ G_7 = \frac{1}{2} - \frac{L_6}{12} \eta^4 - \frac{L_7}{\beta^2} \eta \text{Sinh} [\beta \eta] + \frac{2L_7}{\beta^3} \text{Cosh} [\beta \eta] \]

\[ L_1 = -b(G_1a_1 + G_2a_2 - a_1) \]

\[ L_2 = -b(G_1a_2 + G_2a_1 - a_2) \]

\[ L_3 = \frac{ba_1}{2} \quad L_4 = \frac{ba_2}{2} \]

\[ L_5 = \frac{a_2}{\beta} (G_2\beta^2 + 1) \quad L_6 = -Ep\{(1 + a_{14} - 2a_{11}) \]

\[ L_7 = \frac{1}{\beta} (2G_2\beta^2 - 2G_2a_{11}\beta^2 - 2a_{12} + 2a_{13}) \quad L_8 = \frac{1}{\beta^2} (G_2^2\beta^4 - 2G_2a_{12}\beta^2 + a_{15}) \]
\[ a_1 = G_{1X} \]
\[ a_4 = G_{5X} \]
\[ a_7 = G_{1G_{1X}} \]
\[ a_{10} = G_{2G_{2X}} \]
\[ a_{13} = G_{1XX} G_{2XX} \]
\[ a_{16} = G_{4XX} \]
\[ a_{19} = G_{7XX} \]
\[ a_{22} = L_{3X} \]
\[ a_{25} = L_{6X} \]
\[ a_{28} = L_{1XX} \]
\[ a_{31} = L_{4XX} \]
\[ a_{34} = L_{7XX} \]
\[ a_{37} = G_{2X}^2 \]
\[ a_{40} = C_1C_1X \]
\[ a_{43} = C_3C_3X \]
\[ a_2 = G_{2X} \]
\[ a_5 = G_{6X} \]
\[ a_8 = G_{1G_{2X}} \]
\[ a_{11} = G_{1XX} \]
\[ a_{14} = G_{1XX}^2 \]
\[ a_{17} = G_{5XX} \]
\[ a_{20} = L_{1X} \]
\[ a_{23} = L_{4X} \]
\[ a_{26} = L_{7X} \]
\[ a_{29} = L_{2XX} \]
\[ a_{32} = L_{5XX} \]
\[ a_{35} = L_{8XX} \]
\[ a_{38} = C_1X \]
\[ a_{41} = C_1C_3X \]
\[ a_{42} = C_3C_1X \]
\[ h_1 = \frac{L_2}{2\beta^3} + \frac{17L_4}{4\beta^5} - \frac{5L_5}{4\beta^4} \]
\[ h_2 = \frac{L_4}{6\beta^3} \]
\[ h_3 = \frac{L_5}{4\beta^3} - \frac{5L_4}{4\beta^4} \]
\[ h_4 = \frac{L_1}{2\beta^2} + \frac{2L_3}{\beta^4} \]
\[ h_5 = \frac{L_3}{12\beta^2} \]
\[ h_6 = G_5 - G_1a_2\zeta - G_2a_1\zeta \]
\[ h_7 = h_1 - G_2\beta\zeta a_1 - \frac{Pa_2\zeta}{\beta} \]
\[ h_8 = h_3 - G_3a_2\zeta \]
\[ h_9 = \frac{L_1}{2\beta^2} + \frac{2L_3}{\beta^4} + 3G_3a_1\zeta \]
\[ h_{10} = G_2a_2\zeta \]
\[ h_{11} = G_4 - G_1a_1\zeta \]
\[ h_{12} = \frac{a_4}{\beta} + \frac{a_{21}}{2 \beta^5} + \frac{3 a_{23}}{4 \beta^7} + \frac{7 a_{24}}{4 \beta^6} \]
\[ h_{13} = \frac{7 a_{24}}{4 \beta^5} - \frac{a_{21}}{2 \beta^4} - \frac{3 a_{23}}{4 \beta^6} + \frac{7 a_{24}}{4 \beta^5} \]
\[ h_{14} = \frac{7 a_{23}}{4 \beta^5} - \frac{a_{24}}{4 \beta^4} \]
