CHAPTER III
ULTRAHARMONIC ENTRAINMENT IN THE PRESENCE OF AN INTERFERENCE TONE

3.1. Introduction:

An injection synchronized oscillator whether operating in harmonic or in anharmonic mode finds wide applications in the fabrication of low noise amplifiers and filters at millimeter wave frequencies, in the generation of carrier signal in communication systems, in the reduction of the depth of modulation of an AM signal, in the improvement of the signal/noise ratio of a noisy signal etc.. The performance of the oscillator in all such cases depends on its ability to discriminate against interfering signals, which in practice always accompany the synchronizing signal. The presence of the interfering signals gives rise to distortion at the output of the oscillator and also brings about an instability in the oscillator synchronization. This necessitates the study of injection synchronization in the face of an interference corrupted signal.

Fundamental or harmonic synchronization in the said situation has been dealt with by many a workers [III-1, III-2, III-3, III-4]. Tucker and Jamieson [III-2], in their extensive study of the problem, obtained expressions for the
response of a synchronized oscillator with interfering signals having frequencies far from and close to the synchronizing zone of the oscillator. However, in their work, the selectivity of the oscillator was neglected. Spence and Boothroyd [III-2], on the other hand, did away with any such restrictions and obtained a completely general expression for the discrimination against the interfering signal. They, however, assumed that the synchronizing signal and the interference was of the same order and also the interference component of the oscillator output was rather small. In a comparatively recent work [III-3, III-4], the authors have dealt with the synchronization characteristics of an injection synchronized oscillator (ISO) under the different cases of small interfering tone lying away and close from the oscillator and large interfering tone. Compared to this, very little work has been done on ultraharmonic entrainment of an oscillator in the presence of an interference signal. This is why attention has been paid in this chapter to consider first the physics of the phenomenon occurring in the case of synchronization of an oscillator in the ultraharmonic mode by an interference corrupted signal. Then a mathematical tool has been developed corroborating the physical arguments. Finally, experiment has been performed in line with the theory developed.

3.2. Physics of the Phenomenon:

The case of ultraharmonic entrainment with the injection
of a pure CW signal has already been investigated in the previous chapter. The effect of the interfering tone on the oscillator may be introduced with supposition that the oscillator is initially locked to the synchronizing signal and then the interfering tone influences it without throwing the system out of synchronism.

Let us assume the net input to the oscillator to be given by,

\[ e_i(t) = E \sin(w_1 t + xE \sin(w_2 t) \tag{3.1} \]

where, \( w_1 \) and \( w_2 \) represent the synchronizing frequency and the interfering frequency respectively and \( x \) is a fraction that measures the ratio of interfering to synchronizing signal strengths. Considering ultraharmonic entrainment of order three, the oscillator frequency component of the output may be taken as \( V \sin(3w_1 t + \phi_3) \). Due to the mixing process provided by the nonlinearity of the oscillator (cf. eqn. (2.44)) three components close to the free-running frequency of the oscillator will be generated which are proportional to (i) \( a_3 E^3 \sin 3w_1 t \), (ii) \( a_3 xE^3 \sin (3w_1 t + (w_2 - w_1) t) \) and (iii) \( a_3 xE^2 \sin 3w_1 t + (w_2 - w_1) t + \phi_3 \). The spectral positions of these components with respect to the oscillator free-running frequency may be depicted as in Fig.3.1. The three quadrature parts of these three components will act simultaneously to modulate the instantaneous frequency of the oscillator. Thus in the presence of both the synchronizing signal and the interfering tone
Fig. 3.1. Spectral representation of the positions of different components in the unlocked state of an ultraharmonic oscillator of the third order for the cases: (a) the interfering tone lying just above one-third the oscillator frequency and (b) the interfering tone lying just below one-third the oscillator frequency.
in the unlocked state of the oscillator, the effect of each of the above components on the phase motion of the oscillator depends both on the strength and the position of the component with respect to the oscillator free-running frequency. Now, under favourable conditions, the oscillator may be entrained to the first of these three with the remaining two components modulating the instantaneous phase of the oscillator with a pull proportional to

$$a_3 x B^2 \sqrt{2} \cos \left\{ (w_2-w_1) t + \Theta_3 \right\} + \nu \cos (w_2-w_1) t$$

Evidently, the instantaneous phase of the oscillator, in the locked state, will be modulated at a frequency \( (w_2-w_1) \). The existence of the other two components may be supposed to offer a shielding effect on the oscillator resulting in a reduction of the pull of the synchronizing signal on the oscillator. Obviously, the tendency of the interfering signal will be to lower the normal locking range of the ultraharmonic synchronization of the oscillator.

