CHAPTER 2

FREGE'S THEORY OF SENSE AND REFERENCE
CHAPTER 8

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I. Frege's Theory

Frege approaches the distinction between sense and reference of any expression to explicate the distinction between trivial or analytic identity statements like \( a = a \) and informative or synthetic identity statements like \( a = b \). According to him, the relation of equality, his name for the relation of identity, holds neither between objects, nor between names. For him, the two statements \( a = a \) and \( a = b \) obviously differ from each other as to their cognitive value. Statements of the form \( a = a \) are apriori and analytic, whereas statements of the form \( a = b \) can not always be established apriori but are synthetic in character. Now, if identity statements express a relation between objects denoted by the expressions in question, \( a = b \) would not differ from \( a = a \). Both the statements would express the relation of a thing to itself, i.e. everything is identical with itself and with no other else.

So Frege once held that the statement \( a = b \) means that what is denoted by the sign 'a' is identical with what is denoted by the sign 'b', in other words, 'a' and 'b' denote the same object. The relation of equality would hold between names so far as they stand for some object, i.e. with reference

60. Translations from the Philosophical writings of Gottlob Frege, ed. Peter Geach & Maxblack, p. 57.
to the objects designated. But this relation between signs or names and objects is arbitrary. We may use any sign to designate any object. But then \(a = b\) would no longer refer to the subject matter, but only to its mode of designation and thereby would lack any cognitive value. It will merely mean that we have decided to use the two names 'a' and 'b' for the same object. If \(a = b\) is to state knowledge, the use of the names 'a' and 'b' should be justified by what is present in the object. Otherwise, the cognitive value of \(a = a\) and \(a = b\), if it is true, would be the same. According to Frege a difference can arise only if the difference between the signs corresponds to a difference in the mode of presentations of the object designated.

So, Frege proposes to distinguish between sense and reference of an expression or sign. By reference he means the object denoted by the sign and by sense he means the mode of presentation of the object designated by the expression. We know an individual object, when presented to us by an expression, in and through a certain aspect. This mode of knowing the object constitutes the sense of the expression used for that object. The expressions 'the morning star' and 'the evening star' have the same reference as they are two names for one and the same object, but they have the different senses as they reveal the different aspects of the same object. So, according to Frege, two names having the same reference may differ in sense.
By 'name' and 'sign' Frege understands any designation representing a proper name which has a definite object as its reference, but not a concept or a relation. He says that the sense of a proper name can be understood by everybody who knows the language which the name belongs to. But everybody grasps only a single aspect of the object referred to. When Venus is known as 'the morning star' it reveals one aspect of the planet and when the same one is known as 'the evening star' it reveals a different aspect. We can never have a comprehensive knowledge of the object.

According to Frege, to every sign there corresponds a definite sense and to that a definite reference or object, but to a given object there may correspond different signs. Again the same sense has different expressions in different languages or even in the same language. To every expression there must correspond a definite sense. But in natural languages we find deviations from this fact. Frege holds that every grammatically well-formed expression which represents a proper name must have a sense, but this does not guarantee that to the sense there also corresponds a referent. For example, the expression 'the least rapidly convergent series' has a sense but no reference, since for any given convergent series another less rapidly convergent series can be found. So, any grammatically well-formed expression may have a sense, though it may lack a reference.

According to Frege, the reference and sense of a sign are distinct from the associated idea. The reference of a sign
is an object perceivable by senses, but the idea of it is an internal image which arises from memories of sense impressions and acts which the observerer has performed. Such an idea is subjective, as one man's idea can not be shared by the other. In this respect it differs from the sense of a sign which may be common property of many and, therefore, is not subjective at all. He urges that the same sense is not always connected, even in the same man, with the same idea. Hence we can speak simply of 'the sense' without any allusion to the person bearing it. But to speak of an idea we have to point to its possessor. We may say that just as one man connects this idea and another that idea to a word, so also, one man connects this sense and another that sense to a word. Yet the difference lies in the mode of association. Two men can grasp the same sense, but they can not possess the same idea.

Hence the reference of a proper name is the object denoted by the name and the idea associated with the word is totally something subjective and between the two lies the sense which is neither subjective like the idea nor the object itself. Frege tries to explicate the relation between these three in analogy with somebody's observing the Moon through a telescope. The moon itself may be compared to the reference, as it is the object of observation. The real image is projected in the interior of the telescope and it can be compared with the sense. And it further produces a retinal image of the observer which may be compared with the idea. The optical image, though it depends upon the stand point of observation, is still objective,
since it can be used by several observers. But the retinal image of each observer belongs exclusively to the person concerned.

Thus, Frege points out, there are three levels of difference between expressions or whole sentences. They may differ in the associated idea, or in the sense but not in the reference, or in the reference as well. The difference concerning ideas is subjective, since a difference at this level may hold for one person but not for another. The difference between a translation and the original text represents the difference in the level of ideas.

Now, for Frege, a proper name expresses its sense and stands for its reference. Idealists or sceptics may object that we speak of the Moon as an object, but how do we know the name 'the Moon' has got any reference? Frege's answer to this objection is that when we say 'the Moon' we do not mean by it our idea of the Moon, nor are we content with the sense of the word, but we presuppose a reference. When we use any expression whatsoever, we presuppose a certain reference. Now we may be mistaken in our presupposition. But to speak of a reference of a sign, it is sufficient to point out that we actually referred to something in using the sign, provided such reference exists.

Frege speaks of sense and reference of a declarative sentence also. According to him any declarative sentence contains a thought. By thought Frege does not mean the
subjective act of thinking rather its objective content
which may be shared by the several thinkers using the sentence.
This thought can not be the reference of the sentence, but
constitutes its sense. If we replace one word of a sentence
by another which has the different sense, but same reference,
it would not affect the reference of the sentence. But in
such a case, the thought contained in the sentence varies.
For instance, the two sentences 'the morning star is a body
illuminated by sun' and 'The evening star is a body illuminated
by sun' contain the different thoughts. Whoever does not know
that the two expressions 'The morning star' and 'The evening
star' apply to the same object may hold one thought to be true
and the other false. Now if a sentence contains proper names
without reference, it affects its sense in the least. If we
are concerned only with the sense of the sentence, the thought,
it is irrelevant whether a part of the sentence has reference
or not. For example, in the sentence "Odysseus was set ashore
at Ithaca while sound asleep", whether the name 'Odysseus'
lacks reference or not the thought remains the same. But it
can be said that such a sentence has only sense but no reference,
since the name used in the sentence does not designate anything.
Such a sentence is neither true nor false. If this sentence
is to be true or false we are to ascribe to the name 'Odysseus'
a reference, because it is of the reference of the name that the
predicate is affirmed or denied. So far as we are concerned
at all about the reference of a part of the sentence, it
indicates that we expect a reference for the sentence itself.
we want every proper name to have not only a sense but also a reference, because we are concerned with the truth-value.

Thus, according to Frege, the reference of a sentence is its truth-value, i.e. either the True or the False. Hence, for him, every declarative sentence which is concerned with the reference of its words can be regarded as a proper name and its reference is either the True or the False, if it has any reference at all. These two objects are recognized by any body who judges something to be true. Thus the truth-values are regarded as objects.

It may be argued that the relation of the thought to the True is not like that of sense to reference, rather as that of subject to predicate. We may say that 'The thought, that 5 is a prime number, is true'. But it expresses the same thing what is expressed in the simple sentence '5 is a prime number'. That a declarative sentence is true is indicated by the form of the sentence. And if the sentence lacks its usual force, viz. in the mouth of an actor upon the stage, the sentence 'The thought that 5 is a prime number is true' contains only the same thought contained by the simple sentence '5 is a prime number'. So the relation of the thought to the True is not same as that of subject to predicate. According to Frege, subject and predicate are elements of thought and stand on the same level for knowledge. While combining subject and predicate we never pass from sense to reference or from a thought to its truth-value. He says that a truth-value can never be a part of
thought, as the Sun can not, since truth-value is not a sense but an object.