\[ h_{15} = -\frac{a_{23}}{6 \beta^4} \]
\[ h_{16} = \frac{a_{20}}{6 \beta^2} + \frac{2 a_{22}}{3 \beta^4} \]
\[ h_{17} = \frac{a_{22}}{60 \beta^2} \]
\[ h_{18} = \frac{a_{21}}{2 \beta^5} + \frac{3 a_{23}}{4 \beta^7} - \frac{7 a_{24}}{4 \beta^6} - \frac{a_4}{\beta} + \frac{G_{1a_{12}}}{4 \beta^6} - \frac{a_1 a_2}{\beta} \]
\[ h_{19} = \frac{7 a_{24}}{4 \beta^5} - \frac{a_{21}}{2 \beta^4} - \frac{3 a_{23}}{4 \beta^6} + G_{2a_{11}} - a_1 a_2 \]
\[ h_{20} = \zeta(G_{1a_{11}} + a_{36}^2) \]
\[ h_{21} = \frac{G_{2a_{12}} - a_{37}^2}{2 \beta} \]
\[ h_{22} = G_3 a_{11} \]
\[ f_1 = h_1 - \frac{a_{12}}{\beta} + \frac{a_{29}}{2 \beta^5} + \frac{3 a_{31}}{4 \beta^7} - \frac{7 a_{32}}{4 \beta^6} + G_3 \beta \]
\[ f_2 = \frac{L_2}{2 \beta^2} - \frac{7 L_4}{4 \beta^4} + \frac{3 L_5}{4 \beta^3} + \frac{7 a_{32}}{2 \beta^4} - \frac{a_{29}}{4 \beta^5} + \frac{3 a_{31}}{4 \beta^6} \]
\[ f_3 = \frac{L_5}{4 \beta^2} - \frac{3 L_4}{4 \beta^3} + \frac{7 a_{31}}{4 \beta^5} - \frac{a_{32}}{4 \beta^4} \]
\[ f_4 = \beta h_5 - \frac{a_{31}}{6 \beta^4} \]
\[ f_5 = -2 h_4 + a_{16} \]
\[ f_6 = \frac{a_{28}}{6 \beta^2} + \frac{2 a_{30}}{3 \beta^4} - 4 h_5 \]
\[ f_7 = \frac{a_{30}}{60 \beta^2} \]
\[ g_1 = 2 f_1 f_7 \]
\[ g_2 = 2 f_1 f_5 \]
\[ g_3 = 2 f_3 f_7 \]
\[ g_4 = 2 f_1 f_6 \]
\[ g_5 = 2 f_3 f_6 \]
\[ g_6 = 2 f_2 f_7 \]
\[ g_7 = 2 f_2 f_5 \]
\[ g_8 = 2 f_4 f_7 \]
\[ g_9 = 2 f_4 f_5 \]
\[ g_{10} = 2 f_3 f_4 \quad g_{11} = f_1^2 \quad g_{12} = 2 f_1 f_3 \]
\[ g_{13} = f_3^2 \quad g_{14} = f_3^2 \quad g_{15} = 2 f_1 f_2 \]
\[ g_{16} = 2 f_1 f_4 \quad g_{17} = 2 f_3 f_4 \quad g_{18} = f_2^2 \]
\[ g_{19} = 2 f_2 f_4 \quad g_{20} = f_4^2 \quad g_{21} = 2 f_5 f_7 \]
\[ g_{22} = f_5^2 \quad g_{23} = 2 f_6 f_7 \quad g_{24} = 2 f_5 f_6 \]
\[ g_{25} = f_6^2 \]

\[ R_1 = -\frac{4G_1 a_{25}}{\beta^5} - \frac{a_5}{\beta^2} - \frac{8 a_{25}}{\beta^7} + \frac{7L_7 a_1}{\beta^5} + \frac{8L_6 a_2}{\beta^6} - \frac{2 g_2}{\beta^3} - \frac{12 g_4}{\beta^5} + \frac{720 g_5}{\beta^7} + \frac{2 g_7}{\beta^4} + \frac{72 g_9}{\beta^6} + \frac{2160 g_{11}}{\beta^8} \]

