Chapter-I
INTRODUCTION AND BASIC EQUATIONS
Fluid mechanics is widely used both in everyday activities and in the design of modern engineering systems from vacuum cleaners to supersonic air craft. To begin with, fluid mechanics plays a vital role in the human body. The heart is constantly pumping blood to all parts of the human body through arteries and veins, and the lungs are the sites of air flow in alternating directions. Needless to say, all artificial hearts, breathing machines, and dialysis systems are designed using fluid dynamics. An ordinary house is, in some respects, an exhibition hall filled with applications of fluid mechanics. The piping systems for cold water, natural gas and sewage for an individual house and the entire city are designed primarily on the basis of fluid mechanics. The same is also true for the piping and ducting network of heating and air-conditioning systems. A refrigerator involves tubes, through which the refrigerant flows, a compressor that pressurizes the refrigerant, and two heat exchangers where the refrigerant absorbs and rejects heat. All the components associated with the transportation of the fuel from the fuel tank to the cylinders - the fuel line, fuel pump, fuel injectors or carburetors as well as the mixing of the fuel and air in the cylinders and purging of combustion gases in exhaust pipes are analyzed. Also which plays a major past in the design and analysis of air craft, boats, submarines, rockets, jet engines, wind turbines, bio medical devices, the cooling of electronic components, and the transportation of water, crude oil, and natural gas. It is also considered in the design of buildings bridges and even bill boards to make sure that the structures can withstand wind loading. Numeric natural phenomena such as rain cycle, weather patterns, and the rise of ground water to top of trees, winds, ocean waves, and currents in large water bodies are also governed by the principles of fluid mechanics. Even though considerable
progress has been made in our understanding of the flow phenomena, more works are needed to understand the effects of various parameters involved in non-Newtonian method models and the formulation of an accurate method of analysis for body shapes of engineering significance. Also the boundary layer concept for such fluids is of special importance because of its application to many practical problems, among which we cite possibility of reducing frictional drag on the hulls of ships and submarines. The concepts of couple stresses and microstructure are conceptually different. The first concept has its origins in the way mechanical interactions are modeled, while the second is essentially a kinematic one, and arises out of an attempt to describe point particles having structure. Where as in a general theory of fluids with microstructure, couple stresses and internal spin may be present simultaneously, theories of fluids in which couple stresses are present but microstructure is absent are also possible. Similarly microstructure may be considered in the absence of couple stresses. In this way the main consequences of each of these concepts may be studied before proceeding to the study of more general theories. Number of theories has been proposed to explain the behaviour of fluids which contain microstructure such as additives, suspensions or granular matter. Eringen's [2] micro polar fluid theory defines the rotational fields in terms of kinematically independent rotation vector called micro rotation vector, for setting up of stress-strain rate constitutive equations. However the theory of couple stress fluid is given by Stokes [13] to define the rotational field in terms of the velocity field, thereby reducing considerably the number of material constants in the constitutive equations characterizing the fluid material. This theory introduces the second order gradient of velocity vector, instead of kinematically independent rotation vector in the
constitutive relationship between stress and strain rate. Stokes theory of fluids is the
generalization of classical theory of fluids allows for the polar effects such as the
presence of a non-symmetric tensor, couple stress and couples.

Stokes gave the equation of motion and constitutive equations for a couples
stress fluid on the basis of couple stress in elastic materials. The equation of motion for
the flow of an incompressible fluid with couple stresses are [13].

\[ T_{ji,j} + \rho f_i = \rho \frac{DV_i}{Dt} \]
\[ E_{jk} T_{jk}^s + M_{ji,j} + \rho C_i = 0 \]

Where \( f_i \) is body force per unit mass, \( C_i \) is body moment per unit mass,
\( V_i \) is velocity vector, \( T_{jk}^s \) and \( T_{jk}^A \) are the symmetric and anti-symmetric parts of
the stress tensor \( T_{jk} \) respectively, \( \rho \) is the density of the fluid, \( M_{ij} \) is the couple
stress tensor and the other terms have their usual meaning of tensor analysis
( Eringen’s [2]). The constitutive equations for an isotropic incompressible fluid with
couple stress are [2, 13].