Since the reference of a sentence is its truth-value, if the part of a sentence is replaced by an expression having the same reference, its truth-value does not change. And the truth-value being the reference of a sentence, Frege says, all the true sentences have the True as their reference and all the false sentences have the False. As we can not be concerned only with the reference of a sentence, so also mere thought yields no knowledge. What is required is thought together with its reference. Thus judgements can be regarded as steps from a thought to a truth-value. It might also be said that judgements are distinctions of parts within truth-values. Frege uses the word 'part' in a special sense by calling the reference of a word part of the reference of the sentence, if the word itself is a part of the sentence.

It has been said above that the truth-value of a sentence is not altered when an expression is replaced by another which has the same reference. So also if the truth-value of a sentence is its reference, the truth-value of a sentence containing another sentence as its part should not change when the part is replaced by another sentence which has the same truth-value.

Thus, according to Frege, 'a=a' and 'a=b' have different cognitive values, because for the purpose of knowledge the sense
of the sentence i.e., the thought expressed is equally necessary as its truth-value which is its reference, 'a=b' means that the reference of 'a' and 'b' is the same and, therefore, 'a=b' and 'a=a' have the same truth-value. Nevertheless, the sense of 'b' may differ from the sense of 'a' and accordingly the sense of 'a=b' differs from that of 'a=a' i.e., they express two different thoughts. Hence the two sentences differ in respect of their cognitive value. Frege understands by 'judgement' the step from the thought to its truth-value. So we can also say that the judgements are different.

So far we have discussed about Frege's alleged distinction between sense and reference of a sign which he introduced in order to account for the distinction between analytic identity statement and synthetic identity statement. Frege's exposition has been interpreted variously by different thinkers and it has also received a number of charges against it of which most important one is Russell's which we shall come to discuss lately. Paul D. Wienpahl attacks Frege's view from a certain standpoint. He agrees with Frege in maintaining that the distinction between the sense and the referent of a word is necessary. But, according to Wienpahl, Frege's argument does not supply any specific meaning for 'sense'. He thinks that Frege's argument discounts a certain point and then takes sense to be a subsistent entity.

According to Frege 'a=a' and 'a=b' possess different cognitive values. On his view the relation of identity stated

in 'a=b' holds neither between the objects which 'a' and 'b' are names of, nor between the signs 'a' and 'b'. Because in the former case 'a=b' would say the same thing as 'a=a', provided 'a=b' is true, hence the two would have same cognitive value. And in the latter case the statement 'a=b' would not express any proper knowledge. It would only assert that two signs 'a' and 'b' are attached to the same object i.e. it would only refer to the mode of designation. So in order to explain the cognitive difference between 'a=a' and 'a=b' Frege introduces the notion of sense. According to him, the two signs 'a' and 'b', though stand for the same object, differ in sense, i.e. in the mode of presentation of the object. That is why 'a=b' differs from 'a=a'.

Wienpahl urges that Frege's argument as stated is incomplete. Frege maintains the view that identity holds not between the objects 'a' and 'b' refer to, but it seems that Frege must have changed his mind. Because from his argument it follows that the relation of identity is asserted to hold between that which 'a' and 'b' refer to in 'a=b'. Otherwise, the relation would have to hold between the modes of presentation of 'a' and 'b'. But since on our hypothesis the modes of presentation of 'a' and 'b' are different, the relation of identity can not hold between modes of presentation. In this case Frege's argument is fallacious. On the other hand, if the modes of presentation of 'a' and 'b' are thought to be same, inspite of the hypothesis, then 'a=b' would state the something as 'a=a'.

Wienpahl, furthermore, attacks Frege as he did not introduce the notion of metalanguage. It makes difficult to follow his argument. It appears that, for Frege, 'a=b' and 'a' and 'b' refer to the same object" are equivalent statements. Hence 'a=b' says something about 'a' and 'b' instead of a and b. In that case 'a=b' would express no proper knowledge. Hence the two statements "'a=b' and 'a' and 'b' refer to the same object" are not equivalents. The former is not a statement in the metalanguage and '=' is not a sign in the metalanguage, while the latter statement belongs to metalanguage. Moreover, according to wienpahl, the term 'mode of designation' is superfluous, since it means the samething as 'referent'.

Wienpahl intends to consider what the sense of a sign is. Frege has offered the example of 'point of intersection of b and c', where a, b, and c are lines connecting the vertices of a triangle with the mid points of the opposite sides, which have the same referent but different senses. wienpahl points out that the only observable distinction between the two are following: (i) the physical difference of shape etc., (ii) the difference of referents of their constituents, i.e. specially of 'a', 'b' and 'c'. As, not all signs are composed of parts which are signs themselves (ii) can be ommitted. Frege discounts the first distinction too, since for him if 'a' is distinguished from 'b' by means of its shape, not as sign, 'a=b' would have the same cognitive value as 'a=a', provided 'a=b' is true.

Wienpahl says that though Frege has provided a clear meaning for 'referent', he has not offered a clear meaning for
'sense'. As the sense of a sign is to be distinguished from objective elements i.e. referent; and subjective elements i.e. conception, it has been treated to be a subsistent entity in absence of a proper designation of it. According to Wienpahl this account is unsatisfactory. In first place it multiplies entities, and in second place it anhilates the possibility of verification, as verification, we know, depends on observation. On this account, the sense of a sign being a mysterious entity which is neither subjective nor the object itself does not admit of any observation; hence we can not verify that there is sense. Thus it renders to be an 'entity' like whiteness or chairness, "things" which can not be observed, but only be thought.

Wienpahl proposes to give an account of 'sense' which does not make sense a subsistent entity. He points out that Frege has overlooked an important distinction between 'a=a' and 'a=b' which can be a help to account for the cognitive difference between the two statements. He urges that the difference between 'a' and 'b' as object serves to explain the difference between 'a=a' and 'a=b' as to their cognitive values. The statement 'a=b' expresses proper knowledge, since the signs 'a' and 'b' can be employed without knowing that they stand for the same object. To know that a=b we have to look for something apart from the signs themselves. But to know that a=a we need not have to look for anything beside the signs. Hence 'a=a' is analytic, as only by examination of signs as objects we can grasp its truth. We do not have to consider that which the signs stand for. In other words, to prove 'a=a'
to be true we require no reference to empirical facts. Its truth is necessitated by the semantical rules of our language. But to prove that 'a=b' is true we have to take help of something besides semantical rules, i.e. we have to account for the objects which 'a' and 'b' are signs of. Thus, according to wienpahl, the difference in cognitive values between 'a=a' and 'a=b' consists in the difference between the physical properties of 'a' and 'b'. So he concludes that, as far as Frege's view that signs must have both a sense and a referent is concerned, the sense of a sign consists in the combination of its physical properties so far as it is an object which serves as a sign.

wienpahl claims that this account of sense as the combination of physical properties is consistent with Frege's position in the article 'Sin Und Bedeutung'. For instance, 'morning star' and 'Evening star' have the same referent but different senses, as their respective modes of presentation of the object are different. Now by the difference in modes of presentation of 'morning star' and 'evening star' he signifies the difference in the ways the two expressions present their object. And this amounts to say that 'morning star' and 'evening star' are different. According to wienpahl 'morning star' and 'evening star' are physical objects which serve as signs. And their difference consists in the difference in physical properties, as Joe's bed differs from Joe's coat. The word 'mode of presentation', wienpahl holds, suggests that the referent of 'sense' or 'mode of presentation' is the sign itself viz. the way of presenting the evening star is the sign.
'evening star'. The cognitive value of 'the evening star' is the same as the morning star' is different form that of 'the evening star is the same as the evening star', since 'evening star' is different from 'morning star'.

According to Frege, the phrase 'the least rapidly convergent series' has a sense but no referent. Wienpahl points out that the cognitive value of a complex sign depends not only on its sense, but also on the referents of its constituent parts. A simple sign which has never a referent is not a sign at all and in such a case we can not speak of its sense. According to Frege the sense of a sign is objective. On Wienpahl's view also it is objective, as it can be common to many minds in the same way that any physical objects can be common to many observers. It seems that, Wienpahl says, Frege and others have not taken into account the physical differences between signs because of the arbitrary character of signs.