\[ R_2 = \frac{C_1 a_{25}}{\beta^4} + \frac{9 a_{25}}{\beta^6} - \frac{5L_7 a_1}{\beta^4} - \frac{6L_6 a_2}{\beta^5} + \frac{g_2}{\beta^3} + \frac{18 g_4}{\beta^4} + \frac{600 g_5}{\beta^6} + \frac{4 g_7}{\beta^3} - \frac{96 g_9}{\beta^5} - \frac{2880 g_{11}}{\beta^7} \]

\[ R_3 = \frac{C_1 a_{26}}{16 \beta^3} + \frac{C_2 a_{26}}{64 \beta^5} + \frac{L_8 a_1}{8 \beta^3} - \frac{L_8 a_2}{36 \beta^3} \]

\[ R_4 = -\frac{10 g_5}{\beta^3} + \frac{g_9}{\beta^2} + \frac{90 g_{11}}{\beta^4} \]

\[ R_5 = \frac{a_{25}}{2 \beta^4} - \frac{L_6 a_2}{3 \beta^3} + \frac{g_4}{\beta^2} + \frac{60 g_5}{\beta^3} - \frac{8 g_9}{\beta^4} + \frac{240 g_{11}}{\beta^5} \]

\[ R_6 = \frac{2L_6 a_2}{\beta^4} - \frac{L_7 a_1}{\beta^3} + \frac{g_4}{\beta^2} + \frac{6 g_4}{\beta^3} - \frac{240 g_5}{\beta^5} + \frac{g_7}{\beta^3} + \frac{36 g_9}{\beta^4} + \frac{1080 g_{11}}{\beta^6} \]

\[ R_7 = -\frac{C_3 a_{25}}{8 \beta^4} - \frac{L_7 a_2}{8 \beta^4} + \frac{3 f_3}{16 \beta^4} - \frac{g_{13}}{4 \beta^3} + \frac{3 g_{14}}{4 \beta^5} + \frac{g_{14}}{8 \beta^2} + \frac{15 g_{17}}{8 \beta^2} + \frac{g_{18}}{6 \beta^4} - \frac{3 g_{19}}{4 \beta^5} - \frac{45 g_{20}}{2 \beta^7} \]

\[ R_8 = -\frac{3C_3 a_{25}}{8 \beta^5} - \frac{L_7 a_2}{4 \beta^5} + \frac{3 f_3}{32 \beta^5} + \frac{3 f_3}{8 \beta^7} + \frac{g_{12}}{8 \beta^2} + \frac{3 g_{13}}{16 \beta^4} + \frac{g_{14}}{2 \beta^6} + \frac{3 g_{14}}{8 \beta^3} + \frac{g_{15}}{32 \beta^7} + \frac{15 g_{17}}{\beta^4} + \frac{g_{18}}{8 \beta^6} + \frac{9 g_{19}}{8 \beta^8} + \frac{45 g_{20}}{8 \beta^8} \]

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\[ R_9 = \frac{5c_{a_{26}}}{4\beta^3} - \frac{2a_{27}}{\beta^3} - \frac{3L_{a_{26}}}{4\beta^3} - \frac{L_{a_{26}}}{\beta^3} + \frac{6g_{a_1}}{\beta^3} + \frac{6g_6}{\beta^4} - \frac{2g_6}{\beta^5} - \frac{24g_8}{\beta^7} - 720g_{10} \]

\[ R_{10} = \frac{c_{a_{26}}}{36\beta^3} \]

\[ R_{11} = \frac{a_{27}}{\beta^2} - \frac{c_{a_{26}}}{2\beta^3} - \frac{L_{a_{26}}}{8\beta^3} + \frac{4g_3}{\beta^3} + \frac{g_6}{\beta^4} + \frac{18g_8}{\beta^6} + 600g_{10} \]

\[ R_{12} = -\frac{L_{a_{26}}}{90} \]

\[ R_{13} = \frac{c_{a_{27}}}{6} - \frac{c_{a_{26}}}{12} + \frac{L_{a_{26}}}{12} - \frac{c_{a_{1}}}{6} \]