The rate at which the locking range falls with the strength of the interfering tone is determined by the distance of the generated interfering component from the free-running frequency of the oscillator. This can be made clear by referring to Fig. 3.1. Let us suppose that the oscillator is initially entrained on the upper locking point by the injecting signal only. Now, the interfering tone having a frequency close to
but just above and below one-third the oscillator free-running frequency be applied, the corresponding spectral positions being shown in Fig.3.1a and Fig.3.1b respectively. Let us now consider Fig.3.2a and Fig.3.2b which represent the spectral positions of different components in the locked condition of the oscillator corresponding to cases shown in Fig.3.1a and Fig.3.1b respectively. The spectral distance between the generated interfering component and the locked position of the oscillator is definitely different for the two cases and this again leads to variation in the pulling effect of the generated interfering tone on the locked state of the oscillator. As a result of all these, one may conclude that the presence of the interfering signal causes the state of fall of locking range to be more rapid when the interfering tone has a frequency higher than the centre frequency of ultra-harmonic synchronization than when it lies on the lower side.

3.3. Analytical Approach:

As in the previous chapter, we consider the low driven ultraharmonic synchronization of a class A oscillator in the third order mode of synchronization. Justification for consideration of low level synchronization stems from the fact that in practice the strength of the incoming signal needs to be small in comparison with the free-running amplitude of the oscillator for purposes of better performance of the ISO.
Fig. 3.2. Spectral representation of the positions of different components in the locked state of an ultraharmonic oscillator of the third order when (a) the interfering tone lies just above one-third the oscillator frequency and (b) the interfering tone lies just below one-third the oscillator frequency.
Moreover, strength of the interfering signal accompanying the synchronizing signal is assumed to be a small fraction of that of the injecting CW signal.

Assuming the input to the oscillator as in (3.1), it is logical to take the output of the oscillator as,

\[ e_0(t) = A \sin(w_1 t + \theta_1) + \sqrt{3} \sin(3w_1 t + \theta_3) + A_1 \sin(w_2 t + \theta_2) \]

...(3.2)

where the terms containing A and A_1 indicate the directly transmitted components of signal frequency and interfering frequency respectively, the remaining term representing the oscillator frequency component.

Fitting the class A oscillator loop equation (cf. Eqn. (2.41)) with these values of \( e_i(t) \) and \( e_0(t) \) and using equations (2.40) and (2.44) of the previous chapter, one can easily arrive at the following governing equations of the system:

\[
\frac{1}{4} a_3 A^2 \sin(3\theta_1 - \theta_3) = \frac{9w_1^2 - w_0^2}{3w_1 w_0} + \frac{2Q}{w_0} V \frac{d\theta_3}{dt} \quad \ldots (3.3)
\]

\[ a_1 V - a_3 \left( \frac{1}{2} A^2 V + \frac{3}{4} V^3 - \frac{3}{2} VA_1^2 \right) + \frac{\alpha}{4} A_2 A^3 \cos(3\theta_1 - \theta_3) = \frac{2Q}{w_0} \frac{dV}{dt} + V \]

\[
\ldots (3.4)
\]

and

\*[\frac{3}{4} a_3 A^2 \sin(\theta_3 - 3\theta_1) - \beta \sin \theta_1 + \gamma \sin \left( (w_2 - w_1) t - \theta_1 \right) = \frac{w_1^2 - w_0^2}{w_1 w_0} QA + \frac{2Q}{w_0} A \frac{d\theta_1}{dt} \]

\*[\ldots (3.5)]
If we denote by $\phi$, the equivalent phase difference between the input and output, then $\phi = 3\theta_1 - \theta_3$. The equation (3.3) governing the phase motion of the oscillator then looks like

$$\frac{d\phi}{dt} = \frac{w_o}{2} \left( \frac{3w_1}{w_o} - \frac{w_o}{3w_1} \right) - \frac{w_o}{8Q} \frac{a_3}{V} A^3 \sin\phi + 3 \frac{d\theta_1}{dt} \quad \text{(3.6)}$$