Though it is true that any arbitrarily producible event or object can serve as a sign, once its relation to its referent is established the connection remains no longer arbitrary. The relation becomes necessary, since without it the object or event can not be a sign. So far as it is assumed that any other object can arbitrarily be substituted for 'a', the physical properties of 'a' considered simply as an object is not a matter of importance. But, Wienpahl, says, from the fact that any arbitrarily choosen object will do as a sign we can not infer that the physical properties of any given sign are inessential so far as it functions as sign.
Wienpahl thinks that another reason for the oversight of physical properties of sign is perhaps the influence of the phrase 'thinking of'. It may be objected to the view, on which 'a' and 'b' differs cognitively as regards their difference in physical properties, that it makes objects of our utterances or writings of them our thoughts. But, in fact, when we think a=b we think of something and signs are the devices by means of which we think of this something. Wienpahl suggests that if we notice that there is no difference in ordinary usage between 'I am thinking that a=b', 'I am thinking of a=b', and 'I am thinking a=b', we find that the use of the preposition 'of' has led us to look for something, namely, a thought, a sense etc. He holds that though we use 'think of' like 'have of' and 'look of', it does not imply that in the case of 'think of', 'of' must have an object. In supposing this we are misled by a grammatical form which can be eliminated without any distortion of the statement it is used in.

Again, the use of the phrase 'think of' may not lead to difficulties. For instance, in 'I am thinking of a', the object of 'of' is the sign 'a'. If we can find out the essential characteristic of signs that they have referents and can function as signs when the referents are not present in the situation, it does not appear to be absurd that the sign itself can be the object of 'thinking of'. In other words, Wienpahl says, 'I am thinking of' is a phrase in metalanguage. We fail to acknowledge this fact due to the common confusion of words and things.
Two objections may be raised against Wienpahl's account. We can have (i) different senses for the same sign, (ii) the same sense for different signs. In answer to these objections Wienpahl says that neither of the two can stand, if we regard the sense of a sign as the combination of its physical properties so far as it is an object. It is thought that words in different languages can mean the same thing. Wienpahl points out that in one sense of 'mean' viz. 'refer to' it is true, but in another sense of 'mean' viz. 'sense' it can not be. It follows that the same word can have two different referents. If it is supposed that in one sense of 'mean', 'sense' is used as Frege has employed the word, the sense of a word as different from its referent seems to be the physical shape etc. Because when we want to speak of a different sense of 'mean' we are to use different words. If here 'sense' is equivalent to 'referent' the phrase "different sense of 'mean'" is legitimate. But if it is used according to Frege's usage, the phrase is illegitimate. Thus, according to Wienpahl, we can not speak of different sense of a word. To speak of the different sense is to speak of the different modes of presentation of the same object and no single word can present its object in different ways. So, only different words can have different senses.

However, in the ultimate analysis the first objection does not hold. It may be argued, for instance, that 'tear' is a word which 'has' two different senses, e.g. 'tear (drop)' and 'tear (rent)', yet we use the same sign 'tear' in each case. But it is obvious that 'tear' as 'tear (drop)' and 'tear' as
'tear (rent)' have different referents. It may be said that they have also different senses, but only in a most peculiar way. On Frege's argument if the relation of identity asserted in 'a=b' holds between that which 'a' and 'b' refer to, 'a=b' becomes cognitively equal to 'a=a', provided 'a=b' is true. Because if 'a', 'b' are only taken as referring to some object, then when that object is same for both 'a' and 'b', the two statements say exactly the same thing. 'Tear=tcar' is equivalent to 'a=a', but to construct the equivalent of 'a=b' we have to add some letters to 'tear' viz. 'tear(drop)=tear(rent)'.

According to Wienpahl these two signs differ in sense, but they are also physically different. Moreover, on Frege's argument 'a=b' have the same cognitive value as 'a=a', if 'a=b' is true. In case of 'tear', 'tear(drop)=tear(rent)' will always be false, since two signs have different referents. Thus it violates another condition of Frege's argument. Hence where it is argued that the same sign can have different senses, it is found that signs in question have different referents.

Yet it may be argued that in some situations 'tear' and 'tear' have different senses without addition of more letters to them. In answer to this, Wienpahl says that in such situations whenever we write or say this, we at least say to our own self 'tear(rent)' and 'tear(drop)'. The same sign with different senses can only be used when the context of usage makes it clear in what sense it is being used. And it is same with adding more letters. So, Wienpahl demands, strictly speaking the same word can not have different senses.
Wienpahl further shows that the second objection that different words can have same sense also does not hold. Let us consider the example of 'chat' and 'cat'. In this case we can not construct the equivalent of 'a=a' as it occurs in Frege's argument. Corresponding to 'a=b' we can construct 'chat-eat', but this statement belongs to metalanguage whereas 'a=b' is not a statement of metalanguage as used in Frege's argument. So it does not correspond to 'a=b' of Frege's argument. Thus the second objection can also be ruled out.

However, according to Wienpahl, 'a=a' and 'a=b' differ cognitively, since 'a' and 'b' differ as objects in so far as they function as signs. He does not regard the sense of a word as a subsistent entity. Rather, on his view, it consists in the physical properties of the signs as objects. According to Wienpahl, the sense of a word is the 'thought' we have while using the word, but the word is a necessary part of the 'thought'.

Richard Rudner tries to show that on certain considerations Wienpahl's proposal is defective. Wienpahl has attributed physical properties to linguistic signs and it implies that he treats the expressions of the kind 'a=a' and 'a' to be physical tokens. But, Rudner points out, from Frege's discussion it is not absolutely precise whether the entities to which he ascribes sense are tokens or symbols.

As we know that symbols i.e. classes of similar tokens are not physical things, the entities to which Wienpahl ascribes

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62. 'On Sinn as a Combination of Physical Properties' in Mind LXXI (1952), pp. 82-84, Reprinted in Essays on Frege ed. E.D. Klemke, pp. 219-222.
sense are tokens. But one of the conditions of Frege's argument is that in expressions like \( a = a \), the sign to the left of the identity sign has the same sense as the sign to the right of the identity sign. In fact, the condition indicates that every occurrence of a token of a given symbol has the same sense as every other occurrence of a token of that symbol. However, two tokens can not have the same physical properties. So if the sense of a token is the combination of its physical properties, no two tokens can have the same sense. Hence, on Wienpahl's explication, it can not be said that in \( a = a \) the token on the right has the same sense as the token on the left. If it is thought that Wienpahl has not attributed sense to tokens, rather to symbols, his view is still defective. Because, as symbols do not have physical properties, it can not be said that linguistic sign has a sense which is the combination of physical properties.

Rudner suggests that though classes do not possess physical properties, it may be said that they possess physical like properties. For example, we can speak of the property of order in an ordered class of classes. We can speak of that 1 precedes 2 in the series of positive integers and also that first symbol precedes in order the second one in the ordered class of classes which may constitute the statement 'A rose is a rose'. Then it can be maintained that Wienpahl meant by the 'sense' of a sign not the combination of physical properties of a token, rather the combination of physical like properties of a symbol. But then Wienpahl's account also seems to make sense to be a subsistent entity.
Rudner urges that Wienpahl's account is unacceptable in the other way also. In the series of symbols constituted by the symbol statement 'a=a', the first symbol possesses the property of being first in occurrence in that series. Thus the combination of physical-like properties of the first symbol can not be the same as the combination of properties of the last symbol. Hence they can not have the same sense. Rudner thinks that the above proposal of regarding the sense of a symbol as the combination of physical-like properties is not actually what is intended by Wienpahl. Wienpahl's difficulty arises as he tries to meet a condition of Frege which is too strong, that in 'a=a' the two 'a's have the same meaning.