\[ R_{14} = \frac{c_{a_{25}}}{2} - \frac{c_{a_{25}}}{2\beta^3} - \frac{L_{a_{26}}}{2\beta^3} - \frac{6g_{14}}{\beta^4} + \frac{6g_{19}}{\beta^5} + \frac{12g_{24}}{\beta^6} \]

\[ R_{15} = \frac{a_{26}}{32\beta^3} \]

\[ R_{16} = \frac{3a_4}{32\beta^4} \]

\[ R_{17} = \frac{a_{25}}{\beta^2} - \frac{a_{26}}{40} \]

\[ R_{18} = \frac{a_4}{24} + 15g_{20} + 30g_{25} \]

\[ R_{19} = \frac{g_3}{\beta^5} - \frac{6g_8}{\beta^3} - \frac{240g_{10}}{\beta^5} \]

\[ R_{20} = \frac{g_5}{\beta^2} - \frac{12g_{11}}{\beta^3} \]

\[ R_{21} = \frac{g_8}{\beta^2} + \frac{60g_{10}}{\beta^3} \]

\[ R_{22} = \frac{g_{a_{10}}}{\beta^2} \]

\[ R_{23} = \frac{10g_{a_{10}}}{\beta^3} \]

\[ R_{24} = \frac{g_{a_{11}}}{\beta^2} \]

\[ R_{25} = \frac{g_{a_{13}}}{8\beta^2} - \frac{15g_{a_{17}}}{8\beta^5} - \frac{g_{a_{18}}}{8\beta^2} - \frac{225g_{a_{20}}}{8\beta^6} + \frac{9g_{a_{14}}}{4\beta^4} + \frac{9g_{a_{19}}}{4\beta^4} \]

\[ R_{26} = \frac{g_{a_{14}}}{4\beta^4} - \frac{5g_{a_{17}}}{16\beta^3} + \frac{g_{a_{19}}}{4\beta^2} + \frac{45g_{a_{20}}}{8\beta^4} \]

\[ R_{27} = \frac{g_{a_{16}}}{8\beta^2} - \frac{g_{a_{14}}}{\beta^3} - \frac{g_{a_{19}}}{\beta^3} + \frac{15g_{a_{17}}}{8\beta^4} \]

\[ R_{28} = \frac{g_{a_{17}}}{8\beta^2} - \frac{3g_{a_{20}}}{2\beta^3} \]

\[ R_{29} = \frac{g_{a_{20}}}{4\beta^5} \]

\[ R_{30} = 6g_{23} \]

\[ R_{31} = -\frac{2L_{a_{26}}}{\beta^6} - \frac{L_{a_{26}}}{4\beta^3} - f_{20} + f_5 \]