\[ T_{jk}^s = -PS_{ij} + 2\mu d_{ij} \]
\[ \mu_{ij} = 4\eta W_{ji} + 4\eta' W_{i,j} \]

where \( \mu \) is the shear viscosity which is different from the solvent viscosity,
\( \mu_s, \eta, \eta' \) are the constants associated with couples stress, \( p' \) is pressure, \( \mu_{ij} \) is
deviatoric part of \( M_{ij} \), \( d_{ij} \) is symmetric part of velocity gradient and \( W_i \) is the
vorticity vector.
Incase of incompressible fluids, when the body forces and body moments are absent, the momentum equation in the vector notation become [1, 5]

\[ \rho \left( \frac{\partial q}{\partial t} + (q \cdot \nabla) q \right) = -\nabla p + \rho g + \mu \nabla^2 q - \eta \nabla^4 q \]

The momentum equation for couple stress fluid through sparsely packed porous medium is given by [1]

\[ \rho \left( \frac{1}{\delta} \frac{\partial q}{\partial t} + \frac{1}{\delta^2} (q \cdot \nabla) q \right) = -\nabla p + \rho g + \frac{\mu_1}{k} q + \mu_2 (\nabla^2 q) - \eta \nabla^4 q . \]

Magneto hydro dynamics (MHD) deals with the motion of electrically conducting fluids under the influence of a magnetic field. The birth date of MHD may be identified with the first experiments by Faraday who attempted to measure the electric potential induced between the opposite banks of the Thames River by the motion of the (weakly) conducting water in the Earth’s magnetic field (Faraday [3]). The principle behind Faraday’s (un successful) experiment is the same which under lies modern MHD flow meters. About in the same period, Ritchie developed a rudimentary electromagnetic pumping device, although the first working MHD pump was presented only much later (Northrup.E.F. [10]). Currently MHD effects are widely exploited in different industrial processes ranging from metallurgy to production of pure crystals (Moreau [9]). A field in which MHD will plays an essential role in nuclear fusion, where it is involved in at least two different problems, the confinement and dynamics of plasma, and the behaviour of the liquid metal alloys employed in some of currently considered designs of tritium breeding blankets.

The flow through a composite media in channels in which a clean fluid region is bounded by fluid saturated porous beds adjacent to impermeable boundaries of the
channel has invoked interest of several research workers, owing to its importance in designing and controlling a number of industrial processes. This type of flow is of importance to the petroleum engineer concerned with production of oil and gas from the underground reservoirs and gas fields and also the hydrologist in the study of migration of underground water and to a chemical engineer in the filtration process. This study is widely applicable in soil mechanics, water purification; sewage treatments, ceramic engineering and powder metallurgy. In view of these applications, it is of interest to formulate a mathematical modeling for the flow through composite media based on different macroscopic representations of porous media.

Flow through porous media has been attracted considerable research activity in recent years because of its several important applications notably in the flow of oil through porous rock, the extraction of energy from the geothermal regions, the evaluation of capability of heat removal from particulate nuclear in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion exchange beds, drug permeation through human skin, chemical reactors for the economical separation or purification of mixture and so on. In understanding circulation of blood in lungs, the effect of porosity has to include either in the medium or near the boundary. In lungs, blood can be visualized as flowing between the opposing layers of capillary endothelium held apart by endothelium covered “post” of septal tissue. This capillary endothelium is covered in turn by thin layer (interstitial space) lining the alveoli. The blood space in lung is idealized into a two dimensional channel and the interstitial tissue space into porous medium. An endothelial layer between the two regions is permeable to water and certain other solutes, it can be considered as a permeable membrane of
negligible thickness. The epithelial tissues between the air and vascular space are less permeable and thus it can be treated as impermeable membrane. The irregular ports which keep apart the endothelial walls can be ignored for the time being. Gopalan [4] discussed pulsatile blood flow in the lung alveolar sheets by idealized each of them as a channel covered by porous media.