In order to derive any useful information regarding the lock-range from such a complex equation it is first necessary to express amplitudes $A$ and $V$ in some suitable approximate forms. From (3.5), we note that since $\theta_1$ for ultraharmonic synchronization is nearly equal to $-\pi/2$ and the first term containing $a_3$ is rather small, we can write,

$$A \approx \frac{Ew_1w_o}{(w_2-w_1)Q} \left( 1 + x \cos wt \right) = A_0(1 + x \cos wt) \quad \text{(3.7)}$$

where

$$w = w_2 - w_1 \quad \text{and} \quad A_0 = \frac{Ew_1w_o}{Q(w_2-w_1)^2} \quad \text{(3.8)}$$

Obviously, $A_0$ represents the amplitude of the directly transmitted component in absence of interference. Similarly, from (3.4), if we put $\theta = \pi/2$ on the verge of locking, substitute $\frac{dV}{dt} = 0$ in the locked state and neglect the small term containing $a_3^2 A_1^2$, we get

$$V^2 = V_o^2 - 2A^2 \quad \text{(3.9)}$$

where $V_o^2 = \frac{4}{3} \frac{a_1 - 1}{a_3}$, the square of the free-running oscillator amplitude.
Substituting the value of $A$ from (3.7), we can put $V$ in the form,

$$V = \sqrt{V_0^2 - 2A_0^2(1 + x \cos \omega t)^2} \quad \ldots \quad (3.10)$$

This again can be written as, by expanding in the Fourier series,

$$V = V_0 \sqrt{1 - 2a_0^2(1 + x \cos \omega t)^2}$$

$$= V_0 (P_0 + P_1 \cos \omega t + \ldots \ldots) \quad \ldots \quad (3.11)$$

where,

$$a_0 = A_0 / V_0 \quad \ldots \quad (3.12)$$

$$P_0 = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{1 - 2a_0^2(1 + x \cos \omega t)^2} \cdot d(\omega t) \quad \ldots \quad (3.13)$$

and

$$P_1 = \frac{1}{\pi} \int_0^{2\pi} \sqrt{1 - 2a_0^2(1 + x \cos \omega t)^2} \cos \omega t \cdot d(\omega t) \quad \ldots \quad (3.14)$$

After writing the amplitudes $A$ and $V$ in these simplified forms, the factor \( \frac{w_0^3}{8Q} \frac{a_2}{V} A^3 \) appearing at the phase equation (3.6) assumes the approximate form, given by,

$$\frac{w_0^3}{8Q} \frac{a_3}{V} A^3 = \frac{w_0^3}{8Q} \frac{A_0^3(1 + x \cos \omega t)^3}{V_0(P_0 + P_1 \cos \omega t + \ldots \ldots)}$$

$$= \frac{x}{F_0} (1 + 3x \cos \omega t) (1 - \frac{P_1}{P_0} \cos \omega t)$$

$$= \frac{x}{F_0} (1 - \frac{3x}{2} \frac{P_1}{F_0} + (3x - \frac{P_1}{F_0}) \cos \omega t)$$

$$= f_1(x) + f_2(x) \cos \omega t \quad \ldots \quad (3.15)$$
where,

$$K_0 = \frac{w_0^2 a^3}{8QV_0} A_0^3 \quad \cdots (3.16)$$

$$f_1(x) = \frac{K_0}{P_0} (1 - \frac{3x}{2} \frac{P_1}{P_0}) \quad \cdots (3.17)$$

and

$$f_2(x) = \frac{K_0}{P_0} (3x - \frac{P_1}{P_0}) \quad \cdots (3.18)$$

The relation (3.15) is only approximate, since we have neglected the terms containing \(x^2\) and higher powers of \(x\) as well as the components of frequencies \(2w, 3w\) etc. for simplification. The phase equation (3.6), on applying the relation (3.15), and neglecting the rather small term \(\frac{d\theta_1}{dt}\), now reduces to the form,

$$\frac{d\theta}{dt} = \Omega - \left\{ f_1(x) + f_2(x) \cos wt \right\} \sin \varphi \quad \cdots (3.19)$$

where,

$$\Omega = \frac{w_0}{2} \left\{ \frac{3w_1}{w_0} - \frac{w_0}{3w_1} \right\}$$

In order to assume a solution of the phase equation (3.6) we recall the arguments stated in the theory, wherein we thought of the influence of the interfering tone only after locking was achieved by the synchronizing signal. In such a condition, the instantaneous phase of the oscillator consists of two parts. One is the static phase error \(\varphi_1\), that is guided by the initial detuning between the free-running oscillator and the generated ultraharmonic signal. The other part is the phase modulation introduced by the interfering signal. As a result, one can
reasonably assume a solution of (3.6) as given by,

\[ \varphi = \psi_1 + m \sin(wt - \psi_2) \]  

\[ \cdots (3.20) \]

The factor \( m \) is the degree of modulation due to interference and \( \psi_2 \) is a constant to be determined.

