1. INTRODUCTION

Newtonian fluids, which are defined as those fluids for which the rate of deformation under laminar conditions is directly proportional to the stress. This is the simplest stress-rate of strain relationship and fortunately many fluids do behave in this fashion under the usual conditions of interest. However, a significant number of fluids do not behave in this simple fashion and they are termed as Non-Newtonian fluids and posses commercial importance as well as of theortical interest.

The graphs of shear stresses verses rate of shear for Non-Newtonian fluids called the "flow curves" are non-linear.

Real fluids may be classified on the basis of their rheological behavior. The classification presented by METZNER [17] into four broad types.

1. Purely viscous, or time-independent fluids: The deformation rate depends only upon the stress and is single-valued function of the later.

a) Newtonian: The viscosity is independent of shear stress.

b) Non-Newtonian: The viscosity is a function of shear stress.

2) Time-dependent fluids: The deformation rate and viscosity depend upon both the stress and the duration of the stress.

3) Viscoelastic fluids: The deformation rate and viscosity depend upon both the stress and the extent of deformation.

4) Complex rheological bodies: Fluids exhibiting the characterstics of more than one of (1), (2), (3).

Fluids of the first 1 (b) type whose properties are independent of time may be described by a reheological equation of the form.

$$\dot{v} = f(T) \qquad (1.2.1)$$

This equation implies that the rate of shear at any point in the fluid is a simple function of the shear stress at the point, such fluids are termed as Non-Newtonian viscous fluids. These fluids are subdivided into three district types depending on the nature of the function in equation (1.2.1). These types are

1. Plastic fluids or Bingham plastics
2. Pseudo plastic fluids
3. Dilatant fluids.

(i) Plastic Fluids:

This model shows a relationship of shear stress to shear rate which is linear but which unlike that for a Newtonian fluid, intersects the shear-stress axis at a finite positive value,

This relation ship can be expressed by the equation

$$\tau - \tau y = -\beta \frac{\partial u}{\partial y} \quad \tau \geq \tau_y \qquad \text{..........} \; A$$

$$\frac{du}{dy} = 0 \qquad \tau < \tau_y \qquad \text{..........} \; B$$

where β is called the "coefficient of rigidity" or the coefficient of "plastic viscosity" and τy is the yield stress, which must exceeded in absolute value for flow to taken place. Equation (A) is valid only for shear stresses grater than the yield stress. While for shearstresses less than the yield stress, there is no relative movement of fluid from equation (B). Infact behaviour of this type can also explained by assuming that when the fluid at rest, contains a three-dimensional structure which can resist stresses, as high as the yield stress τy. Many real fluids such as drilling muds, suspensions of chalk, Oil paints, sevage sludges, closely approximate to this type of behaviors.

(ii) Pseudo Plastic Fluids:

This group of Non-Newtonian fluids shows a relationship of shear stress to shear rate which is concave downward. The curve passes through the origin when displayed with linear co-ordinates, and slope decreases with increasing shear rate when displayed on logarthimic co-ordinates. Many of fluids which show pseudoplastic behavior include polymeric solutions or melts, such as rubbers, cellulose acetate and napalm;

suspension such as paints mayonnaise, paper pulp, and detergent slurries, and dilute suspensions of inert solids. This model shows the characteristic shape of a pseudoplastic over the entire range of shear, including the approximate Newtonian behavior at both low and high shear. These fluids are characterized as power law of fluids as proposed by Ostawald [21] and Reiner [24].

(iii) Dilatant Fluids:

Dilatant fluids show rheological behavior essentially opposite to that of pseudo plastic fluids in that the viscosity increases with the increasing of shear stress. The term dilatant has come to usage for all fluids which exhibits the property of increasing apparent viscosity with increasing rate of shear. Dilatant materials over first recognised and named by Reynolds [25]. Many of fluids which are reported as exhibiting dilatancy are pigment-vehicle suspensions, such as paints and printing inks. When the solids content is high.

2. Time-dependent fluids:

The deformation rate and viscosity depend upon both the stress and the duration of the stress. These fluids may be sub-divided in two classes (a) thixotropic fluids (b) rheopetic fluids. Thixotropic fluids are those fluids for which the shear stress decreases with time at a constant deformation rate. There are many examples of this fluids are paints, gelatine solutions, honey shaving cream, etc. Thixotropic behavior can be explained in the same way as pseudo plastic behavior. Rheopectic fluids are those fluids for which the shear stress increases with time at a given deformation rate. Rheopectic fluids are much less common than thixotropic fluids; gypsum suspensions in water, bentonite sols, and vanadium pentoxide sols are few examples of these fluids. Theopetic behavior can be explained in the same way as dilatant behavior, if the time necessary to approach equilibrium at a constant deformation rate is now assumed to be sufficiently large to be observable.