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\[ R_{32} = R_1 + \frac{2G_1a_{27}}{\beta^3} - G_3a_6 \]  
\[ R_{33} = R_9 - \frac{L_8a_2}{2\beta} - \frac{G_6a_2}{\beta} \]  
\[ R_{34} = R_8 - \frac{L_7a_2}{2\beta^3} - \frac{G_3a_{26}}{\beta^3} \]  
\[ R_{35} = R_3 - \frac{L_{1a_{27}}}{4\beta} \]  
\[ R_{36} = R_{11} + \frac{G_3a_{27}}{2} - G_3a_5 \]  
\[ R_{37} = R_2 - \frac{L_7a_2}{2\beta^2} - \frac{G_2a_{26}}{2\beta^2} \]  
\[ R_{38} = R_{16} + \frac{L_8a_1}{2} \]  
\[ R_{39} = R_7 + \frac{L_7a_2}{2\beta^2} - \frac{G_2a_{26}}{2\beta^2} \]  
\[ R_{40} = R_6 + \frac{G_3a_{26}}{2} + \frac{L_7a_1}{\beta} \]  
\[ R_{41} = R_{15} - \frac{G_3a_{27}}{4\beta} \]  
\[ R_{42} = R_5 - \frac{G_3a_{26}}{2\beta^2} - \frac{L_6a_2}{3\beta} \]  
\[ R_{43} = R_4 - \frac{G_2a_{25}}{12} \]  
\[ R_{44} = -\frac{G_3a_{27}}{4\beta} \]  
\[ R_{45} = \frac{L_8a_2}{2\beta} \]  
\[ R_{46} = R_{12} - \frac{G_3a_{25}}{2} \]  
\[ R_{47} = R_{18} - \frac{G_3a_{25}}{3} \]  
\[ R_{48} = R_{13} + \frac{G_3a_{27}}{2} - G_3a_5 \]  
\[ R_{49} = R_{14} - \frac{G_3a_6}{3} \]  
\[ R_{50} = R_{30} + \frac{G_3a_{27}}{2} - G_1a_5 + \frac{L_8a_1}{2} + G_6a_1 \]  
\[ R_{51} = R_{31} - \frac{G_1a_6 - G_2a_{26} + L_7a_2}{\beta^3} \]  
\[ m_1 = -\frac{b a_{40}}{\beta^4} \]  
\[ m_2 = \frac{b a_{43}}{\beta^4} \]  
\[ m_3 = \frac{b a_{38}}{\beta^4} \]  
\[ m_4 = \frac{3b a_{39}}{\beta^6} \]  
\[ m_5 = \frac{b a_{42}}{\beta^4} \]  
\[ m_6 = \frac{b a_{42}}{\beta^3} \]  
\[ m_7 = \frac{b a_{39}}{\beta^5} \]  
\[ m_8 = \frac{b a_{39}}{\beta^4} \]  
\[ m_9 = \frac{b a_{40}}{\beta^2} \]  
\[ m_{10} = \frac{b a_{43}}{2\beta^2} \]  
\[ m_{11} = \frac{b a_{38}}{\beta^2} \]  
\[ m_{12} = \frac{b a_{38}}{\beta^6} \]
\[ m_{13} = \frac{b_a_{41}}{\beta^4} \quad m_{14} = \frac{b_a_{42}}{\beta^2} \quad m_{15} = \frac{b_a_{39}}{\beta^2} \]
\[ m_{16} = \frac{b_a_{41}}{2\beta^3} \quad m_{17} = \frac{b_a_{42}}{2\beta^3} \quad m_{18} = \frac{b_a_{39}}{\beta^3} \]
\[ m_{19} = \frac{b_a_{39}}{\beta^5} \quad m_{20} = \frac{b_a_{40}}{\beta^2} \quad m_{21} = \frac{b_a_{43}}{\beta^2} \]

\[
M_1 = m_1 - m_2 + m_4 + m_5 \quad M_2 = \frac{1}{2} \left(2m_6 + 2m_7 + m_8\right) \quad M_3 = \frac{1}{2} \left(m_3 + m_{11} + 4m_{12}\right) \\
M_4 = m_3 + m_{13} + m_4 \quad M_5 = \frac{1}{2} \left(m_{16} + m_{17}\right) \quad M_6 = -\frac{1}{2} \left(m_7 + m_8\right) \\
M_7 = \frac{5}{2} \left(m_{18} + m_7\right) \quad M_8 = \frac{5}{4} \left(m_{17} + m_7\right) \quad M_9 = \frac{1}{2} \left(m_{20} + m_{21}\right) \\
M_{10} = -\frac{1}{2} \left(m_{11} + m_{16}\right) \quad M_{11} = -\frac{1}{2} \left(m_{17} - 2m_{18}\right) \quad M_{12} = -\frac{1}{24} \left(m_{11} - 24m_3\right) \\
M_{13} = \frac{4}{24} \left(4m_{18} - 15m_8\right) \quad M_{14} = \frac{1}{4} \left(m_7 + 2m_8\right) \\
M_{15} = \frac{-5}{9} m_8 + \frac{17}{8} m_7 + \frac{1}{4} m_{14} + \frac{1}{2} m_8 \quad M_{16} = \frac{-5}{4} \left(m_{17} - m_7\right)
Profiles for temperature in fluid phase
Fig. 6.1 - Effect of $R$ on $\theta$

$\varepsilon = 0.01$, $\delta = 0.01$, $\alpha = 0.1$, $Pr = 0.7$, $S = 0.2$, $\gamma = 1$