Substituting this assumed solution of \( \varphi \) in (3.19) and applying the method of harmonic balance, it is easy to obtain,

\[ \frac{\partial \psi_1}{\partial t} = \Omega f_1(x) J_0(m) \sin \psi_1 + f_2(x) J_1(m) \sin \psi_2 \cos \psi_1 \]  

\[ \cdots (3.21) \]

\[ mw \cos \psi_2 = 2 f_1(x) \sin \psi_2 \cos \psi_1 - f_2(x) J_0(m) \sin \psi_1 \]  

\[ \cdots (3.22) \]

and

\[ mw \sin \psi_2 = -2 f_1(x) \cos \psi_2 \cos \psi_1 \]  

\[ \cdots (3.23) \]

where, \( J_n(m) \) is the Bessel's function of first kind of \( n \)th order and argument \( m \). In deriving the above three equations, Bessel's functions of order higher than one have been left out. It is obvious, that to find out an expression for the upper side locking range in the presence of an interference corrupted signal, one should put \( \psi_1 = \pi/2 - m \) (cf. eqn. (3.20)). Again, for smaller strength of interference, one can approximate \( J_0(m) = 1 \) and \( 2J_1(m) = m \). On inserting these conditions, equations (3.21), (3.22) and (3.23) yield the following relations for the upperside lock-range (\( \Omega_1 \)),

\[ \Omega_1 = f_1(x) \cos m - \frac{m}{2} f_2(x) \sin m \sin \psi_2 \]  

\[ \cdots (3.24) \]
\[
\cos \xi_2 = - \frac{m w f_2(x) \cos \theta}{m^2 + m^2 f_1(x) \sin^2 \theta} \tag{3.25}
\]

and
\[
\sin \xi_2 = \frac{m/2 f_1(x) f_2(x)}{m^2 + m^2 f_1(x) \sin^2 \theta} \tag{3.26}
\]

If we denote by \( \Omega_0 \), the upperside lock-range of the ISO in absence of interference, then considerations of equations (3.24) and (3.17) furnish its expression in the form,

\[
\Omega_0 = \frac{K_0}{P_0} \tag{3.27}
\]

The parameter \( P_0 \) appearing at (3.27) must be calculated under the condition of no interference (i.e., when \( x = 0 \)). From equation (3.13), \( P_0 \) in this situation has the value,

\[
P_0 = \sqrt{1-2a_0^2} \tag{3.28}
\]

Thus,

\[
\Omega_0 = \frac{K_0}{\sqrt{1-2a_0^2}} \tag{3.29}
\]

Normalising the equation (3.24) by this factor \( \Omega_0 \), one can write,

\[
\frac{\Omega_1}{\Omega_0} = \sum f_1(x) \cos \theta - \frac{m}{2} f_2(x) \sin \xi_2 \sum \frac{1}{P_0} \tag{3.30}
\]

Substitution of the expressions for \( f_1(x) \), \( f_2(x) \) and \( \sin \xi_2 \) from (3.17), (3.18) and (3.26) respectively in (3.30) gives the result,
\[
\frac{\Omega_1}{\Omega_0} = \ln \frac{b}{a} \left( (1 - \frac{5x}{2} \frac{P_1}{P_0}) \cos m - \frac{K_0}{P_0^3} \frac{J_0}{P_0^2} \frac{P_1}{P_0} (1 - \frac{3x}{2} \frac{P_1}{P_0}) \sin m \sin 2m \right)
\]

\[\frac{\Omega_1}{\Omega_0} = \ln \frac{b}{a} \left( \frac{P_1}{P_0} (1 - \frac{3x}{2} \frac{P_1}{P_0}) \right) \sin m \sin 2m \]

The term \( w \) which is the frequency difference between the synchronizing signal and the interfering signal can easily be written as difference of frequency detuning between the oscillator and the generated synchronizing signal (i.e., \( \Omega_1 = 3w_1 - w_0 \)) and that between the oscillator and the interfering signal frequency close to it (i.e., \( \Omega_2 = 3w_2 - w_0 \)) in the form,

\[
w = w_2 - w_1 = \frac{1}{3} (\Omega_2 - \Omega_1)
\]