According to Rudner there is no reason for such a strong demand. If the fact that two 'a's stand for the same entity is taken to justify the demand the argument becomes weak, since the two expressions 'morning star' and 'evening star' also stand for the same object. Again, if the argument depends on the assumption of the identity of the expressions, the assumption is evidently false in so far as expressions are regarded as physical inscriptions. According to Rudner, instead of such a strong demand, we can adopt a much weaker assumption that the two expressions possess a certain degree of likeness of meaning. According to Goodman no two different words can have the same meaning. They can possess likeness of meaning to a greater or lesser degree, or of one or another kind. Whenever we say in our ordinary discourse that two terms have same meaning we usually

mean that they have likeness of meaning to a certain degree or of a certain kind which is sufficient for the purpose of discourse. And Rudner thinks that this weaker assumption is sufficient to solve, perhaps more precisely, any of the problems in discourse which requires such a strong demand as made by Frege to be handled.

However, according to Rudner, Wienpahl's account succeed in eliminating the dubious subsistent entities. Moreover, he has elucidated the entire problem by coming to the conclusion that where there is difference in meaning there must be difference in signs.

Frege's theory of sense and reference was severely attacked by Russell. In a passage in "On Denoting", he forwards a criticism to the theory of Frege. Russell's criticism of Frege also receives differing interpretations. We shall at first follow Searle and state his point of view as found in his "Russell's objection to Frege's Theory of sense and Reference".

Russell's argument, as pointed out by Searle, is based on two assumptions as following:

1) Whenever we intend to refer to the sense of a referring expression we enclose the expression in inverted commas.

64. In Logic and knowledge, ed. R.C. Marsh, p-48-51.
2) The sense of a referring expression refers to the referent. Russell takes (2) to be the view of Frege regarding the relation of sense and reference and he tries to prove this view to be absurd. According to Russell, from the above assumption it follows that (2') whenever the sense of an expression occurs in a proposition, the proposition refers to the referent of that sense.

Russell's argument against Frege's theory shows that either the relation between sense and reference is a logical relation, as it is assumed viz. (2), or it is not a logical relation. In the former case the sense of a referring expression can never be referred to. And in the second case, though we may be able to refer to the sense of a referring expression, the relation between sense and reference being wholly mysterious the theory lacks explanatory value. Thus if the theory is explanatory it can not stand and if it is not the theory becomes philosophically worthless.

Russell's argument tries to prove that the sense of a referring expression can never be referred to. Russell attempts to prove this by means of examples and by virtue of the assumption (1) which helps to present examples. Searle points out that(1) is defective in two ways. In the first place as this assumption is false it raises various confusions in its subsequent employment. Searle says that in our ordinary speech enclosing an expression in inverted commas does not signify that the resultant expression
serves to refer to its customary sense. Instead we use the expression "the sense of the expression", as Frege has shown, to refer to the sense of the expression in question. In the second place even if (1) is taken to be true or granted as an arbitrary ruling adopted by Russell in support of his argument, its use in the argument is confusing. Russell has used inverted commas in various ways ... sometimes to indicate that expressions are being spoken of, not being used in their customary way and sometimes to refer to the sense of the expression in question around which inverted commas are put. Searle adopts a variation of the first assumption as follows:

(II) whenever an expression is surrounded by the squiggle signs thus /", the resultant expression refers to the sense of the original expression. Hence, for any expression, e.g. "the dog", the sense of the expression "the dog" is equivalent to / the dog /.

To establish the conclusion that on Frege's theory it becomes impossible to refer to the sense of a referring expression Russell proceeds by offering examples which supports the conclusion. Let us suppose that we like to refer to the sense of the expression "the dog". If for this purpose we use "the sense of the dog", it would not do. Because the expression in question refers to the sense of some dog which being an animal, not a symbol, has no sense at all. We may use the expression "the sense of "the dog", which is same as " / the dog /". But this expression also fails to refer to the sense of "the dog". Because, for Russell, if the sense in question is in fact referred
to it becomes a constituent of the proposition, it occurs in the proposition, and by (2a) if the sense occurs in a proposition it refers to the referent of that sense, i.e. the dog. So here also we can not succeed in referring to the sense. According to Russell, whenever a denoting complex i.e. sense occurs in a proposition, the proposition is about the denotation.

Besides, neither of the phrases (a) "the reference of the dog", (b) "the reference of "the dog"", (c) "the reference of the dog" can be used to refer to the sense in question. Because (a) refers to nothing since the dog being an animal has got no reference, (b) refers to the dog, and by (2), (c) also refers to the dog. So, on Russell's view, we can no way refer to the sense of the expression "the dog".

Hence in a discourse about sense, our subject must not be "x" but some other thing which refers to the sense. Suppose the expression "y" serves this purpose. Then the question arises that what the relation between "y" and its reference is. We would not be able to define "y" by any proposition where "x" occurs, because if "x" occurs without inverted commas it refers to the referent of the sense of "X", not the sense itself. And evidently we can not also go backward from the reference to the sense, for corresponding to any reference there is attached an indefinite number of senses. Thus if "Y" really succeeds in referring to a sense, the relation between sense and reference can not be a logical relation. But then the relation between "Y" and its reference would be a mysterious one. So, according to Russell, either Frege's theory breaks down or it becomes philosophically
worthless being mysterious.

Searle has interpreted Russell's argument in graphic terms in order to make it more intelligible. Suppose, in a game, marbles are being dropped in bowls through pipes. This act of dropping through pipes is called referring in which pipes i.e. senses lead to bowls which are references. According to the rule anything whatever can be referred to. But the difficulty is that we can not stick to this rule, as we can not refer to a pipe. Whenever a marble drops into a pipe it goes through to the corresponding bowl. We can not construct a second pipe leading to the first. For if the second pipe really leads to the first the marble will drop through to the bowl. Again, if it is not connected with the first, then it can not be used to refer to the first according to the rule of the game. Nor can we get to the pipe by going to the bowl, since every bowl has an infinite number of pipes. The way in which the rule of the game is set up involves an absurdity.

Russell has another argument against Frege's theory which emerges from the first. He holds that not only the reference of an expression but the sense also occurs in a proposition. This thesis also he tries to prove by examples. "Scott is Scott" expresses a different proposition from "Scott is the author of waverley", as the former possesses a property which is not possessed by the latter, say, that George IV wanted to know whether it is true. Russell has taken Frege's view to be asserting that only the reference of an expression occurs in a proposition
Then on Frege's view the two propositions would be the same. But as they are not the same, Frege's view is misleading, so false.

Searle thinks that Russell's arguments are fallacious. Because he has misinterpreted Frege's position and also has confused the notion of being referred to by a proposition with the notion of occurring as a part of a proposition. According to Searle this leads to the denial of the very distinction Frege has made and Russell's conclusion can be drawn only from this denial not from the original thesis.

Russell has taken Frege's principle to be that the sense of a referring expression refers to the referent. In fact, according to Frege, what refers is the sign, not the sense, and the sign refers to the referent by virtue of its sense. The sense is the mode of presentation of the referent. A sign expresses sense and refers to its referent. Russell has also falsely assumed that if an object is referred to by a proposition then that object occurs as a part of that proposition. According to Searle, all these assumptions spring from a sheer confusion between the notion of referring to and that of occurring in a proposition. If these two notions are clearly distinguished Russell's argument can hold no longer. Searle thinks that the notion of occurring in a proposition is equivalent to Frege's notion of what is expressed in a proposition. It can not be supposed that a referent occurs as a part or as a constituent of a proposition. For instance, The Tenth King of France is not a part of the proposition about him in the way his elbow is a part of his arm or he himself is a part
of a French wrestling team. So Russell’s view which makes the referent of a proposition literally part of that proposition is nonsensical. Even if we adopt some arbitrary ruling that ‘occurring in’ is equivalent to "referred to", this sense of "occurring in" goes far from the one which is equivalent to the sense of "expressed by". Russell has confused the two. So, according to Searle, Russell’s argument which proposes to develop Frege’s theory actually develops the denial of that theory. Because Russell’s assumption that occurring in a proposition is the same as being referred to by the proposition makes sense and reference equivalents, whereas Frege’s whole thesis aims to establish the distinction between sense and reference.

