CHAPTER III

INTERACTION OF PERISTALSIS WITH HEAT TRANSFER AND WALL PROPERTIES OF VISCO-ELASTIC RIVLIN-ERICKSON FLUID THROUGH A POROUS MEDIUM UNDER MAGNETIC FIELD

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3.1. INTRODUCTION

The Magneto hydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids e.g. the blood and blood pump machines, and with the need for theoretical research on the operation of a peristaltic MHD Compressor. Blood is regarded as a suspension of small cells in plasma. Moreover, it is known that in arteries, blood flows in two layers, a plasma layer near the wall and a core layer consisting of suspension of cells in the plasma. Since the red blood cells, which contain iron, are magnetic in nature, the core may be treated as magnetic field.


The study of magnetic field with porous medium is very important both from theoretical as well as practical point of view, because most of natural phenomena of
the fluid flow are connected with porous medium. For e.g., filtration of fluids, underground water, oil reservoir and fluid through pipes. Agarwal, R.P. [1] has investigated the problem of heat transfer to pulsatile flow of a conducting fluid through a porous channel in the presence of magnetic field. He has observed that the temperature increases throughout the channel as Eckert number increases, even in the presence of magnetic field. D.V. Krishna and Mallikarjuna Goud [15] studied the effect of a magnetic field on the peristaltic flow through a porous medium in a non uniform channel and its application to blood flow. Sobh [29] studied the slip flow of peristaltic transport of a magneto-Newtonian fluid through a porous medium with heat transfer. It has been observed that under the influence magnetic field, flow separation occurred only at high Reynolds number compared to the non-magnetic case.

Keeping the above facts in view, in this chapter we discuss the interaction of peristalsis with heat transfer and wall properties of Visco-elastic Rivlin – Erickson fluid through a porous medium in the presence of Magnetic Field. Assuming that the wave length of the peristaltic wave is large in comparison to the mean-half width of the channel, a perturbation solution has been obtained in terms of the wall slope parameter and closed form expressions have been derived for stream function, velocity, temperature, heat transfer coefficient. The effects of elasticity parameters and magnetic parameter on temperature, heat transfer coefficient, average temperature, average heat transfer, pressure (rise) drop and frictional force have been discussed.
3.2. FORMULATION OF THE PROBLEM

We consider a peristaltic flow of an incompressible visco-elastic Rivlin-Erickson fluid through porous medium under uniform transverse magnetic field in a two dimensional channel of uniform thickness. The channel is symmetric with respect to its axis and walls of are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.

The geometry of the flexible walls are represented by

\[ y = \eta(x,t) = d + a \sin \frac{2\pi}{\lambda} (x - ct) \quad (3.2.1) \]

Where, ‘d’ is the mean half width of the channel, ‘a’ is the amplitude of the peristaltic wave, ‘c’ is the wave velocity, ‘\( \lambda \)’ is the wave length and ‘c’ is the phase speed of the wave.

The equations governing the two-dimensional flow of Rivlin - Erickson fluid are

Equation of Continuity

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.2.2) \]

Equation of Momentum

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \beta \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \frac{v}{k_1} u - \frac{\sigma \mu_0 H^2}{\rho} u \quad (3.2.3) \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \beta \frac{\partial^2}{\partial y^2} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) - \frac{v}{k_1} v \quad (3.2.4) \]
Equation of energy

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \nu \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right)^2 - \beta \left( \frac{\partial^2 u}{\partial y \partial t} + u \frac{\partial^2 u}{\partial y \partial x} + v \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) \] (3.2.5)

Where \( u \) and \( v \) are the velocity components, \( \rho \) is the fluid pressure, \( \beta \) is the coefficient of Visco-elasticity, \( \nu \) is the coefficient of kinematic viscosity, \( k_1 \) is the permeability of the porous medium, \( \mu_e \) is the magnetic permeability and \( H_0 \) is the magnetic field intensity and \( \sigma \) is the conductivity, \( k \) is the coefficient of thermal conductivity, \( C_p \) is the specific heat at constant pressure, \( T \) is the temperature.

The governing equation of motion of the flexible wall may be expressed as

\[ L (\eta) = p - p_0 \] (3.2.6)

Where \( 'L' \) is an operator, which is used to represent the motion of stretched membrane with damping forces such that

\[ L = -T^* \frac{\partial^2 \eta}{\partial x^2} + m \frac{\partial^2 \eta}{\partial t^2} + C \frac{\partial \eta}{\partial t} \] (3.2.7)

Here \( T^* \) is the elastic tension in the membrane, \( m \) is the mass per unit area and \( C \) is the coefficient of viscous damping forces, \( p_0 \) is the pressure on the outside surface of the wall due to tension in the muscles. For simplicity, we assume \( p_0 = 0 \).

The horizontal displacement assumed to be zero, gives the boundary conditions are

\[ u = 0 \quad \text{on} \quad y = \pm \eta = \pm \left[ d + a \ \sin \frac{2\pi}{\lambda} (x - ct) \right] \] (3.2.8)
\[
\frac{\partial}{\partial x} L(\eta) = \frac{\partial P}{\partial x} = \rho \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{v_p}{K} - \sigma \mu_e H_0^2 u
\]

on \( y = \pm \eta \)

(3.2.9)

The conditions on temperature are

\[ T = T_0 \text{ on } y = -\eta, \quad T = T_1 \text{ on } y = \eta \]  

(3.2.10)

Equations (3.2.8), (3.2.9) & (3.2.10) are the required boundary conditions for the present problem.