Fig. 6.2 - Effect of $\alpha$ on $\theta$

$\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $Pr = 0.7$, $S = 0.2$, $\gamma = 0.1$
Fig. 6.3-Effect of $\gamma$ on $\theta$

($\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $P_r = 0.7$, $S = 0.2$, $\alpha = 0.1$)

Fig. 6.4-Effect of $\delta$ on $\theta$

($\varepsilon = 0.01$, $\gamma = 1$, $R = 1$, $P_r = 0.7$, $S = 0.2$, $\alpha = 0.1$)
Fig. 6.5-Effect of $Pr$ on $\theta$

($\varepsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, S = 0.2, \alpha = 0.1$)

Fig. 6.6-Effect of $S$ on $\theta$

($\varepsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, Pr = 0.7, \alpha = 0.1$)
Fig. 6.7 - Effect of \( \varepsilon \) on \( \theta \)

(\( \delta = 0.01, \gamma = 1, R = 1, S = 0.2, P_r = 0.7, \alpha = 0.1 \))
Profiles for temperature in particle phase
Fig. 6.8 - Effect of $R$ on $\theta^P$

$(\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, Pr = 0.7, S = 0.2, \gamma = 1)$

Fig. 6.9 - Effect of $\alpha$ on $\theta^P$

$(\varepsilon = 0.01, \delta = 0.01, R = 1, Pr = 0.7, S = 0.2, \gamma = 1)$
Fig. 6.10 - Effect of $\gamma$ on $\theta^p$

$\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $Pr = 0.7$, $S = 0.2$, $\alpha = 1$

Fig. 6.11 - Effect of $\delta$ on $\theta^p$

$\varepsilon = 0.01$, $\gamma = 1$, $R = 1$, $Pr = 0.7$, $S = 0.2$, $\alpha = 1$
**Fig. 6.12 - Effect of $P_r$ on $\theta^P$**

$(\varepsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, S = 0.2, \alpha = 0.1)$

**Fig. 6.13 - Effect of $S$ on $\theta^P$**

$(\varepsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, P_r = 0.7, \alpha = 0.1)$
Fig. 6.14-Effect of $\varepsilon$ on $\theta^p$

($\delta = 0.01, \gamma = 1, R = 1, S = 0.2, Pr = 0.7, \alpha = 0.1$)
Profiles for heat transfer in fluid phase
Fig.6.15-Effect of R on Z

($\varepsilon = 0.01$, $\delta = 0.01$, $\alpha = 0.1$, $Pr = 0.7$, $S = 0.2$, $\gamma = 1$)

Fig.6.16-Effect of $\gamma$ on Z

($\varepsilon = 0.01$, $\delta = 0.01$, $\alpha = 0.1$, $Pr = 0.7$, $S = 0.2$, $R = 1$)
Fig. 6.17 - Effect of $\alpha$ on $Z$
($\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $Pr = 0.7$, $S = 0.2$, $\gamma = 1$)

Fig. 6.18 - Effect of $Pr$ on $Z$
($\varepsilon = 0.01$, $\gamma = 1$, $R = 1$, $\delta = 0.01$, $S = 0.2$, $\alpha = 0.1$)
Fig. 6.19 - Effect of $\varepsilon$ on $Z$

$(\delta = 0.01, \gamma = 1, R = 1, S = 0.2, P_r = 1, \alpha = 0.1)$

Fig. 6.20 - Effect of $\delta$ on $Z$

$(\varepsilon = 0.01, \gamma = 1, R = 1, P_r = 0.7, S = 0.2, \alpha = 0.1)$
Fig. 6.21-Effect of $S$ on $Z$

($\varepsilon =0.01$, $\gamma =1$, $R=1$, $\delta =0.01$, $Pr=0.7$, $\alpha =0.1$)
Profiles for heat transfer in particle phase
Fig. 6.22 - Effect of $R$ on $Z^p$

($\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, \Pr = 0.7, S = 0.2, \gamma = 1$)

Fig. 6.23 - Effect of $\gamma$ on $Z^p$

($\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, \Pr = 0.7, S = 0.2, R = 1$)
Fig. 6.24 - Effect of $\alpha$ on $Z^p$

