Chapter-II

INTRODUCTION TO QUARKS AND QUANTUM CHROMODYNAMICS

2.1 Quarks:

The existence of quarks was first postulated by Gell-Mann and Zweig [1,2]. They proposed, on the basis of symmetry considerations, that all hadrons are composed of new elementary particles called quarks. The combinations of quarks which give the occupied representations of SU(3) are a quark-antiquark pair for meson multiplets and three quarks for baryon multiplets. The above representation involves three types of quarks - up (u), down(d) and strange (s). These are called the flavours of the quarks. Thus far six flavours have been proposed. Other three flavours are charm (c), top (t) and bottom (b). These six flavours can be grouped into three families in comparison with the three families of leptons. Out of these six flavours, five have been observed. The top quark is yet to be observed. The first family of quarks i.e. up and down quarks dominate the behaviour of nuclear physics. The other families of quarks play an important role in elementary particle physics.

One significant consequence of this scheme is that if
three quarks are to make up each baryon with a baryon number 1, then quarks themselves must have baryon number 1/3. From the formula relating charge to isospin and baryon number, this means that they must also have fractional electrical charges. Also, to ensure that baryons generated are fermions and the mesons are bosons, it is necessary to assign the spin of a quark to be 1/2. A summary of their properties is shown below.

<table>
<thead>
<tr>
<th>Flavour</th>
<th>Spin</th>
<th>B</th>
<th>Q</th>
<th>I</th>
<th>I_3</th>
<th>S</th>
<th>Y</th>
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</thead>
<tbody>
<tr>
<td>u</td>
<td>1/2</td>
<td>1/3</td>
<td>+2/3</td>
<td>1/2</td>
<td>1/2</td>
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<tr>
<td>v</td>
<td>1/2</td>
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<td>-1/3</td>
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<td>s</td>
<td>1/2</td>
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<td>0</td>
<td>0</td>
<td>-1</td>
<td>-2/3</td>
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<tr>
<td>c</td>
<td>1/2</td>
<td>1/3</td>
<td>+2/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
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<td>b</td>
<td>1/2</td>
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<td>0</td>
<td>1/3</td>
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<tr>
<td>t</td>
<td>1/2</td>
<td>1/3</td>
<td>+2/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/3</td>
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</tbody>
</table>

There is one difficulty to explain the baryon spectrum from three quark wave functions. A baryon with 1/2 integral spin is expected to be totally antisymmetric by Pauli exclusion.
principle. But in the ground state of $\Delta^{++}$ or $\Omega^-$, $\Psi_{\text{space}}$ and $\Psi_{\text{spin}}$ are symmetric. As the three quarks are identical so $\Psi_{\text{flavour}}$ is also symmetric. Therefore

$$\Psi_{\text{total}} = \Psi_{\text{space}} \cdot \Psi_{\text{spin}} \cdot \Psi_{\text{flavour}}$$

is not antisymmetric. One can either suppose that quarks do not obey Pauli statistics but instead obey a more complicated statistics called para statistics. Alternatively this can be overcome by introducing an additional degree of freedom or quantum number for the quarks called colour. The colour group which seems to do this job is in fact an SU(3) group and the three colours are designated by red (R), green (G) and blue (B), the three primary colours of the real world. The contradiction with Fermi statistics is removed if the baryon ground states are assumed to be antisymmetric colour singlet states i.e. colour white formed from the above three primary colours. The measured ratio

$$R = \frac{\sigma (e^+ e^- \rightarrow \text{hadrons})}{\sigma (e^+ e^- \rightarrow \mu^+ \mu^-)}$$

and the decay rate of the neutral pion into two photons strongly suggest that the quark colour takes three values.
There is evidence from deep inelastic scattering of electrons on hadrons that for large momentum transfer ($q^2$), u and d quarks are almost massless (less than 10 MeV) and the s quark has mass of about 150 MeV [12]. These are the current masses. It is important to note here that the concept of the quark mass is a dynamical one. The mass has to be understood as a function of $r$, the distance of the probe from its centre and such coordinate dependent mass $m(r)$ is called the running mass. It may be that by the time a probe is as far away as 0.1 fm. from the quark, the mass of the u,d quark as seen by the probe is about 300 MeV. It is sometimes useful to introduce constituent quark masses which incorporate phenomenologically the bulk of strong interaction effects. The connection between current and constituent masses comes through the confinement mechanism. The constituent masses are frequently used in nonrelativistic quark model to obtain a simple phenomenology of hadron masses and static properties.