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function \( \psi \) such that

\[ u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \]  

(3.2.11)

and introducing non-dimensional quantities

\[ x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\delta}, \quad u' = \frac{u}{c}, \quad v' = \frac{v}{c \delta}, \quad \psi' = \frac{\psi}{c \delta}, \quad \tau' = \frac{\tau}{\delta}, \quad \eta' = \frac{\eta}{\delta}, \quad p' = \frac{p}{c \delta^2}, \quad \theta = \frac{T - T_0}{T_1 - T_0} \]  

(3.2.12)

in equations of motion and the conditions (3.2.3) – (3.2.5), (3.2.8) – (3.2.10), we finally get (after dropping primes)

\[
R \delta \left( \frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial y} \right) + \frac{\partial \psi}{\partial y} \left( \frac{\partial^2 \psi}{\partial x \partial y} \right) - \frac{\partial \psi}{\partial x} \left( \frac{\partial \psi}{\partial y} \right) \right) = -\frac{\partial P}{\partial x} + \left( \frac{\partial^3 \psi}{\partial y^3} + \delta^2 \frac{\partial^2 \psi}{\partial x^2 \partial y} \right)
\]

(3.2.13)

\[
SR \delta^2 \left( \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right) + \frac{\partial \psi}{\partial y} \left( \frac{\partial^2 \psi}{\partial x \partial y} \right) - \frac{\partial \psi}{\partial x} \left( \frac{\partial \psi}{\partial y} \right) \right) - \frac{R}{D_a} \frac{\partial \psi}{\partial y} = \left( \frac{R}{D_a} + M^2 \right) \frac{\partial \psi}{\partial y}
\]

(3.2.14)
$$\begin{align*}
R \delta \left( \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \right) &= \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + E \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \\
&- S R E \delta \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x \partial y} \right) + \left( \frac{\partial^2 \psi}{\partial y \partial t} + \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial \psi}{\partial y} \right) - \frac{R \partial \psi}{D_a \partial y} - M \frac{\partial \psi}{\partial y}
\end{align*}$$

(3.2.15)

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{on} \quad y = \pm \eta = \pm [1 + \varepsilon \sin 2\pi (x-t)]$$

(3.2.16)

$$\begin{align*}
\left( \frac{\partial^3 \psi}{\partial y^3} + \delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} \right) &= \frac{R \partial \psi}{D_a \partial y} - \frac{\partial^2 \psi}{\partial x \partial t} + \left( E_1 \frac{\partial^3 \psi}{\partial x \partial t^2} + E_2 \frac{\partial^3 \psi}{\partial x \partial t} + E_3 \frac{\partial^2 \psi}{\partial x \partial t} \right) \eta \quad \text{on} \quad y = \pm \eta
\end{align*}$$

(3.2.17)

$$\begin{align*}
\theta &= 0 \quad \text{on} \quad y = -\eta, \quad \theta &= 1 \quad \text{on} \quad y = \eta
\end{align*}$$

(3.2.18)

Eliminating $p$ from the equations (3.2.13 & 3.2.14), we get

$$\begin{align*}
\delta \left( \frac{\partial}{\partial t} \left( \nabla^2 \psi \right) + \frac{\partial}{\partial y} \left( \nabla^2 \psi \right) + \frac{\partial}{\partial x} \left( \nabla^2 \psi \right) + \frac{\partial^2 \psi}{\partial x \partial y} \right) &= \left( \frac{\partial^3 \psi}{\partial y^3} + \delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} \right) + \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial \psi}{\partial y} + \frac{\partial^2 \psi}{\partial y \partial x} \frac{\partial \psi}{\partial y} + \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial \psi}{\partial y} + \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial \psi}{\partial y} \frac{\partial^5 \psi}{\partial x \partial y^5}
\end{align*}$$

(3.2.19)

Where $\nabla^2 = \frac{\partial^2}{\partial y^2} + \delta^2 \frac{\partial^2}{\partial x^2}$

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The non-dimensional parameters are

\[ \varepsilon = \frac{a}{d} \quad \text{and} \quad \delta = \frac{d}{\lambda} \]

are geometric parameters

\[ R = \frac{c d}{v} \]

is the Reynolds number

\[ S = \frac{\beta}{d^2} \]

is the Visco-elastic parameter

\[ \text{Pr} = \frac{\rho c_p v}{k} \]

is the Prandtl number

\[ D_a = \frac{c k}{v d} \]

is the Darcy number

\[ E = \frac{c^2}{\rho c_p (T_i - T_0)} \]

is the Eckert number

\[ M = \sqrt{\frac{\sigma \mu e^2 H_0^2 d^2}{\mu}} \]

is the Hartman number

\[ E_1 = \frac{T d^3}{\lambda^3 \rho v c}, \quad E_2 = \frac{m c d^3}{\lambda^3 \rho v}, \quad E_3 = \frac{C d^3}{\lambda^3 \rho v} \]

are the elasticity parameters
3.3. METHOD OF SOLUTION

We seek perturbation solution for the stream function ($\psi$), pressure gradient ($p$) and temperature coefficient ($\theta$) in terms of small parameter $\delta$ as follows:

$$\psi = \psi_0 + \delta \psi_1 + \delta^2 \psi_2 + \ldots \ldots \ldots \ldots \ldots (3.3.1)$$

$$p = p_0 + \delta p_1 + \delta^2 p_2 + \ldots \ldots \ldots \ldots \ldots (3.3.2)$$

$$\theta = \theta_0 + \delta \theta_1 + \delta^2 \theta_2 + \ldots \ldots \ldots \ldots \ldots (3.3.3)$$

Substituting (3.3.1-3.3.3) in equations (3.2.13) to (3.2.15) & (3.2.19) and collecting the coefficients of various powers of $\delta$

The zeroth order equations are

$$\frac{\partial^4 \psi_0}{\partial y^4} - \left( \frac{R}{D_a} + M^2 \right) \frac{\partial^2 \psi_0}{\partial y^2} = 0 \quad (3.3.4)$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} - \left( \frac{R}{D_a} + M^2 \right) \frac{\partial \psi_0}{\partial y} \quad (3.3.5)$$

$$\frac{\partial p_0}{\partial y} = 0 \quad (3.3.6)$$

$$\frac{1}{Pr} \left( \frac{\partial^2 \theta_0}{\partial y^2} \right) + E \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 = 0 \quad (3.3.7)$$