$e = 0.01, \delta = 0.01, R = 1, Pr = 0.7, S = 0.2, y = 1$

Fig. 6.25 - Effect of $Pr$ on $Z^p$

$e = 0.01, y = 1, R = 1, \delta = 0.01, S = 0.2, \alpha = 0.1$
Fig. 6.26 - Effect of $\varepsilon$ on $Z^p$

($\delta = 0.01, \gamma = 1, R = 1, S = 0.2, P_r = 1, \alpha = 0.1$)

Fig. 6.27 - Effect of $\delta$ on $Z^p$

($\varepsilon = 0.01, \gamma = 1, R = 1, P_r = 0.7, S = 0.2, \alpha = 0.1$)
Fig. 6.28 - Effect of $S$ on $Z_p$

($\epsilon = 0.01$, $\gamma = 1$, $R = 1$, $\delta = 0.01$, $Pr = 0.7$, $\alpha = 0.1$)
Profiles for average temperature in fluid phase
Fig. 6.29 - Effect of \( R \) on \( \bar{\theta} \)

\[ (\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, \Pr = 0.7, S = 0.2, \gamma = 1) \]

Fig. 6.30 - Effect of \( \gamma \) on \( \bar{\theta} \)

\[ (\varepsilon = 0.01, \delta = 0.01, \alpha = 1, \Pr = 0.7, S = 0.2, R = 1) \]
Fig. 6.31 - Effect of $\alpha$ on $\bar{\theta}$
($\epsilon = 0.01, \delta = 0.01, R = 1, P_r = 0.7, S = 0.2, \gamma = 1$)

Fig. 6.32 - Effect of $P_r$ on $\bar{\theta}$
($\epsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, S = 0.2, \alpha = 0.1$)
Fig. 6.33 - Effect of $\varepsilon$ on $\bar{\theta}$

($\delta = 0.01, \gamma = 1, R = 1, S = 0.2, Pr = 0.7, \alpha = 1$)

Fig. 6.34 - Effect of $\delta$ on $\bar{\theta}$

($\varepsilon = 0.01, \gamma = 1, R = 1, Pr = 0.7, S = 0.2, \alpha = 0.1$)
Fig. 6.35-Effect of $S$ on $\bar{\theta}$

($\varepsilon = 0.01, \gamma = 1, \delta = 0.01, Pr = 0.7, \alpha = 0.1$)
Profiles for average heat transfer in fluid phase
Fig. 6.36 - Effect of R on $\bar{Z}$

$(\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, P_r = 0.7, S = 0.2, \gamma = 1)$

Fig. 6.37 - Effect of $\gamma$ on $\bar{Z}$

$(\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, P_r = 0.7, S = 0.2, R = 1)$
Fig. 6.38 Effect of $\alpha$ on $\bar{Z}$

($\varepsilon = 0.01, \delta = 0.01, R = 1, \gamma = 1, \Pr = 0.7, S = 0.2$)

Fig. 6.39 Effect of $\varepsilon$ on $\bar{Z}$

($\delta = 0.01, R = 1, S = 0.2, \Pr = 0.7, \alpha = 0.1$)
Fig. 6.40 - Effect of $S$ on $\bar{Z}$

$\epsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, \rho_r = 0.7, \alpha = 0.1$}

Fig. 6.41 - Effect of $\delta$ on $\bar{Z}$

$\epsilon = 0.01, \gamma = 1, R = 1, \rho_r = 0.7, S = 0.2, \alpha = 0.1$}
Fig. 6.42-Effect of $Pr$ on $\bar{Z}$

($\varepsilon = 0.01$, $\delta = 0.01$, $\gamma = 1$, $R = 1$, $S = 0.2$, $\alpha = 0.1$)
Profiles for average heat transfer in particle phase
Fig. 6.43 - Effect of $R$ on $Z^p$

\( (\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, P_r = 0.7, S = 0.2, \gamma = 1, \zeta = 1) \)