Searle urges that if we realize what Frege’s intention is, the puzzles about referring to the sense of an expression become cleared out. We can refer to the sense of any expression by such a phrase as "the sense of the expression 'E'". Here the sense in question does not occur as a part of the proposition in which this phrase is used. On the contrary, it is referred to by virtue of the sense of the phrase, i.e. "the sense of the expression "E". In fact, the sense of an expression occurs in a proposition and the proposition refers to the referent by virtue of that sense. It is false that, as Russell has assumed, the referent occurs in the proposition. Therefore, when a sense is referred to, it does not occur as a part of the proposition, only the sense in virtue of which the first sense is referred to occurs. Thus the referent of the first sense can not be referred to at all by the proposition.
Searle further points out that Russell's second conclusion that Frege's theory fails to account for the distinction between "Scott is Scott" and "Scott is the author of Waverly" does not hold also. This conclusion is also a outcome of his confusion between "occur in" and "refer to". When these two notions are distinguished, it shows that how the two sentences express different propositions. They include expressions which differ in sense.

Hence, according to Searle, Russell fails to refute Frege's distinction between sense and reference. He has succeeded merely in conjoining the distinction and its negation.

It is usually thought either that Russell has not succeeded at all to provide any serious argument against Frege's theory as his argument was confused, or that he has merely succeeded in attacking his earlier view as found in *The Principles of Mathematics*, not the theory of Frege. But Simon Blackburn and Alan Code try to show that these assessments are not correct. According to them in 'On Denoting' Russell has presented a serious argument against Frege's doctrine. In support of this claim they proceed with a reformulation of Russell's argument and for the sake of convenience they refer to the relevant eight paragraphs of 'On Denoting' by the italic capitals (A) to (H) respectively.

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Both proper names and definite descriptions are called 'denoting phrases'. They are supposed to have both a 'sense' and a 'reference'. The reference of the word 'Aristotle' is Aristotle. The sense of the word is neither Aristotle nor the word 'Aristotle', but a third entity associated with the word. This view may be called three entity view. Russell's argument points to the fact that we can not refer to the sense of a word. A word refers to its reference and expresses its sense. According to Blackburn and Code the sense determines the thing, since a sense relates to a thing in such a way that word which expresses it refers to that thing.

Geach maintains that Russell's argument can, in fact, be put against his earlier view, not against Frege's. Russell seemed to direct his argument against Frege's exposition as he was mistaken in interpreting Frege. But Geach has not pointed it out where Russell's earlier view differs from Frege's. So Blackburn and code propose to show the difference and also to indicate what is wrong with Geach's exposition.

They point out two important distinctions between the theory of POM, and that of Frege. For Frege it is the definite description itself, the actual expression which refers to the reference. On the other hand, for Russell the denoting concept meant by a definite description denotes the thing referred to or denotation. Hence we can not identify sense with meaning, reference with denotation, referring with denoting, since it

would not map Russell's theory onto Frege's. In Frege an expression refers to, while in Russell a concept denotes. But this difference is trivial. If a relation holds between the meaning of the description and the denotation, we can define in terms of it another relation obtaining between the description itself and the denotation of its associated meaning. Likewise, we may define in terms of the relation between a word and its reference another relation which holds between the sense of the expression and that reference. Thus the first distinction seems to be basically terminological and it does not indicate how Russell's own theory is less acceptable than Frege's.

The second difference consists in the fact that, for Frege, every singular term has a sense and a reference, whereas at the time of POM, Russell's analysis applied only to definite descriptions, not to ordinary names. But Russell has noted this difference in POM. As he lately holds that the three-entity view should not be applied to anything at all, it covers Frege's as well as his own more limited thesis. Hence though the two thesis differs in certain respects we can not say that in 'On Denoting' Russell refutes the one, not the other.

Cassin believes that Russell did not even propose to attack Frege. He holds that on Russell's earlier view only terms can be denoted, but later he came to realize that denoting concepts themselves must be denoted. But this interpretation of Russell's earlier view is mistaken. In POM a term is anything
which can be the subject of a proposition, or an object of thought, or a logical subject and both things and concepts are terms. In POM he criticises Frege for his view that concepts can not be logical subjects.

Cassin tries to indicate another supposed weakness of Russell's earlier view. Let us introduce the following abbreviations to account for Cassin's exposition:

\[ C_1 = \text{Some denoting concept which denotes Aristotle.} \]
\[ C_2 = \text{Some denoting concept which denotes } C_1. \]
Cassin says that it is a part of Russell's earlier view that: (A) any proposition which contains \( C_2 \) as a constituent will be about Aristotle. The proposition is intended to be about \( C_1 \), but since the concept \( C_1 \) determines Aristotle, the proposition ultimately is about Aristotle. Hence, on this view, we can in no way refer to denoting concepts by means of expressions which have higher-order denoting concepts as their sense. In otherwords, no denoting concept can be referred to by descriptions, because any attempt to do this falls through to the original object. From this it follows that denoting concepts of any level can atbest be referred to by names. Searle also, as we have seen earlier, makes this point. But he regards (A) as a outcome of the conjunction of Frege's theory with its negation, and according to him it is that which Russell has succeeded in refuting.

Blackburn and Code point out two major defects of

A.J. Ayer, on the other hand, believes that Russell has a valid argument against Frege. Ayer supports Russell's argument that there is a mystery about identifying senses and their relations to corresponding references. But he fails to acknowledge the force of Russell's argument correctly. Ayer interprets Russell as claiming that 'the first line of Gray's Elegy' and "the meaning of 'the first line of Gray's Elegy'" have different meanings but the same denotation. This goes against Frege's account, since it amounts to the demand that sense and reference be identified. On Ayer's account, Russell makes this demand since otherwise there will be a mystery about what the meaning or sense is. But this account awaits another argument. Blackburn and Code think instead that Russell makes a powerful and precise argument against Frege and they proceed to unveil the force of Russell's argument.

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(Referred to by Blackburn and Code)
In 'On Denoting', for Russell, the meaning of a denoting phrase is called a denoting complex which he has called denoting concept in POM. This denoting complex corresponds to Frege's sense. The thing denoted, for Russell, is denotation and denoting is the relation between the sense and the reference, in other words, between the denoting complex and the denotation. In Russell's exposition it is not the expression which denotes, rather denoting complex determines the denotation, i.e. sense determines the reference. Russell here uses single quotes to indicate the expressions which themselves refer to senses or denoting complexes. In the end of (B) Russell is concerned with the relationship between sense and reference which it determines, i.e. determining.

In (C) Russell urges that if an expression has a distinct element called sense apart from its reference, we have no guarantee for any logical connection between the two. In the subsequent paragraphs Russell proceeds to show this and thereby to prove the absurdity of Frege's doctrine of sense and reference. What he wants to prove is not that a sense itself can not be an object of reference, rather that we can not specify a sense in such a way so that we can show it performing a certain logical role.

Russell holds that the determining relation can not be merely linguistic, i.e. it can not be explained by Frege in terms of expressing and referring. For Frege, sense is a theoretical entity and denoting phrases happen to refer to their own customary sense when they are used in psychological
contexts. Frege made this point to explain the difference between 'George IV wished to know whether Scott is the author of Waverly' and 'George IV wished to know whether Scott is Scott' regarding their truth-values. Russell does not accept this explanation. Because, as there is no theoretical definition of terms which purport to refer to senses, we can not be sure of what logical role senses play. Russell tries to show that we can never specify senses in a way so as to be sure that they play the role as claimed by Frege.