The corresponding boundary conditions are

$$\frac{\partial \psi_0}{\partial y} = 0 \text{ on } y = \pm \eta \quad (3.3.8)$$

$$\frac{\partial^3 \psi_0}{\partial y^3} - \left( \frac{R}{D_a} + M^2 \right) \frac{\partial \psi_0}{\partial y} = \left( E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} \right) \eta \text{ on } y = \pm \eta \quad (3.3.9)$$

$$\theta_0 = 0 \text{ on } y = -\eta, \quad \theta_0 = 1 \text{ on } y = \eta \quad (3.3.10)$$
The first order equations are

$$\frac{\partial}{\partial t} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \psi_0}{\partial x} \right) \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)}{\partial x} \right) - \frac{\partial}{\partial x} \left( \frac{\partial \psi_0}{\partial y} \right) \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)$$

$$- \frac{\partial \psi_0}{\partial x} \left( \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) \right) = \frac{1}{R} \left( \frac{\partial^4 \psi_1}{\partial y^4} \right) - \left( \frac{1}{D_a} + \frac{M^2}{R} \right) \frac{\partial^2 \psi_1}{\partial y^2} +$$

$$+ S \left( \frac{\partial}{\partial t} \left( \frac{\partial^4 \psi_0}{\partial y^4} \right) + \frac{\partial \psi_0}{\partial y} \frac{\partial^3 \psi_0}{\partial y^3} + \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial \psi_0}{\partial y} - \frac{\partial^4 \psi_0}{\partial y^4} \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial y^2} - \frac{\partial \psi_0}{\partial y} \frac{\partial^3 \psi_0}{\partial y^3} \frac{\partial \psi_0}{\partial y} \right)$$

(3.3.11)

$$R \left( \frac{\partial}{\partial t} \left( \frac{\partial \psi_0}{\partial y} \right) + \frac{\partial \psi_0}{\partial y} \left( \frac{\partial}{\partial x} \left( \frac{\partial \psi_0}{\partial y} \right) \right) \right) = - \frac{\partial p_1}{\partial x} + \frac{\partial^3 \psi_1}{\partial y^3} \left( \frac{R}{D_a} + M^2 \right) \frac{\partial \psi_1}{\partial y}$$

(3.3.12)

$$+ S R \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial t} \left( \frac{\partial \psi_0}{\partial y} \right) + \frac{\partial \psi_0}{\partial y} \left( \frac{\partial}{\partial x} \left( \frac{\partial \psi_0}{\partial y} \right) \right) \right) - \frac{\partial \psi_0}{\partial y} \left( \frac{\partial}{\partial y} \left( \frac{\partial \psi_0}{\partial y} \right) \right)$$

(3.3.13)

$$\frac{\partial p_1}{\partial y} = 0$$

$$R \left( \frac{\partial \theta_0}{\partial t} + \frac{\partial \psi_0}{\partial y} \frac{\partial \theta_0}{\partial x} - \frac{\partial \psi_0}{\partial y} \frac{\partial \theta_0}{\partial y} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta_1}{\partial y^2} \right) + 2 \frac{E}{Pr} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)$$

$$- S R \left( \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^3 \psi_0}{\partial y^3} + \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial \psi_0}{\partial y} - \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial \psi_0}{\partial y} \frac{\partial \psi_0}{\partial y} \frac{\partial \psi_0}{\partial y} \right)$$

(3.3.14)

The corresponding boundary conditions are

$$\frac{\partial \psi_1}{\partial y} = 0 \quad \text{on} \quad y = \pm \eta$$

(3.3.15)
\[
\frac{\partial^3 \psi_1}{\partial y^3} - \left( \frac{R}{D_a} + M^2 \right) \frac{\partial \psi_1}{\partial y} - R \left( \frac{\partial}{\partial t} \left( \frac{\partial \psi_0}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \psi_0}{\partial y} \right) \right) - \left( \frac{\partial \psi_0}{\partial x} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) \right) = 0
\]

on \( y = \pm \eta \) \hspace{1cm} .3.16)

\[
\theta_0 = 0 \text{ on } y = -\eta, \quad \theta_1 = 1 \text{ on } y = \eta \hspace{1cm} (3.3.17)
\]

**Zeroth-order problem**

On solving the equations (3.3. 4-3.3.7), subject to the conditions (3.3.8-3.3.10), we get

\[
\psi_0 = A_1 y + A_2 \sinh \alpha y
\]

(3.3.18)

\[
\frac{\partial \psi_0}{\partial x} = -A_1 \alpha^2
\]

(3.3.19)

\[
\theta_0 = \frac{A_9}{4 \alpha^2} (2 \alpha^2 y^2 - \cosh 2 \alpha y) + A_5 y + A_6
\]

(3.3.20)

**First-order problem**

On solving the equations (3.3.11 - 3.3.14), subject to the conditions (3.3.15-3.3.17), we get

\[
\psi_1 = A_3 y + A_4 \sinh \alpha y + \left( \frac{B_1}{2 \alpha^3} + \frac{5 B_2}{4 \alpha^4} \right) y \cosh \alpha y + \frac{B_2}{4 \alpha^3} y^2 \sinh \alpha y + \frac{B_3}{24 \alpha^4} \sinh 2 \alpha y
\]

(3.3.21)

\[
\frac{\partial p_1}{\partial x} = \left( \frac{1}{2 \alpha^2} (2B_1 \alpha^2 + 8B_2 + 2R S \alpha^5 - 2R S \alpha^5) - 2R S \alpha^5 - R d_4 \alpha^3 \right) \cosh [\alpha y] + \frac{1}{\alpha} (B_2 - S R d_1 \alpha^4 + R d_1 \alpha^2) y \sinh [\alpha y]
\]

