CHAPTER 4

CALCULATIONS INCLUDING ISOBAR DEGREES OF FREEDOM
A. Role of $\Delta$ - Isobar:

It is a long standing question whether two nucleon potentials that are consistent with scattering data and deuteron properties, can quantitatively account for the nuclear saturation $^{67,68}$. The Brueckner-Bethe-Goldstone (BBG) theory predicts the saturation point of nuclear matter for any nucleon-nucleon potential $^{16,32,60}$. The lowest order calculations for various potentials show that the calculated saturation points lie on a narrow band $^{69}$, called the Coester band or the saturation band. The saturation band misses the empirical region which extends from $-15$ to $-17$ MeV in energy and from 1.3 to 1.45 fm$^{-1}$ in Fermi momentum. This discrepancy may arise due to various reasons: (i) the higher order terms, viz. three and four-hole line diagrams, should be taken into account to improve convergence of the BBG theory, which can be done by a selective summation of higher order terms considering a different choice of single particle spectrum $^{19,70,71}$ other than the standard gap choice, (ii) assuming the BBG theory with the standard choice of single particle spectrum to be correct, the evaluation of three-hole-line and four-hole-line terms may not be sufficiently accurate $^{16,72,73}$, (iii) the relativistic effects should be considered but a detailed analysis showed them to be negligible $^{74}$, (iv) two-body potential models can not qualitatively account for nuclear saturation $^{65}$ and (v) finally, it may be inadequate to treat the nucleus as a system of point nucleons with no other explicit degrees of freedom. Non-nucleonic degrees of freedom might, therefore, be considered. One such non-nucleonic degree of freedom is the isobar degree of
freedom. Among the different isobar degrees of freedom, the 3,3 resonance \( \Delta \)-isobar having mass 1236 MeV, spin and isospin both 3/2 plays a dominant role because it is the lightest resonance and is the most strongly excited in pion-nucleon scattering.\(^{37}\) The importance of the isobar degrees of freedom has been pointed out in details by Green and his Co-workers\(^ {37,75-77} \). They have studied the influence of \( \Delta \)-isobar on beta-decay of triton, magnetic moments and charge form factors of triton and helium, three-body forces, nucleon-nucleon problem, and binding energies of nuclear matter and neutron matter. The hypothetical system of infinitely extended nuclear matter, because of its relatively large density, seems to require\(^ {56} \) the inclusion of mesonic and isobar degrees of freedom.

The \( \Delta \)-isobar plays a significant role\(^ {37} \) in the intermediate range attraction of the phenomenological nucleon-nucleon potential. In one boson exchange models\(^ {78} \), this intermediate attraction is obtained from the exchange of the fictitious \( \sigma \)-meson while in meson theoretic and dispersion theoretic models\(^ {1} \) it arises from 2\( \pi \)-exchange. Ordinarily nucleon-nucleon potentials are fitted to NN-phase shifts. Hence they implicitly include the effect of coupling to the nucleon-delta (ND) channel. This coupling produces a large attractive component in the usual \( ^{1}S_{0} \) nucleon-nucleon potential. This attractive component is, of course, exactly the same whether the potential is used for scattering or in nuclear matter. But when the ND channel is included explicitly as in the isobar model, the coupling to this channel is less effective in nuclear matter than in the scattering problem because of the Pauli effect and the dispersion effect. When the two-body reaction matrix is
computed in nuclear matter, the Pauli principle for the nucleons excludes certain NΔ intermediate states. This is the Pauli effect. The energy denominators for the intermediate NΔ states are larger in nuclear matter than in the scattering problem. This is the dispersion effect. Rough calculations of the Pauli and the dispersion effects, as function of the density, have been made by Green and Niskanen. They considered NΔ coupling in the 1S0 channel of the Reid soft core potential. For a Fermi momentum $k_F = 1.4 \text{ fm}^{-1}$, they found 5 MeV per particle less binding when NΔ coupling was included explicitly. The same effect was observed by Day and Coester, Dahlblom and Smulter, and Holinde and Machleidt.

Naturally Δ-isobar plays a very important role in the short-range dynamical nucleon-nucleon correlation. The Δ-isobar should be treated explicitly as it behaves as a free-particle in a many body system with no Pauli effect. We, therefore, treat the excited nucleon $\Delta(1236)$ as a stable elementary particle and assume a phenomenological interaction between Δ's and nucleons. The explicit inclusion of Δ-isobars as intermediate states in a nuclear matter calculation is done through coupled channels with a nucleon-nucleon to nucleon-isobar transition potential. The Hamiltonian is given by

$$H = \sum_{k} \frac{p_{k}^{2}}{2M} a_{k}^{+} a_{k} + \sum_{k} \left[ \frac{p_{k}^{2}}{2\Delta} + (\Delta-M)c^{2} \right] a_{k}^{+} a_{k} + \frac{1}{2} \sum_{klmn} \langle kl | V_{1} | mn \rangle a_{k}^{+} a_{l}^{+} a_{n} a_{m} \right]$$

$$V_{1} | mn > a_{k}^{+} a_{l}^{+} a_{n} a_{m} + \sum_{klmn} \langle kl | V_{2} | mn \rangle a_{k}^{+} a_{l}^{+} a_{n} a_{m}$$

$$+ \sum_{klmn} \langle kl | V_{3} | mn \rangle a_{k}^{+} a_{l}^{+} a_{n} a_{m} \text{ and hermitian conjugate}$$

......[4.1]
Here $M$ is the nucleon mass and $\Delta$ is the mass of the $\Delta$-isobar. The operators $a_k$ and $a_k^+$ destroy and create nucleons of momentum $k$, and the $a_k$ and $a_k^+$ are analogous operators for the $\Delta$. The first two terms of the above equation (4.1) are the kinetic energies of nucleons and $\Delta$'s respectively, along with the excitation energy $(\Delta - M)c^2$ of each $\Delta$ relative to the nucleon. $V_1$, $V_2$ and $V_3$ are the two-body potentials causing transitions of the types $\text{NN} \leftrightarrow \text{NN}$, $\text{NN} \leftrightarrow \text{NA}$ and $\text{NA} \leftrightarrow \text{NA}$ respectively. The couplings $\text{NA} \leftrightarrow \Delta \Delta$ and $\text{NN} \leftrightarrow \Delta \Delta$ are neglected because the mass of the $\Delta \Delta$-channel is 300 MeV greater than that of the $\text{NA}$ channel. However, this does not imply that coupling to the $\Delta \Delta$-channel will have a negligible effect on the properties of nuclear matter. But by including only the $\text{NA}$-channel, which is the single most important isobar channel, we expect to find out whether the inclusion of such channels can improve the saturation properties of nuclear matter. The $\Delta \Delta$-channel and all others are, of course, implicitly included in the model through the requirement that the NN phase shifts be fitted.

The coupling potential $V_2$ must conserve two-body angular momentum $J$, parity $P$ and isospin $T$. This implies that the $^1S_0$(NN) channel is coupled only to the $^5D_0$(NA) channel. Since the $^3S_1$ and $^1P_1$ NN channels have $T = 0$, they can not couple to the NA channel which has $T = 1$ or 2. The $^3P$(NN) channels have $T = 1$ and can couple to NA channels. A table given below shows the NN and NA coupled states for $J \leq 2$. 

We have studied in detail the couplings $^1S_0 \leftrightarrow ^5D_0$, $^1D_2 \leftrightarrow ^5S_2$, and $^3P_1 \leftrightarrow ^5P_1$ only. Other couplings are assumed to give very small effects.

The general procedure for including $\Delta$ intermediate states is to choose a nucleon-nucleon potential and then to use coupled channel equations for the NN and N$\Delta$ systems with a NN $\rightarrow$ N$\Delta$ transition potential. The NN-potentials we consider for calculation are the Paris potential and the Reid soft core (RSC) potential. The results with the RSC potential are already well-discussed in the literature. We, therefore, consider RSC results as a standard for comparing the changes produced when isobars are allowed to be formed virtually.

The mass difference of the isobar and the nucleon would enter into the potential range in the static approximation, if the N$\Delta$ intermediate state be on-mass-shell. This would lead to a complex transition potential of infinite range. But Brown
and Riska suggested to take the transition potential of pion range, essentially neglecting the NΔ mass difference, on an argument based on the virtual and off-mass-shell nature of the NΔ intermediate state. This amounts to consider only the longest range box diagrams of time ordered two-pion exchange (Fig.35). The transition potential of Green and Niskanen or Day and Coester are basically of this type and are similar in nature. The Green-Niskanen potential consists of a part involving pion exchange. The ρ-exchange provides a cut off whereas the short range cut off is of monopole type. The use of the transition potential of pion range has also been justified by Smith and Pandharipande. They showed that in free scattering, there is cancellation among 2π exchange crossed diagrams involving NN, NΔ, and ΔΔ intermediate states. However, the validity of this cancellation in higher orders or in presence of the Pauli effect in the many body system has been questioned by Durse et al.32 and Green.37 They suggested to consider all possible time orderings of the unstretched NΔ box diagrams which means that the pion exchange potential should contain terms having ranges equal to or less than the pion range (Fig.36). These diagrams were summed by Durse et al.32 considering a non-local type transition potential. A local equivalent of this potential, along with the ρ-exchange, was given by Green, Niskanen and Sainio.79 It is known that the local equivalent potentials produce essentially the same results as non-local potentials in low energy systems like nuclear matter. We, therefore, use the Green-Niskanen-Sainio transition potential in our calculation. This potential also contains the short range
cut off. An explicit form of the potential is given in the Appendix-F.

Green, Niskanen and Sainio\textsuperscript{79} with their transition potential have found a reduction in binding energy of about 5 MeV per particle at $k_F = 1.4$ fm$^{-1}$. This effect was found using both the Pandharipande method and the Brueckner method with quantitative agreement between the two. However, they neglected the dependence of the Pauli effect on the relative momentum of the two interacting nucleons. Dahlblom and Smulter\textsuperscript{41}, in a similar approach, calculated the loss in binding for the coupling $^1S_0(NN) \leftrightarrow ^5D_0(N\Delta)$ using the Bethe-Brueckner theory. They reported a reduction in binding energy of about 4.7 MeV per particle at $k_F = 1.4$ fm$^{-1}$. They considered different angle averaged Pauli operators for NN and N\Delta interactions. Day and Coester\textsuperscript{40} with a slightly different Pauli term in the lowest order Brueckner theory obtained a reduction in binding energy per particle of about 3.67 MeV in the $^1S_0(NN) \leftrightarrow ^5D_0(N\Delta)$ coupling, of about 1.71 MeV in the $^3P_1(NN) \leftrightarrow ^5P_1(N\Delta)$ coupling and of about 0.8 MeV in the $^1D_2(NN) \leftrightarrow ^5S_2(N\Delta)$ coupling at $k_F = 1.4$ fm$^{-1}$. In all the above calculations Reid soft core potential was used as the NN potential. Molinde and Machleidt\textsuperscript{42} using their own one-boson-exchange potential carried out a Brueckner calculation and have found a substantial reduction in binding. They also found that this reduction in binding shifts the saturation point to a lower density. Dey et al\textsuperscript{25} with the Tourreil-Rouben-Sprung potential\textsuperscript{36} found a decrease in binding of about 3.27 MeV.
per particle at \( k_F = 1.3 \text{ fm}^{-1} \).

Once the transition potential is known it is straightforward to calculate the binding energy of nuclear matter. A very appealing way of calculating the binding energy per particle was given by Jeukenne, Lejeune and Mahaux\(^{19}\) and has been used by us\(^{84}\) to calculate the optical potential from the Paris potential for nucleon projectiles. The method advocates calculation of particle spectrum on the same footing as the hole spectrum in a self-consistent manner. This is closer in spirit to finite nuclei calculations and is more realistic. The method is appealing because it is simple and gives information about optical model parameters and, therefore, it can be used to find the self-energy of isobars in nuclear matter. The study of isobar optical potential is of basic interest in pion physics.

In this chapter we have calculated the self-energy of isobars which has been defined in the same way as the single particle potential. We have also studied the effect of \( \Delta \)-isobar in nuclear matter binding and saturation with explicit inclusion of \( \Delta \)-isobar. We have considered three couplings:

\[ ^1S_0(NN) \leftrightarrow ^5D_0(N\Delta), \ ^1D_2(NN) \leftrightarrow ^5S_2(N\Delta) \] and \[ ^3P_1(NN) \leftrightarrow ^5P_1(N\Delta) \] for our calculations. The contribution to the nuclear binding energy from coupling in the \( ^3P_0(NN) \) channel is expected to be more than ten times smaller\(^{40}\) than from the \( ^3P_1(NN) \leftrightarrow ^5P_1(N\Delta) \) coupling. The coupling to \( N\Delta \) channels with \( L' = 3 \) should be less important than coupling to channels with \( L' = 1 \). This means that the coupling \( ^3P_1(NN) \leftrightarrow ^5F_1(N\Delta) \) can be
neglected. The same is true to the $^3P_2(NN) \leftrightarrow {}^5F_2(N\Delta)$ coupling. The coupling to the higher $L$ channel is expected to be small so that the $^3F_2(NN) \leftrightarrow {}^5P_2(N\Delta)$ coupling has also been neglected.

B. Isobar Self-Energy:

Treating the $\Delta$-isobar on the same footing as a particle the isobar potential in the continuous choice can be defined as

$$ U(\Delta) = \sum_{n<k_F} \langle \Delta n | G(W) | \Delta n \rangle \quad \Delta \ll k_F $$

$$ = \text{Re} \sum_{n<k_F} \langle \Delta n | G(E_{\Delta} + E_n) | \Delta n \rangle \quad \Delta > k_F $$

where $W$ is the starting energy and $E_{\Delta}$ and $E_n$ are particle energies.

In infinite nuclear matter, the single particle wavefunctions or orbitals, given by equation (3.1), are plane waves and have a continuous distribution in momentum space up to $k_F$ so that $\Sigma$ should be replaced by $\Sigma \frac{\Omega}{s_n t_n (2\pi)^3} \int d^3k_n \cdot$ Then the $\Delta$-particle potential is given by

$$ U(\Delta) = \langle \Delta | U | \Delta \rangle = \sum_{n<k_F} \langle \Delta n | G(W) | \Delta n \rangle $$
\[
\begin{align*}
= & \frac{\Sigma}{\Delta} s_{\Delta n} \frac{(2\pi)^3}{\Omega} C_{\eta}(P_{\bar{k}_o}; \bar{k}_o) \left| s_{\Delta n} \right> s_{\Delta n} \\
= & \frac{\Omega}{(2\pi)^3} \frac{(2\pi)^3}{\Sigma} s_{n_{\Delta n}} \int d^3k_n \left< W_{\Delta} \right| G(P_{\bar{k}_o}; \bar{k}_o) \left| W_{\Delta} \right> \left< W_{\Delta} \right> \quad \cdots \cdots (4.3)
\end{align*}
\]

where \( \left| W_{\Delta} \right> = \left| s_{n_{\Delta n}} \right> \) is the spin-isospin part of the wave function. The momenta \( \bar{P} \) and \( \bar{k}_o \) are the center of mass and relative momentum.

Using the partial wave expansion of \( G \), given by

\[
G(P; k_o, k_0) = \frac{2}{\pi} \frac{\hbar^2}{M} \sum_{n} \left< W_{\Delta} \right| G_{\alpha}(P; k_o, k_0) \left| W_{\Delta} \right> \left( y_\alpha^M(k_o) \right) \left| W_{\Delta} \right> \quad \cdots \cdots (4.4)
\]

where \( \tilde{y} \) is the operator in momentum, spin and isospin space, the expression for isobar self-energy is given by

\[
\begin{align*}
U(\Delta) & = \frac{2}{\pi} \frac{\hbar^2}{M} \sum_{\Delta n} \left< W_{\Delta} \right| G_{\alpha}(P; k_o, k_0) \left| W_{\Delta} \right> \left( \tilde{y}_\alpha^M(k_o) \right) \left| W_{\Delta} \right> \\
= & \frac{2}{\pi} \frac{\hbar^2}{M} \sum_{\Delta n} \left< W_{\Delta} \right| G_{\alpha}(P; k_o, k_0) \left| W_{\Delta} \right> \left( \tilde{y}_\alpha^M(k_o) \right) \left| W_{\Delta} \right> \quad \cdots \cdots (4.5)
\end{align*}
\]

We use the relation
\[
\left( \sum_{n} \langle T^{+} \Delta \Delta^{+} T \Delta^{+} n \mid T \Delta^{+} \rangle \right)^{2} = \frac{2T + 1}{2} \tag{4.6}
\]

and get
\[
U(k_{\Delta}) = \frac{2^{1/4} k^{2}}{\pi M} \sum_{s_{n}} \sum_{\alpha L^{'}, M} \frac{2T + 1}{2} \int d^{3}k_{n} G_{LL'}^{\alpha}(P; k_{o}, k_{o}) \cdot \sum_{M_{\Delta}, M_{s} s_{n}} \langle JM \mid L M_{\Delta} \rangle \langle LM_{\Delta} \mid L M_{s} s_{n} \rangle \langle JM \mid L M_{\Delta} \rangle \langle LM_{\Delta} \mid L M_{s} s_{n} \rangle \langle S \Delta s_{n} s_{n} \mid S_{M} \rangle \tag{4.8}
\]

Expanding the above equation in terms of Legendre function and Clebsch-Gordan co-efficients, we have
\[
U(k_{\Delta}) = \frac{2^{1/4} k^{2}}{\pi M} \sum_{s_{n}} \sum_{\alpha L^{'}, M} \frac{2T + 1}{2} \int d^{3}k_{n} G_{LL'}^{\alpha}(P; k_{o}, k_{o}) \cdot \sum_{M_{\Delta}, M_{s} s_{n}} \langle JM \mid L M_{\Delta} \rangle \langle LM_{\Delta} \mid L M_{s} s_{n} \rangle \langle S \Delta s_{n} s_{n} \mid S_{M} \rangle \tag{4.8}
\]

Since the operator for spin flip leaves the Slater determinant of occupied states unchanged, \(U(k_{\Delta})\) commutes with the spin flip operator. Thus introducing a sum over \(s\) with a factor \(\frac{1}{4}\), we get from equation (4.8)
\[
U(k_{\Delta}) = \frac{2^{1/4} k^{2}}{\pi M} \sum_{s_{n}} \sum_{\alpha L^{'}, M} \frac{1}{4} \cdot \frac{2T + 1}{2} \int d^{3}k_{n} G_{LL'}^{\alpha}(P; k_{o}, k_{o}) \cdot \sum_{M_{\Delta}, M_{s} s_{n}} \langle JM \mid L M_{\Delta} \rangle \langle LM_{\Delta} \mid L M_{s} s_{n} \rangle \langle S \Delta s_{n} s_{n} \mid S_{M} \rangle \tag{4.9}
\]
Using the orthonormality of the Clebsch--Gordan co-efficients and the relations

\[ \Sigma < L M_L S_L M_s \mid J M > < J M \mid L' M_L S_L M_s > = \frac{2J + 1}{2L + 1} \delta_{LL'} \]  \hspace{1cm} (4.10)

and

\[ \Sigma Y_{LM_L}^*(k_o) Y_{LM_L}(k_o) = \frac{2L + 1}{4\pi} \]  \hspace{1cm} (4.11)

we get

\[ U(k_\Delta) = \frac{1}{16\pi^2} \frac{k^2}{M} \Sigma (2J + 1)(2T + 1) \int d^3k_n G_{LL}^\alpha (P; k_o, k_o) \]  \hspace{1cm} (4.12)

It is to be noted that \( \langle \Delta \mid U \mid \Delta \rangle \) only depends on the magnitude of \( \vec{k}_\Delta \). All spin, isospin and directional dependence have disappeared. The integration in equation (4.12) is over all \( k_n \) in the Fermi sea. We may change the integral over \( k_n \) to one over center-of-mass and relative momentum by the relation

\[ \int d^3k_n = \oint \delta k_o dK d\phi \]  \hspace{1cm} (4.13)

where \( K = \frac{P}{2} \) and \( \phi \) is the azimuthal angle between \( \vec{k}_\Delta \) and \( \vec{k}_o \).

The Jacobian \( \oint \) can be derived from

\[ \oint = \begin{vmatrix} \delta k_nx/ \delta k_o & \delta k_nx/ \delta K & \delta k_nx/ \delta \phi \\ \delta k_ny/ \delta k_o & \delta k_ny/ \delta K & \delta k_ny/ \delta \phi \\ \delta k_nz/ \delta k_o & \delta k_nz/ \delta K & \delta k_nz/ \delta \phi \end{vmatrix} \]  \hspace{1cm} (4.14)
for
\[ k_{nx} = \frac{f_M}{f_\Delta} k_\Delta - \frac{1}{f_\Delta} k_0 \sin \theta \cos \phi \]
\[ k_{ny} = \frac{f_M}{f_\Delta} k_\Delta - \frac{1}{f_\Delta} k_0 \sin \theta \sin \phi \]
\[ k_{nz} = \frac{f_M}{f_\Delta} k_\Delta - \frac{1}{f_\Delta} k_0 \cos \theta \]

\[ \text{where } f_M = \frac{M}{M + \Delta}, \quad f_\Delta = \frac{\Delta}{M + \Delta}, \]
\[ \cos \theta = \frac{k_\Delta^2 + k_0^2 - 4f^2_\Delta k^2}{2k_\Delta k} \]

\[ \text{and, } M \text{ and } \Delta \text{ are the nucleon mass and the isobar mass.} \]

From relations (4.14) to (4.16), we finally get the relation (4.13) as

\[ \int d^3k_n = \frac{8\pi}{f_\Delta k_\Delta} \int k_0 dk_0 \int K dK \]

We can verify that the limits of integration are

\[ \left| \frac{k_\Delta - k_0}{2f_\Delta} \right| \leq K < \frac{k_\Delta + k_0}{2f_\Delta} \quad \text{for } k_0 \leq f_M k_\Delta - f_\Delta k_F \]
\[ \left| \frac{k_\Delta - k_0}{2f_\Delta} \right| \leq K < \sqrt{\frac{k_F^2}{4f_\Delta} + (1 - \frac{1}{f_M}) \left( \frac{k_0 - f_M k_\Delta^2}{4f_\Delta^2} \right)} \]

\[ \text{for } f_M k_\Delta - f_\Delta k_F \leq k_0 \leq f_M k_\Delta + f_\Delta k_F \]

Therefore the full expression of \( U(k_\Delta) \) becomes
\[ U(\Delta_k) = \frac{1}{2\pi f_{\Delta}} \frac{k^2}{M} \sum_{2J + 1} (2J + 1)(2T + 1) \int k_0 dk_0 \]

\[ K_{\text{lim}} \int K dk \, G^\alpha_{\text{LL}}(K; k_0, k_0) + \int k_0 dk_0 \int K dk \, G^\alpha_{\text{LL}}(K; k_0, k_0) \]

\[ \frac{f_{\Delta}^k - f_{\Delta}^k F}{f_{\Delta}^M - f_{\Delta}^F} K_{\text{max}} \]

where

\[ K_{\text{min}} = \left| \frac{k^\Delta - k_0}{2f^\Delta} \right| \quad , \quad K_{\text{lim}} = \frac{k^\Delta + k_0}{2f^\Delta} \]

\[ K_{\text{max}} = \sqrt{\frac{k_F^2}{4f_M^2} + (1 - \frac{1}{f_M^2}) \left( \frac{k_0^2 - f_{\Delta}^k k^2}{4f^2_{\Delta}} \right)} \]

Now to reduce the dimension of the integral and the number of G-matrix elements involved in the computation of \( U \), we make the approximation that

\[ G^\alpha_{\text{LL}}(K; k_0, k_0) \approx G^\alpha_{\text{LL}}(K_{\text{av}}; k_0, k_0) \quad \cdots \cdots \cdot (4.19) \]

where \( K_{\text{av}} \) is the average value of \( K \) and is taken to be the root mean square value of \( K \) for two particles with the constraints that one of the particles has momentum \( k^\Delta \) and the relative momentum has magnitude \( k_0 \).

Then the expression (4.18) for \( U(k^\Delta) \) becomes
\[
U(k_\Delta) = \frac{1}{8\pi f_\Delta^3} \cdot \frac{k_\Delta^2}{M} \sum (2J + 1)(2T + 1) \int_0^{f_{M\Delta}k - f_{\Delta}k_F} dk_0 k_0^2
\]

\[
G_\alpha^\alpha(K_{av}, k_o, k_0) + \frac{1}{4f_{M\Delta}} \int_{f_{M\Delta}k - f_{\Delta}k_F}^{f_{M\Delta}k - f_{\Delta}k_F} dk_0 k_0 (f_{K_{av}}^2 k_0^2 - k_0^2 - f_{M\Delta}^2 k_0^2
\]

\[
+ 2f_{M\Delta}k_0^2) G_\alpha^\alpha(K_{av}, k_o, k_0)
\]

\[(4.20)\]

C. Numerical Procedure:

The nucleon-nucleon potential considered is the Paris potential\(^1\) and the transition potential is the Green-Niskanen-Sainio potential\(^7^9\). The potential \(V_1\) is taken in the form

\[
V_1 = V(\text{Paris}) + \frac{V_2^2}{\Delta E(k^2)}
\]

\[(4.21)\]

where

\[
V_2 = V(\text{GNS})
\]

The second term in \(V_1\) is needed to avoid double counting since the Paris potential already contains the intermediate attraction needed to fit the \(^1S_0\) state phase-shift. The introduction of \(V_2\) via the coupled differential equations gives a further attraction which is removed in an average way by adding a term \(V_2^2 / \Delta E(k^2)\). The energy denominator is momentum dependent.
and is adjusted for each value of $E_{\text{lab}}$ to ensure that the same NN phase shift is obtained as with the Paris potential. The form of $V_3$ is quite uncertain since it is a quantity that cannot be observed directly. Fortunately in low energy nuclear physics the form of $V_3$ is relatively unimportant and may be taken to be same as $V_1$.

After obtaining the potential matrix elements in this way, the $G$ matrix elements can be obtained by solving the Brueckner-Bethe-Goldstone equation for $G$-matrix in the same procedure discussed in Chapter-3. However, the angle-averaged Pauli operator $\overline{Q}$ and energy denominator $e$ differ from those used for the NN case.

The angle averaged Pauli operator for a NN channel coupled to a NA channel is given by

$$\overline{Q}_{\text{NA}}(k',p) = \begin{cases} 0 & \text{for } k' < k_F - f_M p \\ 1 & \text{for } k' > k_F - f_M p \\ \frac{1}{4f_M k' p} [k_F^2 - (k' + f_M p)^2] & \text{otherwise} \end{cases}$$

The relativistic $\Delta$-particle potential considered is

$$U(k_\Delta, k_F) = U_{\Delta 0}(k_F) + \Delta c^2 - \sqrt{h^2 c^2 k_\Delta^2 + \Delta^2 c^4} + \sqrt{h^2 c^2 k_\Delta^2 + \Delta* c^4} - \Delta* c^2$$

where $\Delta* = \Delta* (k_F)$ is the isobar effective mass.
The energy denominator then is given by

\[ e_{N\Lambda} = \sqrt{\frac{k^2}{2m}_M (k^2 + p^2) + (\Delta^* c^2)^2} + \sqrt{\frac{k^2}{2m}_\Lambda (k^2 + p^2) + M^* c^4} \\
- 2 \sqrt{\frac{k^2}{2m} c^2 (k^2 + \frac{p^2}{4}) + \Delta^* c^4} + U_\Lambda^0 (k_F) - U_0 (k_T) \\
- \Delta^* c^2 + M^* c^2 + (\Delta - M)c^2 \]

\[ \ldots \ldots \ldots (4.24) \]

Above equation (4.24) is obtained under the assumption

\[ f_{M^*} k^2 = f_\Lambda k^2 = \frac{1}{2} (k^2 + p^2) \]

\[ \ldots \ldots \ldots (4.25) \]

The parameters \( M^*, \Delta^*, U_0 \) and \( U_\Lambda^0 \) are to be determined self-consistently.

With these values of angle-averaged Pauli operator and energy denominator, G matrix elements are computed in the manner discussed previously. The self-consistent \( \Delta\)-particle potential is computed from equation (4.20). The G-matrix elements also give the nuclear matter binding energy, two-body correlation function, correlated two-body wave function, defect function and healing integral in the same way as discussed in the previous chapter.

D. Results and Discussions:

The effect of isobars in nuclear matter was expected to be large \(^{37,79}\), because of the Pauli and the dispersion effects.
These appear in the propagator term of the B3G equation

\[ G = V - VPG \]  \hspace{1cm} (4.26)

One can write an exact relation given by Bethe, Braniow and Petschek

\[ G_a - G_b = \Omega_b(V_b - V_a)\Omega_a + G_b(P_b - P_a)G_a \hspace{1cm} (4.27) \]

Where \( V_a \), \( G_a \) and \( \Omega_a \) refer respectively to the potential, the \( G \)-matrix and the wave operator matrices in momentum space between two plane wave states for the \( NN \) case and those with subscript \( b \) refer to the same quantities when the \( N\Delta \) channels are coupled to it. In case of Reid soft-core potential, the second term on the right hand side of equation (4.26) dominates and one can neglect \( (V_b - V_a) \) in equation (4.27). But Dey et al showed that for super soft core potentials like TRS this is not so, and the two terms on the right hand side of equation (4.27) may cancel to a large extent. For a super soft core potential the inclusion of isobar degrees of freedom thus produces a smaller change in binding energy for nuclear matter. In the present section we find that for the momentum dependent Paris potential the effect is even more pronounced when the energies are calculated through the HF method of Mahaux et al. This is gratifying in a sense since the pion-nucleon dynamics put into derivation of the potential already contains isobar effects. The fact that the separate treatment of isobar degrees of freedom in an explicit calculation does not cause
much change in the energy --- may be constructed as a pay off for the inclusion of the \( \pi N \) dynamics in the construction of the potential.

The self-consistent \( \Delta \)-particle energy is calculated upto \( k = 3 \text{ fm}^{-1} \) at four densities corresponding to the Fermi momenta \( k_F = 1.10, 1.36, 1.60 \) and \( 1.75 \text{ fm}^{-1} \). To do the self-consistent calculation, an initial value of \( U_{\Delta 0} \) and \( \Delta^* \) are chosen. With these values of \( U_{\Delta 0}, \Delta^* \) and the self-consistent values of \( U_0 \) and \( M^* \), obtained in Chapter-3, G-matrix elements are computed which then give the \( \Delta \)-particle spectrum and single particle spectrum by numerical solution of equations (4.20) and (3.43). From these values of \( U(k_{\Delta}) \) and \( U(k_m) \), new values of \( \Delta^*, U_{\Delta 0}, M^* \) and \( U_0 \) are obtained by fitting them to equations (4.23) and (3.44). These values now give new values of \( U(k_{\Delta}) \) and \( U(k_m) \) which again yield another set of values of \( \Delta^*, U_{\Delta 0}, M^* \) and \( U_0 \). The process is repeated until we get stable values of \( \Delta^*, U_{\Delta 0}, M^* \) and \( U_0 \). Tables 28-31 show the self-consistency at the four Fermi momenta mentioned for the Paris potential whereas Tables 32-35 show the same for the RSC potential. In Tables 36 we have made a comparison of the self-consistent parameters for the nucleons with and without explicit inclusion of the isobar degrees of freedom for both the potentials. It is found that the inclusion of isobar decreases the nucleon effective mass and the potential depth, the amount of decrease being greater for higher \( k_F \) values. For RSC potential the nucleon effective mass increases at lower \( k_F \) but decreases at higher \( k_F \) if the isobar is included.
Table -37 shows the isobar effective mass and potential depth for the Paris as well as RSC potentials. The isobar potential depth is shallow, only about 17 MeV at \( k_F = 1.36 \text{ fm}^{-1} \) for the Paris potential, compared to the nucleon potential depth which is about five times as large. The potential depth is attractive at all \( k_F \) for the Paris potential but for the RSC potential, it is attractive at lower \( k_F \) and becomes repulsive at higher \( k_F \). The isobar effective mass becomes lower, about 230 MeV at \( k_F = 1.36 \text{ fm}^{-1} \) for the Paris potential, than its free mass of 1236 MeV. Table-38 shows a detailed breakdown of the isobar self-energy at \( k_F = 1.36 \text{ fm}^{-1} \). It is found that almost all of the contribution to the self-energy comes from the \( ^1D_2 \rightarrow ^5S_2 \) coupling. This is not unexpected because this is the only channel in which the \( N\Delta \) is in a \( S\)-state. In Table-39, we display the effect of inclusion of isobar on the nucleon self-energy for the Paris potential at \( k_F = 1.36 \text{ fm}^{-1} \). The single particle potential becomes less attractive in the \( ^1S_0\)-channel and more attractive in the \( ^1D_2\)-channel but more repulsive in the \( ^3P_1\)-channel. This nature becomes reversed beyond \( k_m = k_F \).

The modified nucleon-self-energy including the isobar explicitly can be parametrised as a function of total energy \( E \) and density \( \rho \) in the form

\[
U = \sum_{i,j=1}^{3} a_{ij} \rho E^{j-1} \tag{4.28}
\]

\[
= A + BE + CE^2 \tag{4.29}
\]
Table 40 shows the co-efficients $a_{ij}$ whereas Table-41 shows the optical parameters $A, B, C$ together with the values for the NN-case.

The effect of explicit inclusion of isobar on the potential energy and binding energy per particle of the nuclear matter for the Paris potential as well as for the RSC potential are shown in Tables 42 and 43 respectively. We obtained a reduction of about 0.8 MeV in the $^{1}S_{0}$-channel, of about 0.12 MeV in the $^{1}D_{2}$-channel and of about 0.57 MeV in the $^{3}P_{1}$-channel at $k_F = 1.36$ fm$^{-1}$ for the Paris potential. The total reduction is about 1.49 MeV for the Paris potential and about 0.40 MeV for the RSC potential at $k_F = 1.36$ fm$^{-1}$. The reduction is density dependent. It is clear that in the continuous choice, the reduction in binding energy per particle is less for the RSC potential when isobar is included explicitly. The modification in wound integral for including isobar for the two potentials are presented in Tables 44 and 45 respectively.

The effect of isobar on the correlation function for the Paris potential in the $^{1}S_{0}$-, $^{1}D_{2}$- and $^{3}P_{1}$-channel are respectively shown in Figs.37-39. The saturation curve is shown in Fig.40. For RSC potential the saturation occurs at $k_F = 1.46$ fm$^{-1}$ corresponding to the binding energy per particle of about -17.70 MeV and for the Paris potential the saturation occurs at $k_F = 1.49$ fm$^{-1}$ corresponding to the binding energy per particle of -19.30 MeV. This is very near to the empirical value. Thus the inclusion of isobar shifts the saturation point towards the empirical value with a decrease in binding energy per particle of about 1.70 MeV for the Paris potential but for RSC potential, the saturation point remains unchanged.