Visco elastic fluids have drawn great attention owing to its importance in transportation or processing of substances which do not fit into classical Newtonian hypothesis of linear stress - strain relationship. Visco-elastic fluids have memory in the sense they have tending to go to the original state. As in the case of ordinary viscous fluids the strain rate does not become zero instantaneously if the stress is removed. They exihibits a contribution of properties of both viscous fluids and elastic solids. One of the interesting feature of a visco elastic fluid is the "Weissenberg effect" the climbing of elastic fluids up a shaft rotating in the fluid.

For visco elastic fluids, part of the deformation is gradually recovered when line stress is removed. Therefore the time derivative of both shear stress and shear rate may be included in the governing equations that describe the flow of such fluid. Pastes, Jellies, Emulsion, Polymeric fluids exihibit viscoelastic nature.

Apart four some of the body fluids like blood, paracreas secretion etc are viscoelastic in nature. The type of viscoelastic fluid depends soley in the stress, rate of strain constitutive relation.

Various models of viscoelastic fluids have been proposed and some of them are discussed by Fredrickson [8] and Metzner [17].

Ericksen [3], Eringen [5,6], Oldroyd's [19] generalization of the Maxwell equations for viscoelastic fluid derived on the assumption that Newtons viscocity and Hook's law in elasticity will determine the flow pattern.

W.E.Langloise [14,15] has published a research report "a recursive approach to the theory of slow steady-rate viscoelastic flow" in which he has obtained various closed from solution of Rivilin-Ericksen fluids.

The constitutive equation governing the Rivilin-Ericksen fluid has been developed by Rivilin and

Erickson [5]. It is given by

$$T = -PI + \phi_1 A + \phi_2 B + \phi_3 A^2$$

where

$I = ||\delta_{ij}||$, δ_{ij} is the Kronecker delta

$A = ||a_{ij}||$, $a_{ij} = \frac{1}{2} [u_{i,j} + u_{j,i}]$ is the deformation tensor

$B = ||b_{ij}||$,

$b_{ij} = a_{i,j} + a_{j,i} + 2V_{m,i} V_{m,j}$ is the visco-elastic tensor

$a_{i,j}$, $a_{j,i}$ are the accelaration gradients

$V_{m,i}$, $V_{m,j}$ are the Velocity gradients

ϕ_1, ϕ_2 and ϕ_3 are material constants constants called the coefficient of viscousity, visco-elasticity and cross-viscosity respectively, which are considered to be constants in our thesis.

The equation of continuity is

$$\frac{\partial \rho}{\partial t} = \rho \bar{V}_{i,i} = 0$$

The equation of linear momentum is

$$T_{ij,j} + \rho(f_i - a_i) = 0$$

where ρ is the density of the fluid

$a_i = \frac{\partial V_i}{\partial t} + V_{i,j} V_j$ is the acceleration vector

f_i is the body force per unit mass

and

T_{ij} is the component of the stress.

For incompressible fluids we have

$V_{i,i} = 0.$

The visco-elastic equations are obtained by substituting for T_{ij} from the constitutive equation in the above linear momentum equation. In deriving the equations governing the flow phenomena of visco-elastic fluid, we assume that the fluid is isotropic, homogeneous, incompressible and posses visco-elastic nature.

The maj rity of the transportation of fluids related to either an industrial or a techn logical or biomedical problem takes place through pipes of uniform or non-uniform gap.

In the case of Non-Newtonian fluids a considerable amount of work has been done within the last two decades. In the particular case of circular cross-section Balmer and Fiorna [1] used a finite difference method to obtain a numerical solution for power-law fluids by considering the pressure pulse as a superposition, position of a oscillation and a step. For the very same class of fluids Gorla [9,10,11] and Madden utilized the semidirect variational method of Kantovorich [12] to study the transient response due to several types of pr essure pulses.

Langoise [14,15] and Rivlin [26] have studied slow steady state flow of viscoelastic fluids through Non-circular tubes. Dutta [2] has obtained the solution for viscoelastic Maxwell fluids through a circular annuls.

Krishnamraj Choubey [13] studied unsteady flow of Non-Newtonian viscoleastic fluid between two co-axial cylinders, when inner cylinder is moving from rest for a certain period with linearly growing speed and then stops suddenly.

Ramkiscon [22,23] investigated the flow of a short memory fluid in tubes of circular and rectangular cross-section. These are other relevant and important works in the area of unsteady non-newtonian flows in pipes and between plates.

The unsteady viscoelastic three parameters oldroyd model of fluid flows in tubes of circular and rectangular crossection has also been studied by Ramkiscon etal [22,23] using transform methods. The special cases of a constant pressure gradient and a periodic or oscillatory pressure gradient has been examined.

In this dissertation we investigate the unsteady flow of a second order Rivlin-Ericksen fluid through a rectangular channel under prescribed discharge using Laplace transform method. The third order Galerkin solution has been obtained for the transformed boundary value problem for the following cases of the discharge function q(t).

i) q(t) is a constant throughout the motion.

ii) q(t) is proportional to t.

iii) q(t) is proportional to $te^{-\gamma t}$.

The pressure gradient has been evaluated numerically for different variations of the governing parameters.