(3.3.22)

\[
+ \frac{B_3}{3 \alpha} \cosh [2 \alpha y] - (R d_2 - R d_8 - R d_1 \alpha^2))
\]
\[ \theta_1 = \frac{g_1}{12} y^4 + \frac{g_2}{6} y^3 + \frac{1}{4} \left( 2 g_1 - g_{13} \right) y^2 + \frac{1}{a^4} \left( a^2 g_5 - 2 a g_8 + 6 g_{11} \right) \sinh \alpha y + \frac{1}{4 a^4} \left( 4 a^2 g_4 - 8 a g_9 + 24 g_{10} - a^2 g_{16} - 4 a^2 g_{14} \right) \cosh \alpha y + \frac{g_7}{4 a^2} \sinh 2 \alpha y + \frac{1}{8 a^3} \left( 2 a g_6 - 2 a g_{12} + a g_{13} - g_{13} \right) \cosh 2 \alpha y + \frac{1}{8 a^2} \left( g_8 - 4 g_{11} \right) y \cosh \alpha y + \frac{g_{10}}{a^2} y^2 \cosh \alpha y + \frac{g_{11}}{a^2} \cosh \alpha y + \frac{1}{72 a^2} \left( 9 g_{14} + 2 g_{16} \right) \cosh 3 \alpha y + A_7 y + A_8 \]

(3.3.23)

The heat transfer coefficient in terms of wall slope parameter \( \delta \) is

\[ z = z_0 + \delta z_1 + \ldots. \]  

(3.3.24)

\[ z_0 = \left( \frac{\partial \eta}{\partial x} \right) \left( \frac{\partial \theta_0}{\partial y} \right) \]  

(3.3.25)

\[ z_1 = \left( \frac{\partial \theta_0}{\partial x} \right) + \left( \frac{\partial \eta}{\partial x} \right) \left( \frac{\partial \theta_1}{\partial y} \right) \]  

(3.3.26)

The average temperature, \( \bar{\theta} \) is given by

\[ \bar{\theta} = \frac{1}{t} \int_0^t \theta \, dt \]  

(3.3.27)

The average Heat transfer, \( \bar{z} \) is given by

\[ \bar{z} = \frac{1}{t} \int_0^t z \, dt \]  

(3.3.28)

The pressure rise (drop) over one cycle of the wave can be obtained as

\[ \Delta p = \int_0^t \left( \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + \ldots \right) \, dx \]  

(3.3.29)
The dimensionless frictional force $F$ at the wall across one wavelength is given by

$$F = \int_{0}^{1} \eta \left( -\frac{\partial p}{\partial x} \right) \, dx$$

(3.3.30)

It can be noticed that when Inverse Darcy number $D_a^{-1} = 0$, Visco-elastic parameter $S = 0$ and Hartman number $M = 0$, this problem reduces to Radhakrishnamacharya and Srinivasulu [25].

The constants $A_1, A_2, \ldots, A_8, B_1, B_2, B_3, d_1, d_2, \ldots, d_{11}, g_1, g_2, \ldots, g_{16}$ are given in appendix.
3.4 RESULTS AND DISCUSSION

In this analysis we analyzed effect of temperature variation on the peristaltic action of Visco-elastic Rivlin-Erickson fluid in the presence Magnetic field through porous medium.