Fig. 6.44 - Effect of $\gamma$ on $Z^p$

\( (\varepsilon = 0.01, \delta = 0.01, R = 0.1, \alpha = 1, S = 1, E = 1, P_r = 0.7, \zeta = 1) \)
Fig. 6.45 - Effect of $\alpha$ on $\overline{Z^\rho}$
($\varepsilon = 0.01, \delta = 0.01, R = 1, \gamma = 1, P_r = 0.7, S = 0.2, \zeta = 1$)

Fig. 6.46 - Effect of $P_r$ on $\overline{Z^\rho}$
($\varepsilon = 0.01, \delta = 0.01, \gamma = 1, R = 1, S = 0.2, \alpha = 0.1, \zeta = 1$)
**Fig. 6.47 - Effect of $\varepsilon$ on $Z_p^\delta$**

$\delta = 0.01, \gamma = 1, \beta = 1, S = 0.2, P_r = 0.7, \alpha = 0.1, \zeta = 1$}

**Fig. 6.48 - Effect of $\delta$ on $Z_p^\varepsilon$**

$\varepsilon = 0.01, \gamma = 1, \beta = 1, S = 0.2, P_r = 0.7, \alpha = 0.1, \zeta = 1$}
Fig. 6.49 - Effect of \( \zeta \) on \( \bar{Z}_p \)
(\( \varepsilon = 0.01, \delta = 0.01, \gamma = 1, R = 1, S = 0.2, Pr = 0.7, \alpha = 0.1 \))

Fig. 6.50 - Effect of \( S \) on \( \bar{Z}_p \)
(\( \varepsilon = 0.01, \gamma = 1, R = 1, \delta = 0.01, Pr = 0.7, \alpha = 0.1, \zeta = 1 \))
Profiles for pressure (rise) drop & frictional force
Fig. 6.51 - Effect of $R$ on $\Delta p$

($\varepsilon = 0.01$, $\delta = 0.01$, $R = 0.1$, $\alpha = 1$, $S = 1$)

Fig. 6.52 - Effect of $\alpha$ on $\Delta p$

($\varepsilon = 0.01$, $\delta = 0.01$, $R = 1$, $S = 0.2$, $\gamma = 1$)
Fig. 6.53 - Effect of $\varepsilon$ on $\Delta p$

$(\delta = 0.01, \gamma = 1, R = 1, S = 0.2, \alpha = 0.1)$

Fig. 6.54 - Effect of $\delta$ on $\Delta p$

$(\varepsilon = 0.01, \gamma = 1, R = 1, S = 0.2, \alpha = 0.1)$
**Fig. 6.55 - Effect of $S$ on $\Delta p$**

($\epsilon = 0.01, \delta = 0.01, \alpha = 0.1, R = 1, \gamma = 1$)

**Fig. 6.56 - Effect of $R$ on $F$**

($\epsilon = 0.01, \delta = 0.01, R = 0.1, \alpha = 1, S = 1$)
Fig. 6.57 - Effect of $\alpha$ on $F$

($\varepsilon = 0.01, \delta = 0.01, R = 1, S = 0.2$)

Fig. 6.58 - Effect of $\varepsilon$ on $F$

($\delta = 0.01, R = 1, S = 0.2, \alpha = 0.1$)
Fig. 6.59 - Effect of $\delta$ on $F$
($\varepsilon = 0.01, \alpha = 0.1, \delta = 0.01, R = 1, S = 0.2$)

Fig. 6.60 - Effect of $S$ on $F$
($\varepsilon = 0.01, \delta = 0.01, \alpha = 0.1, R = 1$)
Dear T. Raghunatha Rao,

I am happy to inform you that your paper entitled,

**Influence of Heat Transfer on Peristaltic Transport of Visco-Elastic Rivlin Erickson Fluid**

*T. RAGHUNATHA RAO & D. R. V. PRASADA RAO*

Department of Mathematics, Sri Krishnadevaraya University, Anantapur-515 003, India


With Best Wishes,

Prof. ADEEL AHMAD

Associate Chief Editor, JPAP
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T. RAGHUNATHA RAO & D. R. V. PRASADA RAO

Department of Mathematics, Sri Krishnadevaraya University, Anantapur-515 003, India


With Best Wishes

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