Using this value of \( w \) in (3.31) and denoting a parameter \( Y \) by the expression,

\[
Y = \frac{\Omega_2}{\Omega_0} \cdot \frac{\Omega_1}{\Omega_0}
\]

we have the relation,

\[
\frac{\Omega_1}{\Omega_0} = \sqrt{1 - 2a_0^2} \left( 1 - \frac{3x}{2} \frac{P_1}{P_0} \right) \cos m^n \sin 2m \]

\[
\frac{\Omega_1}{\Omega_0} = \sqrt{1 - 2a_0^2} \left( 3x - \frac{P_1}{P_0} \right)^2 \frac{P_1}{P_0} \sin m \sin 2m
\]

\[\frac{\Omega_1}{\Omega_0} = \sqrt{1 - 2a_0^2} \frac{P_1}{P_0} \sin m \sin 2m
\]

\[\frac{\Omega_1}{\Omega_0} = \sqrt{1 - 2a_0^2} \frac{P_1}{P_0} \sin m \sin 2m
\]

\[(3.33)\]
Evaluation of locking-ranges in the upperside with variation of interference to synchronising signal ratio \( x \) is not possible from this single equation (3.33). To develop another such formulation we turn to equations (3.25) & (3.26).

Applying the relation,
\[
\cos^2 y_2 + \sin^2 y_2 = 1
\]
and the expression of \( w \) given in (3.32) we can write,
\[
\frac{m^2}{61} (\Omega_2 - \Omega_1)^4 + \frac{2}{9} m f_1^2(x) (\Omega_2 - \Omega_1)^2 \sin^2 m + m^2 f_4^2(x) \sin^4 m
\]
\[
= \frac{1}{9} f_2^2(x) (\Omega_2 - \Omega_1)^2 \cos^2 m + \frac{1}{4} f_1^2(x) f_2^2(x) \sin^2 2m
\]
\[
.. (3.34)
\]
Using the values of \( f_1(x) \) and \( f_2(x) \) and normalising both sides by \( Q_0 \) one can easily rearrange this relation (3.34) as a quadratic equation in \( y \) (cf. equation (3.33)) and is given by,
\[
GY^2 + HY + J = 0
\]
\[
.. (3.35)
\]
where, the coefficients \( G \), \( H \) and \( J \) have the expressions,
\[
G = \frac{m^2}{61(1 - 2a_0^2)^2}
\]
\[
H = \frac{2}{9} \frac{m^2 \sin^2 m}{P_0^2(1 - 2a_0^2)} (1 - \frac{3}{2} \frac{P_1}{P_0})^2 - \frac{\cos^2 m}{9P_0^2(1 - 2a_0^2)} (\frac{3}{2} \frac{P_1}{P_0})^2
\]
\[
.. (3.37)
\]
and
\[
J = \frac{m^2}{P_0^4} (1 - \frac{3}{2} \frac{P_1}{P_0})^4 \sin^4 m - \frac{\sin^2 2m}{4P_0^4} (1 - \frac{3}{2} \frac{P_1}{P_0})^2 (3x - \frac{P_1}{P_0})^2
\]
\[
.. (3.38)
\]
For a fixed detuning of the generated interfering signal and the oscillator (i.e., $\frac{Q_2}{Q_0}$), computation of equations (3.33) and (3.35) lead to the expressions for the upper-side locking ranges of the oscillator with varying strengths of the interfering signal. It is clear, that for the determination of corresponding lower-side locking ranges of the oscillator, one has to take $V_1 = -(\frac{3}{2} - m)$ in the above sets of equations and proceed exactly in the similar way. The theoretical values of the normalised locking ranges in the upper side (i.e., $Q_1/Q_0$) with variation in the interference to synchronizing signal strength ratio (i.e., $x$) are plotted in Fig.3.3. In the figure curve (a) represents the case for the interfering tone lying above one-third the oscillator frequency and curve (b) stands for the same plot only with the interfering tone lying below one-third the oscillator frequency.

3.4. Experiment:

The experiment on ultraharmonic synchronization of class A oscillator with a CW signal corrupted with an interference was conducted by simulating an oscillator in an analog computer. The details of the oscillator is described in section 3.4.1.

The oscillator operation, thus simulated, had frequency of 3.3 KHz. Ultraharmonic synchronisation in the third order was achieved by injecting a CW signal from a stable source.
The theoretical variation of the normalized upper-side locking ranges \( \Omega_1/\Omega_0 \) with the interference to sync. signal ratio \( x \) for the cases when the interfering tone lies just \( (a) \) above and \( (b) \) below one-third the oscillator frequency.
Once locking of the oscillator took place, interfering signals having frequencies slightly higher and lower than the sub-
multiple of oscillator centre frequency but always lying within
the synchronization zone were applied. The upper-side locking
ranges of the oscillator were measured in these two cases by
changing the amplitude of the applied interfering signal.
These are shown in Fig. 3.4. A fair agreement between the
theory and the experiment was observed.

3.4.1. The Analog Computer Model:

Experimental observations were carried out with the
electronic differential analyser shown in Fig. 3.5. In this
model \( I_1, I_2 \) are integrators, \( M_1 \) and \( M_2 \) are multipliers, \( A_1, A_2, A_3, A_4 \) are unity gain adders, and \( x, y, a, b \) and \( c \) are poten-
tial dividers. The central portion of the block consisting
of \( A_1, I_1, y, I_2 \) and \( x \) actually forms the model of a linear
oscillator. The nonlinearity of a cubic type is constructed
by the multipliers \( M_1, M_2 \) and the potential dividers 'a' and
'b'. If we represent the injecting synchronizing voltage by
\( e_s \), the interfering signal voltage by \( e_1 \) and the oscillator
output by \( e_0 \), then a mathematical equation of the following
form can be written from the above model,

\[
\frac{de_0}{dt} = e_s + e_1 + x \int e_o \, dt - ye_o + a_3 e_0^3 - a_1 e_o
\]

or,

\[
\frac{d^2 e_0}{dt^2} + xe_o - (y + a_1) \frac{de_0}{dt} + a_3 \frac{d}{dt} (e_0^3) = - \frac{d}{dt} (e_s + e_1)
\]

\[\cdots (3.35)\]
Fig. 5.4. Experimental variation of normalized upper-side looking ranges ($\Omega_1/\Omega_0$) with the interference-to-sync. signal ratio ($x$) for the cases when the interfering tone lies just (a) above and (b) below one-third the oscillator frequency.
FIG. 3.5. Model of the experimental set-up by analog computer simulation.
The equation (3.35) is an almost exact representation of a van der Pol's oscillator, which inherits a cubic-type non-linearity.

3.5. Conclusion:

In our treatment of ultraharmonic synchronization by an interference corrupted signal we have neglected the amplitude perturbation of the ISO and considered only the phase perturbation. However, when the generated interfering tone lies in the close proximity of the tracking zone of an ISO, as in our case, the role of amplitude perturbation on the locking range of the oscillator is quite an important one. In spite of this fact, we can justify our procedure from the point of a large detuning of the interfering tone from the oscillator free-running frequency.

Another point we must mention here. In our analysis of ultraharmonic synchronization in the previous chapter we showed that the shift of the oscillator free-running frequency caused by the forcing signal in this case marks an important change from the corresponding case of harmonic synchronization. But yet, in our present analysis this shift has not been considered. This could be done only because the shift of oscillator frequency, as we have already indicated, is quite small unless the Q factor of the oscillator is very small indeed.
The theoretical as well as the experimental data lead us to conclude that the locking range, upper or lower, in the case of a class A ultraharmonic oscillator tend to decrease with the application of a interfering tone lying either on the upper or on the lower side of the one-third the oscillator free-running frequency. This is not like the case of harmonic synchronization of an ISO by an interference corrupted signal. In the latter case, if the interfering tone is on the upper-side of the free-running frequency of the oscillator, the upper-side locking range increases further decreasing the lower-side one. As a result, the asymmetry in the locking range due to the asymmetry in the tank circuit is enlarged. The situation is otherwise when the interfering tone lies on the lower side of the oscillator free-running frequency.

The values of upper-or lower-side locking ranges are dependent on the placement of the interfering signal with respect to the free-running frequency of the oscillator. When the interfering frequency is situated very close to but below the one-third of the free-running frequency of the oscillator, the upper-side locking range is higher than when the interfering tone lies above. However, the rate of fall of the upper-side locking range with the increase of interference to signal strength ratio is less rapid in the
former case compared to the latter one. The observations for the lower-side locking ranges are just the opposite.

It has been experimentally observed that for large detuning of the interfering tone, the upper-side locking range is almost equal to that of lower-side and either of the ranges do not suffer any appreciable fall with increase of interfering signal strength. This is as expected from theory also.

The theoretical curve shows a good agreement in nature with the experimental plot up to a certain ratio of interference to signal strength. Above that, the rates of fall of locking ranges are different in the two plots. The reason for this lies in the facts that (i) the theory has been developed for rather small value of interference strength and (ii) a number of approximations have been adopted to make the analytical approach comparatively simpler.
REFERENCES