The non-dimensional temperature distribution ‘θ’ is depicted in figs.(3.1-3.10) for different parametric values of R, E₁, E₂, E₃, S, ε, δ, Pr, E and M. The temperature gradually enhances from the central region y = 0 and it attains prescribed value at y =1. From fig.(3.1) we find that temperature distribution ‘θ’ reduces marginally with increase in R in the entire flow region. Figs.3.2 & 3.3 represent of the variation ‘θ’ with increase in membrane tension parameter (E₁) and mass characterizing parameter (E₂). We observed that ‘θ’ reduces marginally with increasing E₁ and E₂ of order 0.2 and further increase in E₁ and E₂, θ experiences a depreciation, while an increase the viscous damping parameter (E₃) enhances ‘θ’ marginally in the entire flow region fig.(3.4). From fig. (3.5), we find that an increase in Visco-elastic parameter S, the temperature marginally increases in the region 0 ≤ y ≤ 0.1 and depreciates in the remaining region. The variation of ‘θ’ with Prandtl number Pr is shown in fig.(3.6). For an increase in the Prandtl number Pr ≤2 reduces the temperature ‘θ’ marginally and for higher Pr ≥3 we notice an appreciable reduction in θ in the entire flow region. From figs 3.7 & 3.8 we observe that the temperature distribution θ depreciates with increase in ε and δ. Fig.(3.9) represents the variation of θ with Eckert number E. We find that θ experiences an appreciable enhancement with increase in E at y = 0 and it depreciates with increase in E and attains the prescribed value at y = 1. From fig.(3.10), we notice that the temperature θ reduces marginally with increase in the Hartman number M ≤11 and for higher M ≥12, θ depreciates remarkably.
Figs.(3.11-3.20) represent the variation of heat transfer coefficient $z$ with different parameters $R$, $E_1$, $E_2$, $E_3$, $S$, $\varepsilon$, $\delta$, $P_r$, $E$ and $M$. The variation of heat transfer coefficient $z$ with Reynolds number $R$ is shown in fig.(3.11). The variation of $z$ depreciates marginally at the central region $y = 0$, insignificant at $y = 0.2$ and further increases in the region $0.2 \leq y \leq 0.6$ with increase in $R$ and again it decreases in the region $0.6 \leq y \leq 0.7$ and slowly rises to attain the maximum on $y = 1.0$. From fig 3.12 & 3.13, we find that an increase in $E_1$ and $E_2$ depreciates remarkably in the region $0 \leq y \leq 0.2$ and increases in the remaining region. The enhancement is appreciably large at $y = 1$ than at the boundary $y = 0$, while an increase the elastic parameter $E_3$ enhances the heat transfer coefficient $z$ in the region $0 \leq y \leq 0.3$ and depreciates in the remaining region and slowly rises to attain the maximum at $y = 1$ fig. (3.14). The variation of heat transfer coefficient $z$ with Visco-elastic parameter $S$ is almost linear in the region $0 \leq y \leq 0.7$ and slowly rises to attain the maximum at $y = 1$ in fig.(3.15). The variation of Heat transfer coefficient $z$ with the Prandtl number $P_r$ is depicted in fig.(3.16). The variation in heat transfer coefficient $z$ is insignificant at the centre of the channel and appreciably large at the boundary $y = 1$ for all variations. Fig.(3.17), shows $z$ is linear in the region $0 \leq y \leq 0.6$ with increase in the amplitude ratio $\varepsilon$ and is appreciably large at $y = 1$. The variation of $z$ with $\delta$ and $E$ is shown figs.3.18 & 3.19 It is observed that $z$ decreases at $y = 0$, increases in the region $0.1 \leq y \leq 0.7$ and then rapidly increases in the remaining region. The heat transfer coefficient $z$ is appreciably large at the boundary $y = 1$. An increase in Hartman number $M$ leads to a marginal increase in the heat transfer coefficient $z$ at $y = 0$ and then decreases in the region $0.2 \leq y \leq 0.7$. It is observed that $z$ fluctuates at $y = 0.8$ and attains the maximum at $y = 1.0$ fig.(3.20).
The average temperature $\bar{\theta}$ with different parameters is depicted in figs.(3.21-3.30). From figs.(3.21-3.23) ,it is observed that average temperature $\bar{\theta}$ depreciates in the region $0.1 \leq y \leq 0.9$ with increase in $R$, $E_1$ and $E_2$ and the enhancement is marginal at the boundary, while we observe that the depreciation in average temperature $\bar{\theta}$ with $E_3$ is more significant at the centre of the channel than at the boundary fig.(3.24). From fig.(3.25) we find that the average temperature $\bar{\theta}$ experiences a depreciation with increase in the Visco-elastic parameter $S$ and is more significant at the centre of the channel. It is noticed from fig.(3.26) that an increase in the Prandtl number $P_r$ depreciates $\bar{\theta}$ and is almost linear in the flow region $0 \leq y \leq 0.9$. Fig.(3.27) represents the variation of average temperature $\bar{\theta}$ with amplitude ratio $\varepsilon$. We notice that average temperature $\bar{\theta}$ enhances marginally in the flow region $0 \leq y \leq 0.9$ with increase in $\varepsilon$. The variation of $\bar{\theta}$ with slope $\delta$ reveals that higher the slope larger $\bar{\theta}$. The effect of $\delta$ on average temperature $\bar{\theta}$ gradually enhances as we move from the centre $y = 0$ to $y = 1$. It is observe from fig.(3.29) that the average temperature $\bar{\theta}$ depreciates with increase in Eckert number $E$. From fig.(3.30) we observe that the average temperature $\bar{\theta}$ gradually enhances with increase in Hartman number $M$. It is more significant at the centre of the channel than at the boundary and is insignificant at $y = 1$.

The average heat transfer $\bar{z}$ is numerically evaluated and graphically depicted in figs.(3.31-3.40) for different parameters. The variation of average heat transfer $\bar{z}$ with Reynolds number $R$ is shown in fig.(3.31). It is observed that the variation of $\bar{z}$ with $R$ is almost linear in the region $y = 0$ to $y = 0.6$ and $\bar{z}$ graphically enhances in magnitude in the remaining region , the variation of $\bar{z}$ at $y = 0$ is comparably smaller than at $y = 1$. From figs.3.32 & 3.33 we find that $|\bar{z}|$ depreciates with increase in of $E_1$.
and $E_2$. The average heat transfer $\bar{z}$ is appreciably large at boundary of the channel $y = 1$, than at the centre of the channel $y = 0$, while the variation of average heat transfer $\bar{z}$ with $E_3$ shows that $\bar{z}$ enhances marginally at $y = 0$ fig.(3.34). Fig.(3.35) represents the variation of average heat transfer $\bar{z}$ with Visco-elastic parameter $S$. The enhancement of $\bar{z}$ is marginal in the entire flow region and is almost linear in the region $0 \leq y \leq 0.7$. The enhancement is more at $y = 1$ than at the centre of the channel. The variation of average heat transfer $\bar{z}$ with amplitude ratio $\varepsilon$ is shown in fig.(3.36). For small values of $\varepsilon$ there is insignificant variation in $\bar{z}$ and is almost linear in the region $0 \leq y \leq 0.6$. For higher $\varepsilon \geq 0.02$, $\bar{z}$ enhances with $\varepsilon$ and the enhancement is appreciably large with higher values in the region $0.6 \leq y \leq 1.0$. From figs.3.37&3.38, the variation of average heat transfer $\bar{z}$ with $\delta$, $P_r$ is almost linear in the region $0 \leq y \leq 0.7$ and then onwards enhances, the variation of average heat transfer $\bar{z}$ with $\delta$, $P_r$ is appreciably large for higher values of $\delta$, $P_r$, the average Heat transfer $\bar{z}$ with Eckert number $E$ is shown in fig.(3.39). It is observed that average heat transfer $\bar{z}$ is linear in the region $0 \leq y \leq 0.7$ and then gradually enhances, this enhancement is more than that the centre of the channel at $y = 0$. From fig.(3.40) we find that $\bar{z}$ increases in the region $0 \leq y \leq 0.6$, for higher Hartman number $M$, smaller the $|\bar{z}|$ decreases.