2.2 Deep inelastic scattering

For completeness we discuss deep inelastic scattering in some detail. Initially it was not clear whether the quarks were just
synonyms for the carriers useful for the group theoretical classification of hadron families or whether they were new subnuclear particles. The deep inelastic scattering of electrons on nucleons was to a large extent responsible for our present conviction that we can determine experimentally the properties of the quarks though they can not be isolated and that we can study the quark structure of hadrons. As the electromagnetic interaction of the electron is explicitly calculable in QED, the electron-nucleon scattering can unambiguously be studied and interpreted in terms of the structure of the probed nucleon. The composite picture of a nucleon was established in the middle of sixties by electron nucleon scattering experiments. The nucleon form factors fell rapidly with increasing momentum transfer. In 1968, the concept of point like constituents emerged from the results of the first small inelastic e-p scattering experiments conducted at the Stanford Linear Accelerator Centre (SLAC) [13]. The SLAC experiments were the first of its kind to mark the beginning of high energy physics. It led to the parton model and hence the idea of point like quarks as the fundamental hadron objects and to the modern gauge theories. In the simple case of nonrelativistic elastic scattering of an electron on a spin zero target, the cross section is given by
\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} \left| F(q^2) \right|^2 \]

where \( q = (k - k') \) is the momentum transfer.

The form factor \( F(q^2) \) is related to the charge density \( \rho(r) \) through the Fourier transform

\[ F(q^2) = \int e^{iqr} \rho(r) \, d^3r \]

But the simple nonrelativistic approach can not be employed to probe the charge structure of hadrons when \( q^2 \) is very large. Then it becomes relativistic; however, the nonrelativistic aspects still prevail. In the more general process of elastic or inelastic electron proton scattering, square of the momentum transfer \( Q^2 = -q^2 \) and energy transfer \( \nu = E-E' \) become prominent. The differential scattering cross section for one photon exchange is given by the Rosenbuth formula
\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} \left[ \frac{g_E^2 + \frac{q^2}{4M^2}g_M^2}{1 + \frac{q}{2} \frac{2}{M} \left( \frac{q}{4M^2} \right)} + \frac{q^2}{2} g_M \tan^2 \theta/2 \right]
\]

where \(M\) is the mass of the proton, \(\theta\) is the electron scattering angle and \(\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}}\) is the differential cross section for the scattering of point particles, called Mott scattering. The form factors reflect the spatial charge (\(G_E\)) and magnetic moment (\(G_M\)) in the nucleon. However at large momentum transfers the elastic form factors are small and inelastic scattering is more probable. For inelastic processes

\[
q^2 = -Q^2 = 2M\nu - W^2 + M^2
\]

and

\[
\frac{d^2\sigma}{dE\,d\Omega} = \frac{4\,a^2E'}{Q^4} \left[ 2\,F_1(q^2,\nu)\sin^2(\theta/2) + F_2(q^2,\nu)\cos^2(\theta/2) \right]
\]

where \(\alpha\) is the fine structure constant and \(F_1\) and \(F_2\) are called the structure functions for the two polarization states of the mediating photon viz. longitudinal and transverse
respectively.

In the deep inelastic scattering process, the projectile loses a large amount of energy to the target much more than the separation energy of its constituents. The process in the lowest order of QED is shown in figure 4.1, where it is described by the exchange of one virtual photon. QED allows one to calculate unambiguously what happens at the leptonic vertex but the problem at the hadronic vertex can be studied in the inclusive scattering $l + N \to X$ with no reference to any particular hadronic final state i.e. the only relevant four vectors are $p$ and $q$ and the only Lorentz invariant quantities are $q^2$ and $p.q$ and $p^2(=m^2_n)$. The momentum and energy transfer can be measured from the initial and the final lepton energies, $E$ and $E'$ and the scattering angle

$$Q^2 = -q^2 = 4 E E' \sin^2 \theta/2$$

$$\nu = \frac{p.q}{M} = E - E'$$

The deep inelastic limit is $Q^2 \gg \nu \gg M$ where variables $\nu$ and $Q^2$
are independent and the variable $x = Q^2/2M\nu$ is finite.

The measurement of inclusive cross sections yields information about the structure functions $F_{1,2}$, which are the strong interaction quantities characterizing the response and hence the structure of the target nucleon to the electromagnetic probes. As $Q^2$ increases, one would normally expect a decrease of $F_{1,2}$ for a spatial extension of the target and a constant value for point-like structures. At the large momentum transfer the lepton-nucleon inelastic cross section displays scale invariant behaviour consistent with the simplest type of impulse approximation wherein the electrons scatters directly against point-like partons (quarks) of the target. Comparing with the QED formulae describing the scattering of an electron with an electrically charged particle of spin $1/2$, it is possible to derive the relationship between the structure functions

$$2xF_1(x) = F_2(x)$$

The experimental evidence is firmly in favour of spin $1/2$ partons. Another lesson to be learned by comparing the deep inelastic formula with the simple formula from QED is that the
structure functions essentially measure the distribution of electric charge within the nucleon which in turn supports the assignment of fractional electric charges of quarks.

2.3 Quantum Chromodynamics:

The gauge theory, which is responsible for the interaction between quarks, is Quantum Chromodynamics (QCD). It is a renormalizable theory, with the non-Abelian SU(3), called colour SU(3) as its gauge group. Quark fields are represented by the Dirac spinors

$$\psi_i = \begin{pmatrix} g_i^R \\ g_i^B \\ g_i^G \end{pmatrix}$$

with $i$ running over the flavour index and $R, B, G$ standing for the colour label. The quarks with different flavours have also different masses.

Let us introduce the QCD Lagrangian by comparing it with the familiar Lagrangian $\mathcal{L}_{\text{QED}}$ of quantum electrodynamics, which describes a charged fermion with field $\psi(x)$ interacting with the
electromagnetic field. The latter is written in terms of the Maxwell field tensor $F_{\mu\nu}$, which can be expressed in terms of the four-potential $A_\mu(x)$. The relativistically invariant form of $L_{\text{QED}}(\psi, A)$ is

$$
L_{\text{QED}}(\psi, A) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2} \bar{\psi}(x) \gamma_{\mu} [i \partial^\mu + e A^\mu(x)] \psi(x)
$$

$$
- m \bar{\psi}(x) \psi(x)
$$

where

$$
F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)
$$

Apart from its manifest relativistic invariance (i.e. it is Lorentz scalar), the most significant property is its symmetry under the local gauge group $U(1)$, i.e. under the following transformations for each point in space-time:

$$
\psi'(x) = e^{ie\theta(x)} \psi(x)
$$

$$
A'_\mu(x) = A_\mu(x) + \partial_\mu \theta(x)
$$
Under the above transformations neither $F_{\mu\nu}$ nor $\mathcal{L}$ is changed. This gauge principle determines many of the physical properties of electrodynamics, since the gauge field $A_\mu$ is explicitly a four-component object, whereas the Maxwell field only has two independent degrees of freedom. The above gauge transform can be used to remove one component by the appropriate choice of $\partial_\mu \theta$ and it turns out, in the free theory without electrons, that there is one more gauge constraint which reduces $A_\mu$ to two components. This principle also ensures that the electromagnetic field couples universally to all charged particles.

One further observation that can be made as we go from the free Dirac Lagrangian

$$\mathcal{L}_{\text{free}} (\psi) = \frac{1}{2} \bar{\psi} (\mathbf{x}) \left[ i \gamma_\mu \partial_\mu + m \right] \psi (\mathbf{x})$$

to the U(1) gauge invariant form, is that the ordinary derivative $\partial_\mu = \frac{\partial}{\partial x^\mu}$ is replaced by the so-called covariant derivative $(\partial_\mu + e A_\mu)$, which is precisely the way to couple the electromagnetic field to the electron in the equation.

The QCD Lagrangian is one in which we couple a gauge field to the colour charge of the quarks in the above gauge invariant way. However we now have an SU(3) algebra of charges, so that the above
gauge transformation will be replaced by the matrix transforms

\[ A_\mu^a(x) T^a \rightarrow A_\mu^a(x) T^a + U(x) \partial_\mu U^{-1}(x) \]

\[ \psi_i(x) + U_{ij}(x) \psi_j(x) \]

\[ U_{ij}(x) = \left[ e^{ig\lambda^a(x)} \right]_{ij} \]

where \( \lambda^a \) represents the Gell-Mann SU(3) matrices and the quarks are taken to be the lowest triplet representation labelled by colour indices \( i = 1,2,3 \) (or R,Y,B). The gauge fields transform like generators of the group, so they are labelled by an index \( a \) which takes eight values, one for each generator. The gauge invariant Maxwell field tensor has the form

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \]

and the Lagrangian is

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} f^{a}_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \bar{\psi}(x) \gamma_\mu D^\mu_{ij} \psi_j(x) + m \bar{\psi}_i(x) \psi_i(x) \]

where the covariant derivative \( D^\mu_{ij} = \partial_\mu + g A^a_\mu(x) \lambda_i^a \), \( g \)
being the strong interaction coupling constant or charge scale.

The field $A^{a}_{\mu}$ is responsible for the nuclear force between the quarks and it gives rise to their binding into hadronic systems. For this reason it is referred to as the gluon field and the corresponding exchange quanta as gluons, in analogy to photons. This theory has one essential fundamental property which distinguishes it from QED, namely its non-Abelian character. The slightly richer gauge principle requires the gluons to also interact with themselves, which means they also carry the fundamental strong interaction charge. As a consequence, QCD has quite different charge screening properties than QED and one can expect some different physical properties of QCD systems to emerge. We shall conclude this chapter by briefly describing the two most important properties attributed to QCD viz. asymptotic freedom and confinement.

2.4 Asymptotic freedom:

One of the most important properties of non-Abelian gauge theories is the fact that the effective strength
of the interaction between the elementary particles gets increasingly weaker as we restrict the process to smaller space-time domains (i.e. to short distances), which in practice corresponds to going to large momentum transfers. The Bjorken limit for deep inelastic electron proton scattering being a classic example i.e. when the momentum transferred by the probe is very high then the force between quarks is surprisingly weak and they behave rather like free particles. On the other hand no free quark has ever been observed, so it is sure that over long distances, the forces between quarks become increasingly strong. To explain how this comes about let us start from the electromagnetic force.

In classical physics the electrostatic force between two charged particles is given by Coulomb’s inverse square law. When the distance separating the two charged particles becomes small, then classical physics is no longer adequate and quantum mechanical effects must be taken into account. These quantum mechanical effects can be described as the polarization of the vacuum of the virtual electron positron pairs in the environment of an electric charge. In the region of an electron, say, the virtual positron is attracted towards it and the virtual
electron is repelled away from it. This leads to a cloud of virtual positive charge shielding the bare negative charge of the real electron (figure 2.1). The effect of this is that from a distance the negative charge is much reduced compared to its bare charge. This effect is known as the renormalization of the bare electric charge and can be calculated by evaluating the probabilities of the various quantum mechanical processes as shown in figure 2.2. The change in the effective value of the electric charge with distance is shown in figure 2.3.

Like QED, QCD postulates field particles, which mediate interactions. Coloured quarks interact through the exchange of entities called gluons, just as charged particles interact through photons. Whereas QED recognizes only one kind of photon, QCD admits eight kinds of gluons. In contrast to the photon of QED, which do not alter the charge of the interacting particles, the emission or absorption of a gluon can change a quark's colour. The mediating gluon is itself coloured bearing both a colour and an anticolour. The fact that the gluons are colour charged, in contrast to the electrically neutral photons of QED, accounts for the different behaviours over distance of the electromagnetic and the strong interactions. Just
as the vacuum can be considered to be a sea of virtual electron-positron pairs, so too can it be considered as a sea of virtual quark-antiquark pairs and gluons. The bare colour charge on a single quark may then be shielded by the polarization of this vacuum sea of virtual quarks, antiquarks and gluons. The resulting renormalization of the bare colour charge may be calculated by evaluating the corresponding probabilities of occurrence of the various quantum mechanical processes, such as those shown in figure 2.4.

The essential new feature in the QCD case is the presence of gluon shielding, possibly because of gluon self-interactions. Whereas in QED, the single variety of electron-positron leads to a decrease in the effective electric charge compared to its bare charge, the presence of the gluon shielding effect in QCD provides a greater opposite effect and leads to an increase in the effective colour charge relative to the original bare charge. Conversely stated, the effective colour charge on a quark appears to decrease as the distance from which it is viewed decreases. This striking feature was described independently by Politzer at Harvard [14] and Gross and Wilczek at Princeton in 1973 [15]. It was shown that
the running coupling strength as a function of $Q^2$ is

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3} n_f) \log (Q^2/\Lambda^2)}$$

where $n_f$ is the number of flavours. The term related to $2/3n_f$ reflects the screening from virtual quark-antiquark pairs which is turned into antiscreening by gluonic polarization effects. $\Lambda$ is the QCD scale parameter which is difficult to determine precisely but seems to have the value around 200 Mev. Asymptotic freedom results in the limit $Q^2 \to \infty$ (or $r \to 0$) i.e. in this limit, constituents of the hadrons are moving freely inside the hadrons. Above equation also shows that the number of families $n_f$ cannot become larger than 16. So the property of asymptotic freedom means that in the high energy or short distance limit, perturbation theory becomes increasingly better approximations to QCD.

2.5 Confinement:

The free quark and gluon are not observed in nature.
This is called confinement. But confinement in QCD is more general than just having the quarks confined, since no states with a colour charge have been observed as free particles despite many careful searches. This led theorists to conjecture that quarks are permanently confined within the hadrons due to the very nature of chromodynamic forces. Just as the Abelian QED gives rise to Coulomb’s inverse square law, it may be that the non-Abelian nature of QCD gives rise to a confining force which does not decrease with increasing distance. It is assumed that QCD gives rise to confining forces that probably increase as the quarks are drawn apart which also follows from the asymptotic freedom. This phenomenon is known as the 'Infrared Slavery'. It is not yet known whether QCD gives rise to a permanent confinement or whether after a period of rising, the chromodynamic force tends to a constant strength or even decreases as the quarks are separated. If the force does eventually begin to drop off, then the quark will eventually be separable and confinement is only a temporary phenomenon. Though perturbation theory is valid in the asymptotic freedom domain as the forces are weak in this regime it can not be applied when confinement is attained as the size of the coupling constant then makes a little sense.