Frege suggests that in order to speak of the sense of an expression "A", the phrase "the sense of the expression "A"" would do. But such a specification of sense does not guarantee its logical role. If a sense may be specified by merely mentioning an expression and describing our entity as the sense of that expression, the relation between sense and reference can be explicated only by saying, for example, that it is the relation holding between the sense of the word "Aristotle" and the man Aristotle. Hence the relation will be merely linguistic through the phrase. But this would not suffice. Let us consider following two pairs:

(1) Aristotle taught philosophy.
(2) Jones believes that Aristotle taught philosophy and
(3) Aristotle, the magnet, married Mrs. Kennedy.
(4) Aristotle, the philosopher, wrote books.
In the second pair the relation between the two things named Aristotle, is merely linguistic. But evidently there is a close connection between two things, named Aristotle, in the first pair. In fact, Russell holds rightly that there must be a logical connection between (1) and (2), and hence between the reference of 'Aristotle' in each. For they two together yield the conclusion that Jones believes something true. Therefore, unless we are able to specify the nature of this connection, any theory of sense will give rise to mysteries. Hence sense can not be introduced or defined linguistically. In paragraph (c) also Russell makes this point by saying that there will be a difficulty if meaning can only be arrived at by means of denoting phrases. So either 'the sense of "Aristotle"' or some other phrase which purports to refer to the sense of 'Aristotle' requires to be defined.

In the paragraph (D) Russell proposes to rule out a false move. As it has been objected that the sense of an expression can not be introduced by means of mentioning the expression a Fregean might make the mistake of using it and try to give the definition of 'the sense of "Aristotle"'. He may define it as the sense of Aristotle. Russell tries to show the defect of this procedure. He considers the case where the thing referred to is a linguistic expression and seems to have a sense by itself. In this case also its sense can not be the same as the sense of our original denoting phrase which we intend to specify. So in order to specify the meaning we want, we must speak not of "the meaning
of C", rather of "the meaning of "C"". Thus we are led back to the method of introduction which is simply linguistic.

Blackburn and Code point out that Russell has made us face a certain kind of dilemma. To give a proper introduction to the notion of sense we can either mention expressions or use them. The first alternative will not do, since it renders the relation between sense and reference to be merely linguistic. On the other hand, to use the denoting phrase is to refer. We can not introduce the sense as the sense of the thing referred to, because it is not what is intended. In paragraph (1) Russell has discussed about the initial stage of the "use" horn of the dilemma. And in (C) he has merely hinted at its other horn, i.e. "mention" horn. In the beginning of (E) Russell has said that when we use the denoting complex in a proposition, the proposition is about the denotation. And if we construct a proposition in which "the meaning of C" stands as the subject, the subject is the meaning of the denotation which is not what we want to speak about. At the end of (E) Russell speaks again of the "use" horn. He urges that the meaning has denotation and is a complex and is supposed to possess both meaning and denotation.

In the next paragraph (F) Russell discusses the same problem of specifying the sense, meaning or complex. Hence it has been explained that when the sense is referred, the expression which we use should have its own sense which determines the first sense we intend to refer to. Though this is applicable to any expression used to refer to the
sense, Russell is here primarily concerned with the original defining expression i.e. 'the sense of "Aristotle"'. But then arises the problem that we can in no way specify the secondary sense, viz. the sense of 'the sense of "Aristotle"' as a function of the original sense. For, sense can never be identified in terms of the corresponding reference. In this passage Russell shows that no denoting phrase can specify the sense of a phrase by mentioning its reference and by identifying the sense in terms of some fixed function of that reference. As there is no backward road, we can in no way explain this function. Thus we can not define any phrase which is supposed to refer to the sense of, say, 'Aristotle'. Hence it can not be ascertained that the relation between sense and reference is logical, not mere linguistic.

Blackburn and code try to reveal the force of Russell's argument through a consideration of the following three expressions:

\[(E_1) \ 'Aristotle' \]
\[(E_2) \ 'The sense of "Aristotle"' \]
\[(E_3) \ 'The sense of E_2' \]

Hence \(E_2\) is an expression which refers to the sense of the name 'Aristotle'. To introduce the sense properly there must be some denoting phrases, viz. \(E_2\), which is intelligible to us and whose reference is grasped through our understanding of it. A second possibility is that we may recognize senses directly without a definition and so no such phrase is required.
But it makes the notion of sense obscure. But to grasp senses through our understanding of some denoting phrase like $E_2$ requires that we should grasp the sense of $E_2$ itself. Now the problem arises how can we recognize the sense of $E_2$. It can be done, again, either directly or through our understanding of some denoting phrase, viz. $E_3$, which refers to it. In this level, in turn, since there is no backward road from reference to senses, to understand $E_3$, i.e. to grasp its sense, we are to follow the same procedure and thus will ad infinitum. The regress can be stopped if we admit that in a certain level we do not require an understood definition, instead we can recognize the sense straight off.

Dummett interpretes Russell in a different way. According to him, Russell's criticism of Frege is extremely confused and through it we can find at least a valid criticism of Frege's doctrine of indirect sense and reference. But Dummett holds that through a simple change in Frege's doctrine, consistent with his other views, it can avoid Russell's criticism. According to Frege a name expresses indirect sense in the context in which it refers to its customary sense. On Dummett's account, Russell is taken to be objecting that there is no way of specifying this indirect sense, since there is no backward road. So Dummett suggests that Russell's objection will be superfluous if we take indirect and ordinary sense to be one and the same entity, so that in such contexts a word expresses just its ordinary sense. According to Dummett, the sense of a word can not vary from context to context, since it is a property of the word itself independent of my context.

71. *Frege: Philosophy of language* p. 267
But Dummett's suggestion will not do against the difficulty as represented earlier. Because the problem about the sense of a denoting phrase such as $E_2$ is not the problem about indirect sense. The two can be confused if we suppose that the sense of a phrase such as $E_2$ itself may be the indirect sense of the name "Aristotle". But on Dummett's view it is not possible. Let us consider the two sentences 'Jones knows that Aristotle=Aristotle' and 'Jones knows that Aristotle=the sense of "Aristotle"'. In the context of the second sentence $E_2$ refers to its normal sense. If this is the indirect sense of "Aristotle" and is to be identified, as Dummett has suggested, with the ordinary sense of "Aristotle", then $E_2$ in this context refers to the same as the name 'Aristotle' in this context. Each denoting phrase refers to the customary sense of "Aristotle". Therefore the two sentences should have the same truth-value. Because, for Frege, if two sentences contain denoting phrases which in the context of those sentences refer to the same thing, they can not differ as regards their truth-values. Thus, as the first sentence is true it follows that Jones knows that Aristotle is the sense of 'Aristotle', in which Aristotle is the sense of 'Aristotle', hence two become one and the same as Russell has claimed.

Blackburn and Code point out that what Russell through his argument aims at is to make us notice the absence of a proper denoting phrase specifying the ordinary sense which we understand and the understanding of which admits explanation.
A phrase, namely, 'the sense of "Aristotle"' does not satisfy these conditions. A phrase which uses the word in question instead of mentioning it, will not also do. Although we can apparently understand such a phrase, it can not serve as a definition; since it presupposes that we already know what we are speaking about. It does not tell us what we are talking about, since we have no idea of how the word 'Aristotle' functions in such a phrase.

We have already mentioned the possibility that senses can be named straight off instead of being introduced with a definite description which refers to them by means of their defining properties. But, on the theory of denotation as propounded in PM, a sense or denoting concept can not be named directly. Because, according to this theory, (i) when a name is used in a sentence, the thing named is a constituent of the proposition expressed by the sentence; (ii) if a proposition contains a denoting concept as a constituent the proposition is about the object denoted, not about the denoting concept. It may be supposed that Russell came to see that point while writing 'On Denoting' and it made him insisting that senses were to be introduced by means of definite descriptions.

Thus the central point, as suggested by Blackburn and Code, is that whether or not the lack of explicit definition of sense will prevent us from having a specific notion of sense. According to Dummett, in saying what the reference
of a word is we show what its sense is. we can not directly state what the sense of an expression is and we require only a sufficient grasp of the notion what some one can do when he has grasped a sense. But Frege's theory demands that we, in fact, refer to the senses of denoting phrases. According to him, we refer to them when we use names in indirect contexts. But our understanding of any such context is as confused as our understanding of what senses precisely are. If we consider the common explanation in terms of "a way of finding a reference", "a mode of presentation of a reference", we find that the nature of the logical connection between Aristotle and mode or way of finding him remains obscure to us. Russell's argument in 'On Denoting' makes us to see this weakness. Thus Russell tries to point out the difficulties in Frege's doctrine which have seldom been found out.