In conventional magneto hydro dynamics poiseuille flow has been the subject of many investigations. Most of these studies pertain to Newtonian fluids. However, a generalization of Newtonian fluid theory has been introduced which takes care of the presence of couple stresses in fluids. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure. A model for such a fluid has been proposed by Stokes [13]. As an example to illustrate the effects of couple stresses on viscous incompressible fluids, Stokes has solved the problem of channel and couette flow also. In order to study the effect of a transverse magnetic field on the flow of an electrically conducting, viscous, incompressible fluid with couple stresses, Stokes [12] presented the problems of MHD channel and MHD couette flow. Further extension to his work has been done by Soundalgekar and Aranake [11, 12]. Later Lalitha Jayaraman and Ramanaiah [8] discussed the unsteady laminar flow of an electrically conducting couple stress fluid between parallel insulating plates subjected to a transverse magnetic field and due to a constant pressure gradient taken into account. Couple stress fluid theory has been used a model for flow in small arteries. Later D.V.Krishna et.al [7] discussed a three dimensional flow in a parallel
plate channel through a composite medium in parallel plate channel under the influence of an inclined magnetic field.

Recently D.V.Krishna et.al [6] discussed an initial value investigation of unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field. The exact solutions for the velocities in the clean fluid region and porous region have been obtained analytically and their behaviour has been discussed computationally with reference to different variations in the governing parameters. Veera Krishna. M. et.al [20] discussed an initial value investigation of unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field making use of Darcy-Lapwood model with Beaver Joseph conditions. Later Veera Krishna. M. et.al [17] discussed an initial value investigation of unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field taking hall current into account. The exact solutions for the velocities in the clean fluid region and porous region have been obtained analytically and their behaviour has been discussed computationally with reference to different variations in the governing parameters with the help of graphs. Veera Krishna et.al [19] discussed the steady hydro magnetic flow of a couple stress fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field making use of Brinkman’s model. Veera Krishna et.al [18] discussed the steady hydro magnetic flow of a couple stress fluid through a porous medium in a rotating parallel plate channel
under the influence of a uniform transverse magnetic field making use of Brinkman’s model. Recently Suneetha S.V. et.al [15] discussed the steady hydro magnetic flow of a couple stress fluid in a parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field, here the flow takes place with uniform axial pressure gradient and under the influence of periodic body acceleration. Later Veera Krishna et.al [16] discussed the steady hydro magnetic flow of a couple stress fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field taking hall current into account. Keeping the above mentioned facts in view in this an attempt has been made to discuss the steady and unsteady magneto hydro dynamic three dimensional flows of couple stress fluid through a porous medium in a parallel plate channel. Since the flow is three dimensional in view of secondary flow arising normal to the axial flow due to the component of the inclined magnetic field in the direction normal to the flow and lying parallel to the boundary plates. These velocity components are coupled in a manner similar to the flow in rotating parallel plate channels and ultimate governing equations in both the clean fluid and porous region are expressed in terms of complex velocity obtained through coupling.

In this dissertation we discuss the hall current effects on an analytical study of unsteady magneto hydro dynamic flow of an incompressible electrically conducting couple stress fluid through a porous medium between parallel plates, taking into account pulsation of the pressure gradient effect and under the influence of a uniform inclined magnetic field of strength $H_0$ inclined at an angle of inclination $\alpha$ with the normal to the boundaries. The solution of the problem is obtained with the help of perturbation
technique. Analytical expression is given for the velocity field and the effects of the various governing parameters entering into the problem are discussed with the help of graphs. The shear stresses on the boundaries and the discharge between the plates are also obtained analytically and their behavior computationally discussed with different variations in the governing parameters in detail.
References:


