CHAPTER-VIII
ON THE PERFORMANCE OF A NEW PHASE LOCKED LOOP

8.1. Introduction:

To improve the performance of a conventional phase locked loop (PLL) in its variety of applications, different structural modifications of the PLL have been suggested by different workers [VIII-1-6]. Knowing that the loop phase detector (PD) transfer characteristic has an important role in different modes of system operation, many of the earlier works attempted to modify the shape of the system phase error function. It is wellknown that the acquisition range of a PLL is primarily determined by the shape of PD transfer characteristic [VIII-1] and in the tracking mode of system operation, the number of cycles slipped by the loop due to loss of lock impulses (LLI) is determined by the range of phase error after which PD sensitivity changes sign throwing the loop into the regenerative state. For better performance of the PLL in the tracking mode the extension of the dynamic zone of a conventional PD characteristic can be hopefully asked for and so the previous workers on this problem suggested methods of obtaining PD characteristics having larger linear dynamic range than that of the conventional sinusoidal PD. Different workers obtained the extended range PDs by different methods, which can be classified as carrier signal wave shaping, postdetection
linear synthesis, postdetection nonlinear synthesis etc. 

In the carrier signal wave shaping technique, which is of limited practicality, the wave shape of the input signal and (or) of the reference signal are (is) so processed that the low frequency version of the multiplier type PD output is a function of relative phase of the input signal having larger dynamic zone. The examples of this technique are the systems with sawtooth type or rectangular type PD transfer characteristics \( \sqrt{\text{VIII-2}} \). By the method of postdetection linear synthesis one can have the error signal as a weighted sum of the harmonics of the sinusoidal phase error function, simply by generating the individual sinusoidal terms these comprise the Fourier series. The resultant output will be the transfer function of an extended range PD (ERPD). The examples of this technique \( \sqrt{\text{VIII-3}} \) are ERPD by phase error feedback method and ERPD by quadrature detection and trigonometric multiplication method. The method of generating ERPD transfer function by postdetection nonlinear synthesis is best illustrated in the reference, the well-known example of this class of PDs is a tan-lock PD \( \sqrt{\text{VIII-5}} \); the experimental studies of the PLL system with this PD have been reported asserting the better threshold performance of the system. In the analysis of noise performance of these PLL systems, the statistical character of the additive loop noise is very much difficult to assert and in almost all cases the equivalent loop noise
has been considered to be of the same form as in the case of a conventional loop. Besides these synthesis and shaping techniques, another method for realising the extended range phase detector is the application of phase feedback as suggested by Acampora and Newton [VIII-6]. In the noise free situation, the system operation with this PD has been explored indicating the extension of dynamic range but in the noisy environment the system behaviour is analytically unknown since Viterbi's [VIII-7] classical analysis on phase error dynamics of a conventional PLL cannot be applied for this ERPLL system. The search for an optimum PD transfer characteristic is still on and recently some authors [VIII-9] have considered the problem of implementing a PD as a parallel combination of two different functions of phase error and solved the problem by finding the phase error functions which minimise the system phase error variance.

In the present chapter of this dissertation a technique of obtaining a new PD has been described and the merits and demerits of a PLL incorporating this PD has been analytically discussed. Also has been given the experimental results in support of theoretical anticipations. The basic idea leading to the organization of the new PD is as follows. The elaborate study on the noise response of different PDs [narrated in chapter II of this dissertation] points out that the sinusoidal and sawtooth PDs of equal maximum gain in noise-free environment
have got superiority over one another in two different modes of system operation; in the acquisition mode, where the gain characteristic of the PD as a function of phase error is of prime importance, the sinusoidal PD is better than a sawtooth PD; while, in the tracking mode, where the linearity of the PD transfer characteristic is important for faithful operation, the sawtooth PD is better than a sinusoidal PD. From these observations, one can ask for a PD having the qualities of a sinusoidal PD in gain performance and of a sawtooth PD in linear operation. The most natural way of implementing such a PD is to have the parallel combination of sinusoidal and sawtooth PD characteristics with proper weight factors. In designing the PD, the maximum value of the PD transfer characteristic should have to be taken same as that of a conventional PD (sinusoidal). This is done to have comparative performance of different PLLs (with different PDs) having same hold-in range which is dependent on the maximum loop gain. The shape of the PD transfer characteristic is important in determining the PLL performance in the context of the acquisition range and tracking performance with lower rate of loss of lock impulses in face of noisy input signal.

The organization of the present chapter has been made in the following manner. The section 8.2 describes how the transfer characteristic of the new PD has been formulated and evaluates the equivalent noise bandwidth of the PLL designed with the new PD. In the section 8.3, the open loop
noise performance of the new PD has been given. The first part of the section 8.4 considers the character of the loop noise of the PLL with new PD, while, in the second part, the linear analysis of the system equation has been performed to calculate the phase error variance of the loop phase error. In the section 8.5, the nonlinear analysis of system equation has been described. The acquisition analysis, the study on PLL performance of the system and the reports of experimental studies have been described in section 8.6, 8.7 and 8.8 respectively. The chapter concludes in section 8.9 with certain additional comments on the behaviour of the new PLL.

8.2 General Features of a PLL with Sine-O-Linear PDs

The transfer characteristic of the new PD has been considered to be a weighted sum of sinusoidal and sawtooth transfer function as

$$f(\phi) = x \sin \phi + y \frac{\phi}{\pi}$$

where $\phi$ has been considered within the range $-\pi$ to $\pi$, $x$ and $y$ are the weight factors. Obviously, $f(\phi)$ would be an odd periodic function of $\phi$. Now imposing the conditions that (i) the maximum value of $f(\phi)$ would be unity (or minus unity) and (ii) it would be obtained at $\phi = \pi$ (or $-\pi$), one finds the values of the weight factors to be $x = 1/\pi$ and $y = 1$. Hence the new PD transfer function is of the form,

$$f(\phi) = \frac{1}{\pi} \left[ \sin \phi + \frac{\phi}{\pi} \right], \quad -\pi \leq \phi \leq \pi$$

$\cdots (8.1)$
The mathematical modelling of a sinusoidal PD is done as a multiplier and that of a sawtooth PD has been described in the chapter II. Incidentally, the output of a sawtooth PD is a linear function of the difference \( \psi \) of input and reference phase, lying between \(-\pi\) to \(\pi\). The new PD, named as a sine-o-linear PD, has been shown in the block diagram, Fig. 8.1. Referring to the Fig. 8.2, one can write the phase governing equation of the PLL with the sine-o-linear PD in the noise-free condition as

\[
\frac{d\phi}{dt} = \Omega \cdot \text{AKF}(p) f(\psi) \tag{8.2}
\]

where \( \psi = (w_1 - w_2) t + (\theta_1 - \hat{\theta}) = \Omega t + \theta_1 - \hat{\theta} \), \(w_1 t + \theta_1\) and \(w_2 t + \hat{\theta}\) being the input phase and the reference phase respectively.

For a first order loop \( F(s) = \frac{1}{s} \), one can see that the maximum hold-in range of the PLL with the sine-o-linear PD is \( \text{AK} \) which is same as that of a conventional PLL. This is because of the fact that \( |f(\psi)|_{\text{max}} \) and \( |\sin\psi|_{\text{max}} \) are both equal to unity.

Again from the loop equation (8.2), with zero static frequency error (\( \Omega = 0 \)) one can have the phase estimate of the loop VCO as,

\[
\hat{\theta} = \frac{\text{AKF}(p)}{xp} (\psi + \sin\psi) \tag{8.3}
\]

For a first order loop \( F(s) = 1 \) in the linear condition \( \psi \) small, \( \sin\psi = \psi \) one can write the relation as,

\[
\hat{\theta} = \frac{2\text{AK}}{x} (\theta_1 - \hat{\theta}) \tag{8.4}
\]
Fig. 8.1: Block diagrammatic representation of the realisation of sine-ϕ-linear phase detector.

Fig. 8.2: The phase locked loop structure with sine-ϕ-linear phase detector.
and so the loop transfer function in the complex frequency
domain be,

\[ L(s) = \frac{\theta}{\phi} = \frac{2AK}{s + \frac{2AK}{\pi}} \]  \hspace{1cm} \text{(8.3)}

Thus the two-sided noise bandwidth \((W_L)\) of the loop is,

\[ W_L = 2B_L = \frac{1}{2\pi} \int_{-\infty}^{\infty} |L(jw)|^2 dw \]
\[ = \frac{AK}{\pi} \]  \hspace{1cm} \text{(8.6)}

So, the one-sided noise bandwidth of the new PLL is, \(B_L = \frac{AK}{2\pi}\)
which is less compared to that of a conventional PLL with a
sinusoidal PD, known to be equal to \(\frac{AK}{4}\) \(\sqrt{\text{VIII-7}}\). From this
observation it can be anticipated that the noise filtering
capability of the new PLL would be better than that of a
conventional PLL.

8.3. Additive Noise Performance of a Sine-o-Linear PD:

The noise performance of a sine-o-linear PD can be
studied by the analytical technique established in section 2
of the chapter II of this dissertation. The odd periodic
transfer function of the sine-o-linear PD can be written as

\[ f(\phi) = (C_1 + \frac{1}{\pi}) \sin \phi + \sum_{n=2}^{\infty} C_n \sin n\phi \]  \hspace{1cm} \text{(8.7)}

where

\[ C_n = (-1)^{n+1} \frac{2}{n\pi} \]
and the utilising the result of (2.13) one can write the average PD output as a function of phase error \( \psi \) as,

\[
\langle e_0(\psi) \rangle = \sum_{m=0}^{\infty} c_{2m+1} \frac{\sqrt{\xi}}{2} \exp\left( -\frac{\psi}{2} \right) I_m(\frac{\psi}{2}) + I_{m+1}(\frac{\psi}{2}) \sin(2m+1)\psi \\
+ \sum_{m=1}^{\infty} c_{2m} \frac{\exp(-\frac{\psi}{2}) \rho^m}{2} \frac{(m-1)!}{(2m-1)!} \sqrt{\Gamma(2m)} I_{m+1}(2m+1) - \\
\frac{(m+1)^{\rho}}{2m(2m+1)} I_{m+2}(2m+1) \sin 2m\psi \\
+ \frac{1}{\pi} \frac{\sqrt{\xi}}{2} \exp\left( -\frac{\psi}{2} \right) I_0(\frac{\psi}{2}) + I_1(\frac{\psi}{2}) \sin \psi \quad \ldots (8.8)
\]

where \( \rho \) denotes the input signal-to-noise power ratio. The computed average transfer characteristic of the PD has been shown in Fig.8.3, for different \( \rho \), along with the same for a sinusoidal PD for comparison. It can be observed from the figure that for small phase errors the gain of the sin-o-linear PD is less than that of a sinusoidal PD but higher values of \( \psi \) (specifically more than \( \pi/2 \)) the new PD has got greater gain than the gain of a sinusoidal PD. So far the linearity of the transfer characteristic is concerned the sin-o-linear PD is superior. But for large noise condition (smaller values of \( \rho \)), the sin-o-linear PD performance degrades in all respects and it reduces to a sinusoidal type with much reduced gain.
Fig. 8.3: The additive noise response of the sine-o-linear phase detector in the open loop condition (solid curve). The dotted curves give the same for a conventional sinusoidal PD under identical conditions.
8.4.1 Characterization of the Noise in the Loop with Sine-0-Linear PD:

The noise inside the loop is considered to be composed of two components, one through sinusoidal channel and the other through sawtooth channel. Let the input signal plus noise of the PLL be written as,

\[
x(t) = s(t) + n(t) = \sqrt{2} A \sin(w_1 t + \Theta_1) + n(t)
\]

where \( n_s(t) \) and \( n_c(t) \) are Gaussian, stationary, statistically independent, zero mean noise processes, each having one sided spectral density \( N_0 \) (same as that of \( n(t) \)). Then the variances of the input noise process is,

\[
\sigma^2 \text{(say)} = N_0 W
\]

where \( W \) is the one sided bandwidth of the input noise spectrum, large compared to the loop band, so that white input noise approximation is valid. Again, writing (8.9) as

\[
x(t) = \sqrt{2} \left( \sqrt{(A+n_s)^2 + n_c^2} \sin(w_1 t + \Theta_1 + Y(t)) \right)
\]

\[
\leq \sqrt{2} A \sin(w_1 t + \Theta_1 + Y(t))
\]

in the low noise condition, where

\[
Y = \arctan \frac{n_c(t)}{A + n_s(t)} = \frac{n_c(t)}{A}, \quad \ldots (8.12)
\]

the output of the sinusoidal PD is \( A \sin(\Theta_1 Y) \). Writing the PD
output as $A\sin\varphi N_1(t)$, the sum of signal part and noise part, 
one can get the noise introduced through the sinusoidal 
channel as,

$$N_1(t) = n_s(t)\sin\varphi + n_c(t)\cos\varphi.$$  \hspace{1cm} (8.13)

Here $\varphi$ is the loop phase error. Thus $N_1(t)$ is Gaussian and has 
the spectral density $N_0$, same as that of $n(t)$. Also, the output 
of the sawtooth PD is

$$A(w_1t+\phi_1+w_2t-\hat{\theta}) = A\phi + A\dot{\varphi}$$  \hspace{1cm} (8.14)

which gives the noise introduced through the sawtooth channel as

$$N_2(t) = A\dot{\varphi} = n_c(t)$$ \hspace{1cm} (8.15)

Thus, $N_2(t)$ has the one-sided spectral density $N_0$, same as that 
of the input noise. The system modelling with the noisy input 
signal is given in Fig.8.4. As a first approximation we assume 
complete non-correlation between $N_1(t)$ and $N_2(t)$. This is 
reasonably valid for very high CMR conditions.

8.4.B. Linear Analysis on the Noise Performance of the PLL 
with Sine-$\varphi$-Linear PD:

The phase governing equation of the PLL with the sine-$\varphi$-linear PD can be had from the system model depicted in 
Fig.8.4. The VCO estimate of the input phase can be written as,

$$\hat{\varphi} = \frac{KF(p)}{\pi p} \int [\dot{\varphi} + \sin\varphi] + N_1(t) + N_2(t)$$  \hspace{1cm} (8.16)

where $\hat{\varphi} = \omega t + \phi_1 - \varphi$, $\omega$ is the static frequency detuning between
Fig. 8.4: Equivalent representation of the phase locked loop with noisy input signal.
the signal frequency and the VCO frequency. In the linear
analysis the phase error is assumed to be small and so one
can put \( \sin \phi = \phi \). Then, with a bit simplification of (8.16),
one can write

\[
\phi = - \left[ \int \frac{2AKF(p)}{p} \frac{N_1(t)}{2A} + \frac{2AKF(p)}{p} \frac{N_2(t)}{2A} \right]
\]

where \( \Omega \) has been assumed to be zero, without any loss of
generality. With the characteristic of \( N_1(t) \) and \( N_2(t) \) as
discussed in the previous subsection, that \( N_1(t) \) and \( N_2(t) \)
are completely un-correlated and each being of two sided
spectral density \( \frac{N_0}{2} \) Watts/Hz, the phase error variance for a
first order loop (with \( P(s) = 1 \)) can be written as,

\[
\sigma_a^2 = \frac{N_0K}{4\pi A} \tag{8.18}
\]

In terms of loop CNR \( a \) defined as,

\[
a = \frac{\text{Signal power}}{\text{Noise spectral density} \times \text{Loop noise bandwidth}} \tag{8.19}
\]

one can write,

\[
\sigma_a^2 = \frac{1}{2a} \quad (a = \frac{2\pi A}{K\lambda}) \tag{8.20}
\]

Considering \( \beta \) as the CNR at the system input, i.e., \( \beta \) being
defined as,

\[
\beta = \frac{A^2}{N_0} \tag{8.21}
\]
it is easy to show that, \( \alpha = 2\pi \frac{\varphi}{\frac{W}{AK}} \), and hence,

\[
\varphi^2 = \frac{1}{4\pi \frac{\varphi}{\frac{W}{AK}}} \quad \cdots (8.22)
\]

Now for a conventional PLL it is quite well known that the loop phase error variance is,

\[
\sigma^2_{\varphi} = \frac{1}{8\frac{\varphi}{\frac{W}{AK}}} \quad \cdots (8.23)
\]

when it will be expressed in terms of CNR at the system input. Comparing (8.22) and (8.23) it can be noted that the phase error variance of a PLL having sine-o-linear PD would be less than that of a conventional PLL, for identical values of system input CNR's \( \beta \). For all practical purposes \( \frac{\varphi}{\frac{W}{AK}} \) should be greater than 2 and then the white noise approximation of the input noise would be valid.

8.5. Nonlinear Analysis of Noise Performance of the PLL with the Sine-O-Linear PD:

The approximate nonlinear analysis of the performance of the new PLL can be done by finding the probability density function (pdf) of the loop phase error \( \varphi \), \( p(\varphi) \), by solving the Fokker Planck Equation of \( p(\varphi) \). From (8.16) one can write the phase governing equation of the system as,

\[
\frac{\text{d}\varphi}{\text{d}t} = -\frac{K_p(p)}{\frac{\varphi}{\frac{W}{AK}}} \varphi \left( \varphi + \sin \varphi \right) + \Pi_1(t) + \Pi_2(t) \quad \cdots (8.24)
\]

Here \( \Pi_1(t) \) and \( \Pi_2(t) \) have been characterised as zero mean.
Gaussian processes with one sided spectral density $N_0$ Watts/MHz.

To write the Fokker-Planck Equation of $p(\varphi)$, we consider the PLL system to be a first order one and open loop frequency error is zero. Then following [VIII-7], the Fokker-Planck equation to be solved to get $p(\varphi)$ is,

$$
\frac{\partial}{\partial t} p(\varphi, t) = -\frac{3}{\partial \varphi} \left[ A_1(\varphi) p(\varphi, t) \right] + \frac{3^2}{\partial \varphi} \left[ A_2(\varphi) p(\varphi, t) \right]
$$

...(8.25)

where $A_1(\varphi)$ and $A_2(\varphi)$ are intensity co-efficients to be obtained from (8.24) as,

$$
A_1(\varphi) = -\frac{AK}{\pi} (\varphi \sin \varphi)
$$

...(8.26)

and

$$
A_2(\varphi) = \frac{N_0 K^2}{\pi^2} (1 + \cos \varphi)
$$

...(8.27)

The solution of (8.25) with $A_1(\varphi)$ and $A_2(\varphi)$ given by (8.26) and (8.27) is very much involved, if not impossible. Here, keeping practical situations in mind, we would look for an approximate solution. To start with, we consider that the dynamics of $\varphi$ is like a Gaussian random variable and the time variation of $\varphi$ is much slower than that of the additive noise process. Then we can replace $\cos \varphi$ in the expression of $A_2(\varphi)$ by its quasi-linear equivalent $\exp(-\sigma^2 / 2)$. Again, considering fluctuations of $\varphi$ to be small, we replace $\sigma^2$ in the expression of $\exp(-\sigma^2 / 2)$ by the value obtained in linear analysis i.e., we put $\sigma^2 = \frac{1}{2a}$, $a$ being loop CNR as defined earlier. Thus, to make the problem tractable, we approximate $A_2(\varphi)$ as,
\[ A_2(\psi) = \frac{N_0 k^2}{\pi} \int \left( 1 + \exp \left( - \frac{1}{4 \alpha} \right) \right) \, d\psi \]  
\[ \cdots (8.28) \]

which is independent of \( \psi \).

Now in the steady state \((t \to \infty)\), the time variation of \( p(\psi, t) \) vanishes and so the Fokker-Planck equation reduces to

\[ \frac{AK}{\pi} \frac{d}{d\psi} \left( (\psi + \sin \psi) p(\psi) \right) + \frac{N_0 k^2}{2\pi} \int \left( 1 + \exp \left( - \frac{1}{4 \alpha} \right) \right) \frac{d^2}{d\psi^2} p(\psi) = 0 \]
or

\[ \frac{d^2}{d\psi^2} p(\psi) + \gamma \frac{d}{d\psi} \int g(\psi) p(\psi) \, d\psi = 0 \]  
\[ \cdots (8.29) \]

where

\[ g(\psi) = (\psi + \sin \psi) \]  
\[ \cdots (8.30) \]

and

\[ \gamma = \frac{\alpha}{1 + \exp \left( - \frac{1}{4 \alpha} \right)} \]  
\[ \cdots (8.31) \]

The solution of (8.29) is known as \[ \sqrt{\text{III-7}} \].

\[ p(\psi) = \frac{\exp \left( - \gamma h(\psi) \right)}{\pi} \int_{-\pi}^{\pi} \exp \left( - \gamma h(\psi) \right) \, d\psi \]  
\[ \cdots (8.32) \]

where,

\[ h(\psi) = \int_{-\pi}^{\psi} g(x) \, dx \]  
\[ \cdots (8.33) \]

and \( p(\psi) \) is subject to boundary conditions given as,

\[ p(\pi) = p(-\pi) \]  
\[ \cdots (8.34a) \]

and \( \int_{-\pi}^{\pi} p(\psi) \, d\psi = 1 \)  
\[ \cdots (8.34b) \]

Then the pdf of \( \psi \) is,
\[ p(\phi) = \frac{\exp\left(-\frac{\phi^2}{2}\right) \cdot \exp(\nu \cos \phi)}{\sqrt{2\pi}} I_0(\nu) + 2 \sum_{n=1}^{\infty} I_n(\nu) \exp\left(-\frac{n^2}{2\nu}\right) \quad -\pi \leq \phi \leq \pi \]