Russell tries to solve this problem by reducing sentences containing definite descriptions to a set of sentences in which the original definite description does not occur; and thus giving them a quantifier-plus-propositional-function interpretation. Kaplan has suggested that we may identify the Russellian propositional function with the Fregean sense. Thus we can meet any Russellian qualms about the ontological and logical status of senses.
According to Bradley, there can be no judgement where there is no knowledge of truth or falsehood. Truth or falsehood is the outcome of the relation of our ideas to realities. So, in the strict sense, we can have no proper judgement without ideas. Ideas are not realities. They are symbols standing for the existing reality. Everything, which we know, has two aspects. (i) existence and (ii) content; in other words, that it is and what it is. But a symbol has a third aspect too, i.e. its signification or what it means.

On Bradley's view, a judgement is the act which refers an ideal content to a reality beyond the act. The ideal content is the logical idea which serves as a symbol, and thus it can transcend itself and gain universality being capable of standing for other things. The ideal content is not a fact, but a wandering adjective. When we make a judgement we assign this adjective to a real substantive. The whole judgement is the predicate or unified ideal content. Reality is the subject of all judgements and it exists outside the structure of judgement.

In the description of judgement, we have found that there is only one idea being asserted in the judgement. So there is no subject and copula. But the traditional view

72. 'The Principles of Logic', vol. 1, p. 2
73. Ibid, p. 10
upholds that in a judgement we have always two ideas and that one of them is the subject. Bradley does not admit of the association of two ideas in a judgement. For him, every judgement is a unified whole. We may admit of plurality of ideas in a judgement. For example, in the judgement 'The wolf is eating the lamb', there are three distinct ideas corresponding to the three terms 'wolf-eating-lamb'. But this is supposed to be so, as in the whole there exist distinctions and we are led to make such groupings of attributes. But if we thus deny the singleness of every idea which embraces other, we shall find that wolf himself is not also a single idea, being the synthesis of a number of attributes. In the end we shall find no idea as one. So, according to Bradley, any content whatever which the mind takes as a whole, however, complex it may be, is one idea and its manifold relations are embraced in a unity.

According to Bradley, the relations within the content of any meaning are not relations between mental existences. There is a wolf and a lamb. In the judgement 'The wolf is eating the lamb', a relation is being suggested between the wolf and the lamb, but that relation is not a factual connection between events in my head. What is meant is not psychical conjunction of images. It is a one unified ideal content. It is not the case that in a judgement one idea is the subject and another is referred to it. For example, in the case, say, 'A is in south of B', 'B follows A', it is unnatural to take A or B as the subject and residue as the predicate. In every
judgement, there is a subject which the whole ideal content is referred to and which falls outside it. Otherwise, it would be the idea attributed to itself. The subject is, in ultimate analysis, no idea, but always reality.

Bradley rejects some erroneous views which depend on the false notion that in a judgement we have a pair of ideas. According to Bradley, in their ordinary acceptance the traditional subject, copula, and predicate are mere superstitions. The ideal content which is affirmed in the judgement may possess internal relations and may be arranged as the subject and the attribute. In 'the wolf eating the lamb' the relation remains the same whether I affirm or deny or ask. So the content is the same both in the assertion and out of it. Hence this internal relation can not itself be the judgement, rather can atleast be the condition of judging. By virtue of it we can not make any distinction between subject and predicate within the structure of a judgement.

According to Bradley, judgement is not inclusion in or exclusion from a class. He says that the doctrine that 'A is equal to B' or 'B is to the right of C', or 'Today precedes Monday' implies that we have in mind a class, either a collection or a description, of 'things equal to B' or 'to the right of C', or 'preceding Monday' is opposed to fact. It is as absurd as the saying that in 'It is our son John', 'It is my best coat', we think of a class of 'our sons John', or 'my best coat'.
The second view that Bradley rejects is that judgment is the inclusion or exclusion from the subject. Here the term 'subject' does not signify the ultimate subject which the whole ideal content is referred to, but the subject which lies within that content, i.e. the grammatical subject. In 'A is simultaneous with B', 'D is equal to F', we can not naturally consider A, and D, as sole subjects and the rest as the attributive, since the positions can be easily reversed. We can say instead 'A and B are synchronous' and 'D and F are equal'. Usually the ideal content is arranged in the form of a subject with adjectival qualities; but in certain instances it takes the form of two or more subjects with adjectival relations existing between them:

Bradley also rejects the view that the judgement is an assertion that the subject and the predicate are equal or identical. This is also an outcome of two-ideas theory. It is incredible that what we mean by 'A is north of C' or 'B follows D' is a relation either of equality or of identity.

Equality in its proper sense means an identity in respect of quantity. But, if so, if 'Negroes are men', when written 'All negroes = some men', is on a level with '2 = 12-10', there is no difference whatever we can at once pass on. But the sign '=' does not seem to mean equality, but absolute identity, i.e. the units of the subject and the predicate are identical both in quantity and quality.
The identity is not likeness. It is not a relation consisting in a partial quantitative identity. Nor it is partial identity consisting in sameness in some particular point or points of quality. Rather the sign '=' means total sameness to the exclusion of all differences. But then it will not be true that 'Negroes = some men', since 'some men' are not '=' negroes'. It is not also true that negroes are equal to a certain stated fraction of mankind. That stated fraction is an universal adjective which might be applicable to other men as well as to negroes. The men that are negroes are not any and every set of men who have a certain number. They are those men who are negroes and this is the predicate. Hence, Negroes = negroe-men, and iron = iron-metal. The predicate here really seems to be the subject and can be substituted for it. But if 'iron-metal' is the same with 'iron' it is wrong to set down the two sides as different terms. But, on the other hand, if there is really a difference between the two then the statement is false, as by '=' we deny the difference. It may be assumed that both sides of assertion are just the same, i.e. 'Negroes-men are negroes-men' and 'iron-metal is iron-metal'. But it only asserts that where there is no difference there is no difference. But, according to Bradley, a judgement which asserts no difference is nothing. Thus in removing the difference of subject and predicate we have removed the whole judgement.

From these errors we can draw certain important results. The ideal content asserted is always complex and must always
involve relations between distinguishable elements. The predicate must be always a universal, since every idea is universal, though it is false that predicate is a class in which the subject is inserted. Again, though assertion is not attribution to a subject in the judgement, nevertheless in every judgement there must be a subject. The ideal content becomes real by union with a substantive. Thirdly, the object of judgement is to assert the synthesis of different attributes within the identity of a subject. Whenever we write '=' there must be a difference. In "S = P", we do not mean that S and P are identical, rather we mean to say that the diverse attributes of S and P are united in one subject.

According to Bradley, an identity must underlie every judgement. It has been already said that every judgement is the attribution of an ideal content to reality and this reality is the subject of which the ideal content is predicated. For example, in 'A precedes B', the whole relation A-B is the predicate and is treated as an adjective of the real world. Hence, the reality to which the adjective A-B is referred is the subject of A-B and is the identity underlying this synthesis of differences. Again, in the judgement we have an opposition in time of A to B. Hence the subject of which A-B is asserted is subject to these differences and is then different in itself, while remaining the same.

According to Bradley, a judgement says something about some fact or reality. A judgement must be true or false, and its truth or falsehood can not lie in itself. They involve a
reference to something beyond. A truth is necessary, when in some way, it is compelled to be true. And compulsion can not exist unless there is something that compels. Hence it is the real, which exerts this force, of which the judgement is asserted. It is not the case that the suggestion $S-P$ itself is categorically true of the fact. The actual judgement asserts that $S-P$ is forced on our minds by a reality $\mathcal{X}$. And this reality, according to Bradley, is the subject of the judgement. It is the same with objectivity, i.e. it is the object itself which is this or that.

