CHAPTER - 1
CHAPTER - 1
INTRODUCTION

The concept of fuzzy set was introduced by L.A. Zadeh in his classical paper [46]. The fuzziness of a symbol lies in the lack of well-defined boundaries of the set of the objects to which this symbol applies. More specifically, let \( X \) be a reference set, also called a universe of discourse, covering a definite range of objects. Consider a subset \( A \) where transition between membership and non membership is gradual rather than abrupt. A fuzzy subset \( \mu \) of \( X \) is a mapping from \( X \) into unit interval \([0,1]\). G.A. Goguen [14], considered fuzzy sets whose grade of membership lies in complete distributive lattice. If the fuzzy set \( \mu \) takes values '0' or '1', then \( \mu \) is an ordinary subset of \( X \). The most common example of fuzzy sets are natural, for let \( X \) be a city and \( x \) is a citizen of \( X \). Then ‘\( x \) is tall in the city’ is an imprecise statement. Here ‘tall’ is a vague description. How tall is tall? No well-defined boundary exists between being tall and not being tall. Similarly ‘\( x \) is beautiful’ and ‘\( x \) is ugly’, ‘\( x \) is good’ etc, are all vague concepts. We can explain these kinds of concepts in real life situations with the help of fuzzy sets. The theory of fuzzy sets has proved itself to be of importance in pattern recognition problems, both using statistical decision theoretic and syntactic approaches.

C.L. Chang [4] was the one who applied the concept of fuzzy sets to topological spaces and generalized many of the concepts of general topology to what might be called fuzzy topological spaces.
In his paper, C.L. Change defined fuzzy topology on $X$, as family of fuzzy subsets $T$ of $X$ satisfying the following conditions

(a) $\emptyset, X \in T$

(b) If $A, B \in T$, then $A \cap B \in T$

(c) if $A_i \in T$ for each $i \in I$, then $\bigcup A_i \in T$.

R.H. Warren [42] showed that fuzzy continuous functions could be characterized by closure of fuzzy sets, a sub basis for a fuzzy topology and fuzzy neighbourhoods. Warren [42] also introduced the concepts of the derived fuzzy set and relative fuzzy topology. He proved that the collection of all fuzzy topologies on a fixed set is a complete lattice. Some characterizations of open fuzzy set, the neighbourhood of a fuzzy set and basis for fuzzy topology were given. Warren [42] proved that neighbourhood systems are an equivalent method of determining fuzzy topologies. Lowen [20] introduced a subcategory of fuzzy topological spaces and proved basic properties of the subcategory. M.H. Chanin and N.N. Morsi [5], studied pseudo-closure operators in fuzzy neighbourhood spaces. They also established relationship among the pseudo-closure operators of fuzzy neighbourhood base spaces, their fuzzy closure operators and their level topologies. R. Lowen [20], studied compactness notions in fuzzy neighbourhood spaces. In the Class of fuzzy neighbourhood spaces Lowen made a comparative study between the notions of compactness, $\alpha$-compactness, strong compactness and ultra compactness.

Michael D. Weiss [43] studied about induced topologies fixed point theorems. He Generalized Schauder fixed point theorem to fuzzy sets and a revised version of "Fuzzy separation theorem" has also been proved.


The concept of fuzzy smooth preproximity space is induced and studied by R. Badared, A.S. Mashhour and A.A. Ramadin [2]. They gave some particular constructions of fuzzy smooth preproximities. They established a natural link between fuzzy smooth preproximity spaces and the concept of fuzzy pre topological spaces, fuzzy preuniform spaces. M. A. Erceg [10] introduced metric spaces in fuzzy set theory. He proved that a pseudo-quasi metric on a set may be regarded as a distance function between subsets of that set. This equivalent definition is generalized to fuzzy set theory where points need not have Boolean properties and hence in which a natural generalization of a pseudo-quasi metric is unsatisfactory. Additional axioms were introduced which corresponds to pseudometrics and metrics in fuzzy set theory. Zike Deng [8] defined fuzzy metric and fuzzy pseudometric on the space of all fuzzy points by defining distance between two fuzzy points as a real number. Zike Deng [8] continued the study of fuzzy pseudometric spaces and introduced Separation axioms and modified version of fuzzy unit interval and proved Urysohns Lemma. He also proved Baire theorem and contraction mapping theorem in Pseudometric spaces.

Osmo Kaleva and Seppo Seikkala [26] introduced the concept of fuzzy metric space. They defined the distance between two points in a fuzzy metric space as a non-negative upper semicontinuous, normal and convex fuzzy number. They studied some properties of fuzzy metric spaces and proved some fixed point theorems. Osmo Kaleva [27] proved that some fuzzy metric spaces have completion.
Fuzzification of algebraic systems is a recent topic of interest in Mathematics. In this direction Kumbhojkar [16] has studied some properties of fuzzy primary ideals. Madana Swamy [28] has studied some results on general theory of algebraic fuzzy systems.

In 1968 Maia [21] has considered the concept of a bi metric spaces and generalized the Banach contraction principle by taking two metric spaces on a set $X$ and obtained a unique fixed point for a contraction mapping. In this direction we would like to mention the works of Mishra S.N. [29] and Ravi Dawar [31].

The concept of two metric spaces was introduced by Gahler in the Year 1969. In his paper [12] he proved many interesting properties of two metric spaces and this theory was further developed by many mathematicians. Meera Srivatsava and Ghosh [30], Singh, Sharma [33,34] have extended some fixed point theorems in metric spaces to two metric spaces.

Again in 1992 B.C. Dhage [3] has introduced the concept of a $D$-metric space, which is a recent topic of interest, and has studied the fixed point theorems on compact $D$-metric spaces. Anil Rajput [1], S.V.R. Naidu, Veerapandi [38] continued the study of fixed point theorems and also the topological properties of $D$-metric spaces.

We in our present work we consider the concept of fuzzy $D$-metric space and prove some of its properties. We also prove some fixed point theorems in $D$-metric spaces, Fuzzy $D$-metric spaces and bitwo metric spaces which generalize some of the existing results in Metric and $D$-metric spaces.