Now with the knowledge of pdf of $\phi$, the variance of $\phi$ can be obtained, by the relation,

\[ \sigma_\phi^2 = \int_{-\pi}^{\pi} \phi^2 p(\phi) d\phi \]

as,

\[ \sigma_\phi^2 = \frac{1}{\nu} \int_{-\pi}^{\pi} \left[ \sum_{n=1}^{\infty} \frac{I_n(\nu)}{I_0(\nu)} \left( \frac{\nu^2 - n^2}{2\nu} \right)^{\nu^2} \exp\left(-\frac{n^2}{2\nu}\right) \right] \]

In Fig. 8.5 the pdf of the loop phase error ($\phi$) for a particular input signal-to-noise power ratios (CNRs) has been given for the new PLL. The same for a conventional PLL has also been given for identical input conditions. A comparison of the two pdfs confirms the better performance of the new PLL. The computed variation of $\sigma_\phi^2$ with input CNR for the new PLL as well as for the conventional PLL has been shown in Fig. 8.6.

8.6. Acquisition Analysis of the PLL with Sine-O-Linear PD:

It is quite well known that in the acquisition mode of system operation the centre frequency of the local VCO is driven towards the input frequency by the average d.c. voltage at the PD output. Hence so far as the pull-in or the acquisition
Input CNR = .5

Fig. 8.5: Computed pdfs of loop phase error of PLLs with sine-o-linear PD and sinusoidal PD under identical input conditions.
Fig. 8.6: Computed variations of loop phase error variance of the new PLL and the conventional PLL with input carrier-to-noise power ratio (CNR).
behaviour of the PLL is concerned the average d.c. voltage at the PD output matters a lot and in this respect the shape of the PD transfer characteristic plays an important role. In this section the average d.c. voltage at the PD output would be obtained in terms of that at the output of a sinusoidal PD in a conventional PLL. And from the knowledge on the acquisition range of a conventional PLL, the same for the new PLL will be predicted.

In the noise free condition the phase equation of the first order PLL is given by,

$$\frac{d\phi}{dt} = \Omega - AKf(\phi) \quad \cdots (8.37)$$

where $f(\phi) = (1/\pi)(\phi + \sin \phi)$. Therefore, the d.c. voltage at the PD output in the steady beating condition is

$$Af(\phi) = A\left[\frac{\Omega}{AK} - \frac{1}{AK} \langle \frac{d\phi}{dt} \rangle \right]$$

or

$$Af(\phi) = A\left[\frac{\Omega}{AK} - \frac{2\pi}{AK} \left\{ \int_{\phi_0}^{2\pi+\phi_0} \frac{d\phi}{\Omega - AKf(\phi)} \right\}^{-1} \right] \quad \cdots (8.38)$$

Now expanding the integrand within the braces in a power series of $\frac{AK}{\Omega} f(\phi)$ and neglecting the fourth and the higher order terms, one gets,

$$Af(\phi) = A \frac{\langle f^2(\phi) \rangle}{(\Omega/\Delta K)} \quad \cdots (8.39)$$

where

$$\langle f^2(\phi) \rangle = \frac{1}{2\pi} \int_{\phi_0}^{2\pi+\phi_0} f^2(\phi) d\phi \quad \cdots (8.40)$$

$f(\phi)$ being odd periodic function of $\phi$, odd power terms do not appear.
The transfer characteristic for the sine-o-linear PD is considered as $x \sin \phi$, where $x$ is a constant to be determined from the condition that the average d.c. at the PD output is the same in the cases when PD transfer characteristics are $f(\phi)$ and $x \sin \phi$. With this equivalent PD, the first order loop equation is,

$$\frac{d\phi}{dt} = \Omega - AKx \sin \phi$$

and so, the average d.c. output of the PD is,

$$\overline{Ax \sin \phi} = A \cdot \frac{x^2}{2(\Omega/Ax)}$$

Equating (8.39) and (8.42) one gets,

$$x = \sqrt{2 \langle f^2(\phi) \rangle}$$

For the sine-o-linear PD the value of $\langle f^2(\phi) \rangle$ is equal to 586 and hence $x$ is equal to 1.08. This means that the new PLL is nearly equivalent to a conventional PLL so far acquisition behaviour is concerned. Specifically speaking the acquisition range of the new PLL will be 1.08 times that of a conventional PLL. The validity of this approach can be tested by the following way. It is well known [VIII-11] that the normalised acquisition range of second order PLL with generalised PD is equal to $2 \sqrt{F_0 F_2}$ where $F_0$ is the high frequency gain of the loop filter and $F_2$ is the averaged value of the square of the PD transfer function. Approximations...
leading to this result are \( F_0 \neq 0 \) but \( F_0 \ll 1 \). Now considering that \( F_2 = 0.5 \) for a sinusoidal PD and \( F_2 = 0.586 \) for a sine-cosine linear PD, the ratio of the normalised acquisition ranges of the new PLL to the conventional PLL with identical loop filter will be,

\[
\frac{\text{Normalised acquisition range of the new PLL}}{\text{Normalised acquisition range of the conventional PLL}} = \sqrt{\frac{0.586}{0.5}}
\]

\[= 1.08 \quad \ldots \,(8.43)\]

which has been stated earlier. This aspect has been studied experimentally and the results are in support of the above observations.

Thus the acquisition properties of the new PLL with noisy and noisy-fading input signals can be obtained directly from the studies on the acquisition behaviour of a conventional PLL by putting \((x \sin \phi)\) in place of \(\sin \phi\).

8.7. Partial Coherent PSK Detection Performance:

Usually in the partial coherent techniques of phase shift keyed (PSK) signal detection the information bearing signal is correlated with the regenerated carrier signal, the phase of which is imperfectly known, and then correlated output is filtered using a matched filter. It is of interest to know the performance of a correlator and matched filter type of receiver which uses the carrier signal regenerated by the new PLL using the sine-cosine linear PD. The pdf of phase error the carrier signal is given by (8.35). Fig. (8.7) shows the
Fig. 8.7: A block diagrammatic representation of a correlator and matched filter type receiver.
structure of the receiver in block diagram. The correlator output would be integrated and dumped over a period T, the bit period; the phase error of the regenerated carrier, \( \phi(t) \), is assumed to be nearly constant within period T. Then, referring to the literature [VIII-12], the probability of error in determining an 'one' bit in presence of noise for a fixed \( \phi \) will be,

\[
P_E(\phi) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{A^2 T}{N_0}} \cos \phi \right)
\]
or

\[
P_E(\phi) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{2E}{N_0}} \cos \phi \right)
\]  \hspace{1cm} (8.44)

where \( E = \frac{1}{2} A^2 T \) is the bit energy, \( \frac{E}{N_0} \) is the bit-to-noise power ratio and \( \text{erfc}(x) \) is usually defined as,

\[
\text{erfc}(x) = 1 - \text{erf}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy
\]  \hspace{1cm} (8.45)

Now knowing the pdf of \( \phi \) process one can obtain the error probability in demodulating binary PSK signals as a function of bit-to-noise power ratio \( (E/N_0) \) for different input CMRs at the carrier tracking loop by the relation,

\[
P_E = \int_{-\pi}^{\pi} P_E(\phi) p(\phi) d\phi
\]  \hspace{1cm} (8.46)

The same can be calculated for a system using a conventional PLL for carrier reconstruction with identical input CMRs. Comparison of the two error probabilities for the same bit-to-noise power ratio shows that the new system has got better
Fig. 8.8(a): Error probability of PSK reception with the carrier signal generated by the phase-locked loop having sinusoidal PD.
Figure 8(6): Error probability of PSK reception with the carrier signal generated by the phase locked loop having sin-e-linear type phase detector.
performance than the conventional system. Fig. 8.8 shows the computed variation of $P_E$ with $R$ for different system input CNRs with conventional as well as new carrier reconstructing loop.

8.8. Experiment:

The performance of the new PLL incorporating the sine-o-linear PD has been experimentally studied. To realize the sine-o-linear PD, a sinusoidal PD and a linear (sawtooth) PD have been constructed in the usual way. An analog multiplier (1495) acts as a sinusoidal PD and the sawtooth PD has been deviced by using two monoshots (74121) and a R-S Flip-flop (using 7400) by the method narrated in chapter II. First the maximum gain of the two PDs have been adjusted to the same value at the phase difference $\pi/2$ and $\pi$, respectively, for sinusoidal and sawtooth PD. Then using the two PDs in parallel the output voltages have been added after passing through identical low pass loop filters and then properly attenuated to give the maximum gain as before at the phase difference $\pi$ between the input signal and the reference. This error signal has then been used to control the loop VCO. The block diagram of the experimental set up and the actual circuit arrangement have been shown in Fig. 8.9 and Fig. 8.10. The performance of the new loop has been experimentally studied in the acquisition mode of operation with noise free and noisy input signal. At the same time the performances of PLLs with
Fig. 8.9: Experimental block diagram of the set up used to study the properties of a PLL with sine-o-linear PD.
Fig. 8.19: Experimental diagram of sawtooth PD. R_T and C_T are the timing resistor and capacitor of the monostable. IC pin numbers have been given inside respective blocks.
sinusoidal and sawtooth PDs have been studied under similar conditions. The experimental results are in fair agreement with the theoretical anticipations.