Figs.(3.41-3.44) represent the variation of $\Delta p$ with elastic parameters and Hartman number. It is found that the variation of $\Delta p$ is linear in the region $0 \leq y \leq 0.8$ for smaller values of $E_1$ and $E_2$ and for higher $E_1$ and $E_2 > 0.2$, the $\Delta p$ gradually enhances, this enhancement is appreciably large for higher values of $E_1$ and $E_2$ at the boundary $y = 1$, while for an increase in $E_3$, there is no change in $\Delta p$ in the region $0 \leq y \leq 0.7$ and gradually enhances in the remaining region. Fig.(3.44) shows that an
increase in the Hartman number $M$ reduces $\Delta p$ gradually and is almost linear in the region $0 \leq y \leq 0.5$ and then onwards $\Delta p$ gradually enhances and this enhancement is appreciably large at $y = 1$.

Figs.(3.45-3.48) represent the frictional force $F$ with $E_1$, $E_2$, $E_3$, and $M$. It is observed that the variation of $F$ is almost linear in the region $0 \leq y \leq 0.6$ for $E_1$ & $E_2 \leq 0.3$ and then onwards the enhances $F$ is much larger at $y = 1$, while an increase in $E_3$ the frictional force $F$ depreciates in the region $0 \leq y \leq 0.4$ and then gradually enhances, this enhancement is larger at $y = 1$ than at $y = 0$. 
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APPENDIX:

\[ \alpha = \sqrt{\frac{R}{D_a} + M^2} \]

\[ \eta = 1 + \varepsilon \sin 2\pi (x - t) \]

\[ A_1 = \frac{\varepsilon}{\alpha^2} \left[ 8\pi^3 \cos 2\pi (x - t) (E_1 + E_2) - 4 E_3 \pi^2 \sin 2\pi (x - t) \right] \]

\[ A_2 = -\frac{\varepsilon}{\alpha^3} \text{Sech} \alpha \eta \left[ 8\pi^3 \cos 2\pi (x - t) (E_1 + E_2) - 4 E_3 \pi^2 \sin 2\pi (x - t) \right] \]

\[ A_3 = \frac{1}{\alpha^4} (\alpha B_1 - B_2 - \alpha^2 (d_4 + d_{10} + d_9)) \cosh \alpha \eta + \frac{1}{\alpha^3} (\alpha^3 d_{10} - B_2) \eta \sinh \alpha \eta + \frac{B_3}{4\alpha^3} \cosh 2\alpha \eta - \frac{1}{\alpha^2} (d_2 + d_8 + d_{11}) \]

\[ A_4 = \frac{1}{\alpha^3} (d_2 + d_8 + \alpha^2 d_{11}) \text{Sech} \alpha \eta - \frac{1}{4\alpha^4} (4 \alpha^3 d_{10} + 7B_2 + 2 \alpha B_1) \eta \tanh \alpha \eta - \frac{B_3}{3\alpha^4} \text{Sech} \alpha \eta \cosh 2\alpha \eta - \frac{B_2}{4\alpha^3} \eta^2 + \frac{1}{4\alpha^5} (9B_2 - 6\alpha B_1 + 4\alpha^3 (d_4 + d_{10} + d_9)) \]

\[ A_5 = \frac{1}{2\eta} \]

\[ A_6 = \frac{1}{2} - \frac{A_9}{4\alpha^2} (\cosh 2\alpha \eta - 4\alpha^2 \eta^2) \]

\[ A_7 = \frac{1}{2\eta} + \frac{g_2}{6} \eta^2 + \frac{1}{\eta \alpha^4} (\alpha^2 g_5 - 2 \alpha g_8 + 6 g_{11}) \eta \sinh \alpha \eta + \frac{g_7}{4\eta \alpha^2} \sinh 2\alpha \eta + \frac{g_{11}}{\alpha^2} \eta \sinh \alpha \eta \]

\[ A_8 = \frac{1}{2} - \frac{g_3}{12} \eta^4 - \frac{1}{4} (2g_1 - g_{13}) \eta^2 - \frac{1}{4\alpha^4} (4\alpha^2 g_4 - 8\alpha g_9 + 24 g_{10} - \alpha^2 g_{16}) - 4\alpha^2 g_{14}) \cosh \alpha \eta - \frac{1}{8\alpha^3} (2 \alpha g_6 - 2 g_{12} + \alpha g_{13} - g_{15}) \cosh 2\alpha \eta - \frac{1}{\alpha^3} (\alpha g_9 - 4g_{10}) \eta \sinh \alpha \eta - \frac{1}{\alpha^3} (\alpha g_8 - 4g_{11}) \eta \cosh \alpha \eta - \frac{g_{10}}{\alpha^2} \eta^2 \cosh \alpha \eta - \frac{1}{8\alpha^2} (2g_{12} + g_{15}) \eta \sinh 2\alpha \eta - \frac{1}{72\alpha^2} (9g_{14} + 2g_{16}) \cosh 3\alpha \eta \]
\[
A_9 = \frac{d_{20} a^4 A_2^2}{2}
\]

\[
B_1 = a^2(d_4 - d_9 - d_{10}) - S d_4 a^4 \quad B_2 = R d_{10} a^3 (S a^2 - 1)
\]

\[
B_3 = R d_{11} a^3 (2 S a^2 - 3)
\]

\[
d_1 = A_{1x} \quad d_2 = A_{1t} \quad d_3 = A_{2x}
\]

\[
d_4 = A_{2t} \quad d_5 = A_{5t} \quad d_6 = A_{6t}
\]

\[
d_7 = A_{9t} \quad d_8 = A_1 A_{1x} \quad d_9 = A_1 A_{2x}
\]

\[
d_{10} = A_2 A_{1x} \quad d_{11} = A_2 A_{2t} \quad d_{12} = A_5 A_{1x}
\]

\[
d_{13} = A_5 A_{2x} \quad d_{14} = A_9 A_{1x} \quad d_{15} = A_9 A_{2x}
\]

\[
d_{16} = A_2 A_{2t} \quad d_{17} = A_2 d_{10} \quad d_{18} = A_2 d_{11}
\]

\[
d_{19} = p_t R \quad d_{20} = p_t E \quad d_{21} = d_{19} d_{20}
\]

\[
g_1 = p_t \left( R d_6 + E A_3 a^2 \right)
\]

\[
g_2 = d_{19} \left( d_5 - E d_{12} \right)
\]

\[
g_3 = \frac{d_{19}}{2 a^2} \left( d_7 - 2 d_{14} \right)
\]

\[
g_4 = \frac{d_{20}}{4 a^2} \left( 4 A_4 a^5 + 2 a B_1 - 5 B_2 \right)
\]

\[
g_5 = \frac{p_t}{2 a^3} \left( 4 E B_2 + 2 E B_1 a - 2 E A_4 a^5 - 2 R d_{13} a^3 \right)
\]

\[
g_6 = \frac{p_t}{12 a^2} \left( E B_3 a - 6 R d_7 \right)
\]

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\[ g_7 = -\frac{d_{20} B_3}{6 \alpha^2} \]

\[ g_9 = \frac{1}{4 \alpha} \left( 2 d_{20} B_1 \alpha - 3 d_{20} B_2 - 8 R d_{15} \alpha \right) \]

\[ g_{11} = -\frac{g_{10}}{\alpha} \]

\[ g_{13} = d_{21} d_{16} \alpha^4 \]

\[ g_{15} = -d_{21} d_{17} \alpha^5 \]

\[ g_8 = \frac{d_{20}}{4 \alpha^2} \left( B_2 - 2 \alpha B_1 \right) \]

\[ g_{10} = \frac{d_{20} B_2}{4} \]

\[ g_{12} = \frac{d_{19} d_{14}}{\alpha} \]

\[ g_{14} = \frac{d_{19} d_{15}}{\alpha} \]

\[ g_{16} = -d_{21} d_{18} \alpha^5 \]
Profiles for temperature
Fig. 3.1 - Effect of $R$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_a=10000$)

Fig. 3.2 - Effect of $E_1$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $R=10$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_a=10000$)
Fig. 3.3-Effect of $E_2$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $R=10$, $E_1=0.1$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_s=10000$)

Fig. 3.4-Effect of $E_3$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $R=10$, $E_1=0.1$, $E_2=0.2$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_s=10000$)
Fig. 3.5 - Effect of $S$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $\Gamma=10$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $E=0.5$, $P_r=0.7$, $M=10$, $D_\infty=10000$)

Fig. 3.6 - Effect of $P_r$ on $\theta$

($\varepsilon=0.01$, $\delta=0.01$, $\Gamma=10$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $M=10$, $D_\infty=10000$)
Fig. 3.7 - Effect of $\varepsilon$ on $\theta$

$(\delta=0.01, R=10, E_1=0.1, E_2=0.2, E_3=0.3, \alpha =10, P_r=0.7, S=0.5, E=0.5, M=10, D_a=10000)$

Fig. 3.8 - Effect of $\delta$ on $\theta$

$(\varepsilon=0.01, R=10, E_1=0.1, E_2=0.2, E_3=0.3, \alpha =10, P_r=0.7, S=0.5, E=0.5, M=10, D_a=10000)$
Fig. 3.9 - Effect of $E$ on $\theta$

($\epsilon=0.01$, $\delta=0.01$, $R=10$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $P_r=0.7$, $M=10$, $D_x=10000$)

Fig. 3.10 - Effect of $M$ on $\theta$

($\epsilon=0.01$, $\delta=0.01$, $R=10$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $D_x=10000$)
Profiles for coefficient of heat transfer
Fig. 3.11 - Effect of $R$ on $Z$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $Pr=0.7$, $M=10$, $Da=10000$)

Fig. 3.12 - Effect of $E_1$ on $Z$

($\varepsilon=0.01$, $\delta=0.01$, $E_2=0.2$, $E_3=0.3$, $R=10$, $\alpha=10$, $S=0.5$, $E=0.5$, $Pr=0.7$, $M=10$, $Da=10000$)
Fig. 3.13-Effect of $E_2$ on $Z$

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_3=0.3, R=10, \alpha=10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000$)

Fig. 3.14-Effect of $E_3$ on $Z$

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, R=10, \alpha=10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000$)
Fig. 3.15 - Effect of $S$ on $Z$

$($$e=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, E=0.5, P_r=0.7, M=10, D_a=10000)$

Fig. 3.16 - Effect of $P_r$ on $Z$

$($$e=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, E=0.5, S=0.5, M=10, D_a=10000)$
Fig. 3.17 - Effect of $\varepsilon$ on $Z$

$(\delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, P_r=0.7, E=0.5, S=0.5, M=10, D_a=10000)$

Fig. 3.18 - Effect of $\delta$ on $Z$

$(\varepsilon=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, P_r=0.7, E=0.5, S=0.5, M=10, D_a=10000)$
Fig. 3.19 - Effect of E on Z

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, P_r=0.7, S=0.5, M=10, D_a=10000$)

Fig. 3.20 - Effect of M on Z

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, R=10, \alpha=10, E=0.5, P_r=0.7, S=0.5, D_a=10000$)
Profiles for average temperature
Fig. 3.21 - Effect of $R$ on $\bar{\theta}$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_4=10000$)

Fig. 3.22 - Effect of $E_1$ on $\bar{\theta}$

($\varepsilon=0.01$, $\delta=0.01$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_4=10000$)
Fig. 3.23 - Effect of $E_2$ on $\bar{\theta}$

$\bar{\theta}$ vs $\gamma$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_3=0.3$, $\alpha=10$, $R=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_r=10000$)