Experimentally the locking ranges of the new PLL with noise free input signal have been measured for different values of the high frequency gain \( F_0 \) of the loop filter, keeping the filter constant unchanged. The ranges have been normalised in terms of the same for the first order PLL \( (F_0=1) \). The identical measurements have been carried out for a conventional PLL with sinusoidal PD. Now theoretical analysis tells that the ratio of the normalised locking ranges for the new PLL and the conventional PLL should be 1.08 (vide section 3.6) when \( F_0 \) is very small. The following table (Table 3.1) gives the results of the experiment which support the theoretical observation.

<table>
<thead>
<tr>
<th>High frequency gain of the loop filter* ( (F_0) )</th>
<th>Normalised locking range of the new PLL ( (X) )</th>
<th>Normalised locking range of the conventional PLL ( (Y) )</th>
<th>Ratio ( (X/Y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.54</td>
<td>.8101</td>
<td>.8095</td>
<td>1</td>
</tr>
<tr>
<td>.44</td>
<td>.7731</td>
<td>.7748</td>
<td>1</td>
</tr>
<tr>
<td>.2</td>
<td>.7453</td>
<td>.7359</td>
<td>1.012</td>
</tr>
<tr>
<td>.066</td>
<td>.7175</td>
<td>.6796</td>
<td>1.055</td>
</tr>
</tbody>
</table>

*Time constant of the loop filter is fixed.
Thus as $F_o$ tends to small values, the ratio $(X/Y)$ approaches the theoretical value, 1.08.

With noisy input signal the measurement of locking ranges of the new PLL, the PLL with sinusoidal PD and the PLL with sawtooth PD have been carried out for different values of the input signal to noise power ratio. The experimental results have been depicted in Fig. 8.11. Here also the relative superiority of the new system has been established.

8.9. Conclusion:

A new PD has been proposed which can be designed very easily using conventional PDs. Actually the new PD is a combination of two well known PDs, viz., sinusoidal and linear (sawtooth) PD. The sine-o-linear PD has the merits of both of its constituent PDs up to certain extent. A new PLL structure has also been designed using the sine-o-linear PD. The performances of the new PLL have been analytically examined and experimentally verified in several modes of system operation. The superiority of the new PLL has been established by comparing its performances with those of a conventional PLL. To make the comparison logical, the overall loop gains of the two systems in the tracking mode have been kept identical. The results obtained in the study can be summarised as follows:

(i) The open loop noise performance of the sine-o-linear PD is better than that of a sinusoidal PD, because its phase
transfer characteristic is linear over a range larger than the linear range of operation of a sinusoidal PD. Though the PD gain for the sine-o-linear one is less than the same for a sinusoidal PD when phase error is small, but for larger phase error (greater than π/2) the new PD has higher gain.

(ii) The noise bandwidth of the PLL with sine-o-linear PD is less than the same for a conventional PD. This ensures that the new PLL will have better noise filtering capability.

(iii) The loop phase error variance of the PLL is less than that of the conventional PLL, as has been analytically established in the case/system operation with low to moderately high noise strength.

(iv) The acquisition performance of the new PLL is slightly better than that of the conventional systems. To establish this property of the new PLL analytically, a method of findings the acquisition range of a PLL with general PD has been put forward. The result of this analysis is exactly same as that of the other studies reported elsewhere.

(v) The performance of quasi-coherent PSK reception systems using the carrier reconstructed by the new PLL has been studied. This also indicates the better performance of the new PLL system than the conventional one.

(vi) Experimental verification of the performance of the new PLL has been made and the observed results support theoretical anticipations.
It should be mentioned here that the analytical study reported in this chapter assumes that the input noise to the system is not high. If it be so, the quasilinear approximation made in the text would not hold good and one has to look for more general analytical tool for studying the system performance. The performance of the new system in the face of noisy fading signals has not been studied; but it can be noted from the analysis made with additive noise that the new system would have better performance with noisy fading signal than the conventional ones in the same situation. The idea of getting a new PD by combining two different PDs can be extended; one can design a PD with a number of usual PDs combining them with different suitable weight factors. This will complicate the system design no doubt, but, one can have better noise performance, acquisition behaviour with the new system.