Fig. 3.24 - Effect of $E_3$ on $\bar{\theta}$

$\bar{\theta}$ vs $\gamma$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $\alpha=10$, $R=10$, $S=0.5$, $E=0.5$, $P_r=0.7$, $M=10$, $D_r=10000$)
Fig. 3.25 - Effect of $S$ on $\bar{\theta}$

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, R=10, E=0.5, P_r=0.7, M=10, D_a=10000$)

Fig. 3.26 - Effect of $P_r$ on $\bar{\theta}$

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, R=10, E=0.5, S=0.5, M=10, D_a=10000$)
Fig. 3.27 - Effect of $\varepsilon$ on $\bar{\theta}$

($\varepsilon=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, R=10, E=0.5, P_r=0.7, S=0.5, M=10, D_\alpha=10000$)

Fig. 3.28 - Effect of $\delta$ on $\bar{\theta}$

($\varepsilon=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, R=10, E=0.5, P_r=0.7, S=0.5, M=10, D_\alpha=10000$)
Fig. 3.29: Effect of $E$ on $\bar{\theta}$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $P_r=0.7$, $S=0.5$, $M=10$, $D_a=10000$)

Fig. 3.30: Effect of $M$ on $\bar{\theta}$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $P_r=0.7$, $S=0.5$, $E=0.5$, $D_a=10000$)
Profiles for average heat transfer
Fig. 3.31 - Effect of $R$ on $Z$

($\varepsilon = 0.01$, $\delta = 0.01$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$, $\alpha = 10$, $S = 0.5$, $E = 0.5$, $Pr = 0.7$, $M = 10$, $D_s = 10000$)

Fig. 3.32 - Effect of $E_1$ on $Z$

($\varepsilon = 0.01$, $\delta = 0.01$, $E_2 = 0.2$, $E_3 = 0.3$, $R = 10$, $\alpha = 10$, $S = 0.5$, $E = 0.5$, $Pr = 0.7$, $M = 10$, $D_s = 10000$)
Fig. 3.33 - Effect of $E_2$ on $\bar{Z}$

$(\varepsilon=0.01, \delta=0.01, E_1=0.1, E_3=0.3, R=10, \alpha=10, S=0.5, E=0.5, Pr=0.7, M=10, D_a=10000)$

Fig. 3.34 - Effect of $E_3$ on $\bar{Z}$

$(\varepsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, R=10, \alpha=10, S=0.5, E=0.5, Pr=0.7, M=10, D_a=10000)$
Fig. 3.35 - Effect of $S$ on $\bar{Z}$
($\varepsilon = 0.01$, $\delta = 0.01$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$, $\alpha = 10$, $R = 10$, $E = 0.5$, $P_r = 0.7$, $M = 10$, $D_s = 10000$)

Fig. 3.36 - Effect of $\varepsilon$ on $\bar{Z}$
($\delta = 0.01$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.3$, $\alpha = 10$, $R = 10$, $E = 0.5$, $S = 0.5$, $P_r = 0.7$, $M = 10$, $D_s = 10000$)
Fig. 3.37 - Effect of $\delta$ on $\bar{Z}$

$\bar{Z} \times 10^2$

$y$

$\delta=0.01$
$\delta=0.02$
$\delta=0.03$
$\delta=0.04$
$\delta=0.05$

$\varepsilon=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $E=0.5$, $S=0.5$, $Pr=0.7$, $M=10$, $D_\varepsilon=10000$

Fig. 3.38 - Effect of $Pr$ on $\bar{Z}$

$\bar{Z} \times 10^2$

$y$

$Pr=0.7$
$Pr=0.8$
$Pr=0.9$
$Pr=1.0$
$Pr=1.1$

$\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $E=0.5$, $S=0.5$, $M=10$, $D_\varepsilon=10000$
Fig. 3.39 - Effect of $E$ on $\bar{Z}$

($e=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $P_r=0.7$, $S=0.5$, $M=10$, $D_a=10000$)

Fig. 3.40 - Effect of $M$ on $\bar{Z}$

($e=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $R=10$, $E=0.5$, $P_r=0.7$, $S=0.5$, $D_a=10000$)
Profiles for Pressure (rise) drop & Frictional force
Fig. 3.41 - Effect of $E_1$ on $\Delta p$

($\varepsilon=0.01, \delta=0.01, E_2=0.2, E_3=0.3, \alpha=10, S=0.5, E=0.5, R=10, M=10$)

Fig. 3.42 - Effect of $E_2$ on $\Delta p$

($\varepsilon=0.01, \delta=0.01, E_1=0.1, E_3=0.3, \alpha=10, S=0.5, E=0.5, R=10, M=10$)
Fig. 3.43 - Effect of $E_3$ on $\Delta p$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $\alpha =10$, $S=0.5$, $E=0.5$, $R=10$, $M=10$)

Fig. 3.44 - Effect of $M$ on $\Delta p$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_2=0.2$, $E_3=0.3$, $\alpha =10$, $S=0.5$, $E=0.5$, $R=10$)
Fig. 3.45 - Effect of $E_1$ on $F$

($\varepsilon=0.01$, $\delta=0.01$, $E_2=0.2$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $R=10$, $M=10$)

Fig. 3.46 - Effect of $E_2$ on $F$

($\varepsilon=0.01$, $\delta=0.01$, $E_1=0.1$, $E_3=0.3$, $\alpha=10$, $S=0.5$, $E=0.5$, $R=10$, $M=10$)
Fig. 3.47 - Effect of $E_3$ on $F$

$\epsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, S=0.5, E=0.5, R=10, M=10$

Fig. 3.48 - Effect of $M$ on $F$

$\epsilon=0.01, \delta=0.01, E_1=0.1, E_2=0.2, E_3=0.3, \alpha=10, S=0.5, E=0.5, R=10$