But the question is how ideas are related to realities. The real is that which is known in presentation or intuitive knowledge. It is what we encounter in perception and it is that which appears in the series of events that occur in space and time. The real is self-existent and individual. And ideas are merely symbols. They are general and adjectival. Their essence lies within their meaning and beyond their existence. So long as judgements are confined to ideas, their reference to fact is a mere implication. It is presupposed outside the assertion.

But judgement is not merely confined to ideas, it is the mere synthesis of ideas. According to Bradley, judgements can exist without any copula and with but one idea. In the simplest judgement, an idea is referred to what is given in perception and it is identified therewith as one of its adjectives. There is no need for an idea to appear as the subject. The present reality is the actual subject and the genuine
substantive of the ideal content. where 'this', 'here', 'now' seem to stand as subjects, the real subject is the actual fact appearing in perception which is indicated by those phrases. For Bradley, all judgements predicate their ideal content as an attribute of real which appears in presentation.

Hence, according to Bradley, judgement is not the synthesis of ideas, but the reference of an ideal content to reality. From this basis he tries to interpret three kinds of judgements - (i) analytic judgements of sense in which we simply analyse the given, e.g. 'I have a toothache', 'There is a wolf' etc., (ii) synthetic judgement of sense, which state either some fact of time or space, or some quality of the matter given which we do not directly perceive at the present, e.g. 'This road leads to London'; (iii) The third class of judgements are concerned with a reality which is never a sensible event in time, e.g. 'God is a spirit'.

The first kind of judgements may further be of two types. They may have neither grammatical subject nor copula, or again may possess one or both. In the judgements that have neither copula nor subject, an idea is referred (a) to the whole sensible reality, or (b) to some part of it. In the 'wolf' or 'Rain', the subject is the unspecified present environment to which is attributed the ideal content 'wolf' and 'Rain'. Again, when in some moment of both inward and outward wretchedness, we say 'miserable', the subject is here the whole given reality. Utterance of these single words convey a statement about fact and
we are indeed judging without any other subject than the whole sensibly present. Again, when seeing a visible wolf, we predicate the words 'asleep' or 'running', the real subject is one piece of the present. Even here no subject or copula appears.

Secondly, we have analytic judgements where a subject is expressed. The ideal content of the predicate is here referred to another idea which stands as a subject. But in this case also, the ultimate subject is not an idea, but the real in presentation to which the content of both ideas, along with their relation, is attributed. In such judgements as 'Now is the time', 'The present is dark', an idea intervenes between the reality and the predicate and takes the place of immediate subject. But the immediate subject is the sign of a reference to the whole given reality. In such judgements as 'This is a bird', 'There is a wolf', 'this', 'there' are certainly ideas and stand for the immediate subject of judgement. But they are also signs of reference, though not to a whole given reality, but to a part of it. If we consider such a judgement as 'This bird is yellow', it is not the bare idea, symbolized by 'this bird', of which we affirm the predicate. Rather it is the fact distinguished and qualified by 'this bird' to which the adjective 'yellow' is really attributed. Bradley holds that in this case the genuine subject is the thing as perceived, the content of which our analysis has divided into 'this bird' and 'yellow', and of which we predicate indirectly those ideal elements in their union.
In the second class of judgements we make some assertion about that which appears in a space or time, not perceivable by us, and we predicate of a presentation something not got by analysis of its content. For example, if we say 'this road leads to London', the judgement is synthetic, since it goes beyond perception. According to Bradley, in synthetic judgements there is always an inference, for an ideal content is connected with the sensible qualities that are given to us.

According to Bradley, in synthetic judgements the ultimate subject is still the reality. But this reality is not confined to the momentary appearance. The real can not be identical with the content that appears in presentation. It always transcends the presentation. Yet synthetic judgements are possible only by being connected with what is given at this very instant. We seem to find contact with reality nowhere outside the presented. How then is content referred to reality if it can not be referred to the real as perceived? According to Bradley, the content is referred indirectly. It is not attributed to the given as such. But, by establishing its connection with what is presented, it is attributed to the real which appears in that given. Though it is not and can not be found in presentation, it is true because it is predicated of the reality, and unique since it is fixed in relation with immediate perception.

Thus, Bradley, holds, synthetic judgements cease to be merely adjectival. They express a series of unique events by
indirect reference to the real which appears in unique presentation. They are connected by an inference with the content of this appearance and so far are directly related to perception. But their ideas are never referred to the presentation itself. The content of our perception and the content of our ideal constructions are both referred to as the adjective to one reality which both appears in presentation and transcends beyond.

The third kind of judgements consists in assertions not confined to an event or a number of events in time. Any judgement we make about the Universe or God or the soul belongs to this class. Our ideas are here identified with the real that we find in perception, but they do not attach themselves to anyone part of the phenomenal series. This third class of judgements differs from others in the fact that its ultimate subject is not the real as it appears in 'this' or in any one event in the series.

Hence, according to Bradley, the ultimate subject is always reality which is qualified by adjectives of ideal content. The whole judgement is a unified predicate asserted of the reality.

For Bradley, we have seen, reality is the ultimate subject of every judgement. The whole judgement is a predicate of that reality. Bosanquet also upholds that judgement is an act of reference to reality. But he differs from Bradley in an important respect that, for him, this reality is not mere
transcendent to the act of judging. According to him, reality is given for us in present sensuous perception and the direct awareness of our own existence. And it is through this given perceptual reality the reference of ideal content to the ultimate reality is affected.

For Frege, on the other hand, judgement does not consist in such an ideal act. For him, judgement is passing from the sense of a sentence to its reference. Frege, as we have seen, speaks of both sense and reference of a sentence. By sense of a sentence he means the thought expressed by it. By thought he does not mean the subjective act of thinking, but its objective content which may be shared by all who use the sentence. By reference of a sentence Frege means its truth-value, i.e., either the True or the False. For Frege, in making a judgement we pass from a thought to its truth-value. A mere thought can not give us knowledge. When a thought expressed by a sentence is ascertained either to be true or to be false, we have judgement. Hence, whereas in Bradley's exposition reality is the subject of a judgement, on Frege's view the reality or the True is the referent of a sentence.

According to Bradley ideal contents are meanings. The ideal content is not a fact, it is the logical idea which serves as a symbol and thus can transcend itself. Everything has two aspects - 1) existence and ii) content. But a symbol as we have seen, has a third aspect too, i.e. its signification or what it means. There are two senses of 'idea'. Each idea, we know, exists as a psychical fact with particular qualities and
relations. This character is assigned to an idea when confined to the two aspects of existence and content. But each idea involves a third aspect also, i.e. its meaning. Suppose we have the idea of a horse. It is a fact in my mind and so far it is a unique individual differing from others. But, for logic, the idea of horse is associated with the attributes we recognise as belonging to a horse, and its meaning is a part of the content considered apart from the existence. An idea is thus a symbol in respect of one quality of its entire consciousness. Hence 'idea' in the judgement is abstract and the universal meaning. They mean something beyond themselves.

For Frege meaning of a name or a sentence consists of two parts, i.e. its sense and reference. Frege draws an important distinction between sense and reference of a name. By reference of a name he means the object designated by the name, and by sense he means the mode of presentation of the object designated by the name. We know an object in and through a certain aspect. This mode of knowing the object constitutes the sense of the name which designates the object. Frege mentioned also of, as said before, the sense and reference of a declarative sentence. What he meant by the 'sense' of a sentence seems to be analogous to the 'ideal content' as used by Bradley.

According to Bradley, the relation between the ideal content and reality is one of predication. For him, the ideal content is a unified whole and it is predicated of a transcendent reality which lies beyond the act of judgement. But, for Frege,
the relation between the proposition which is the sense of a sentence and the True or the False is not a predication, but is that between sense and reference. Because, we have already seen, the sense of a sentence is the thought or proposition expressed by the sentence and its reference is either the True or the False. According to Frege, subjects and predicates are elements of thought and the subject-predicate distinction is a distinction within a thought. This distinction is restricted to a thought, i.e., to the ideal content. The act of passing from thought to the True or the False is not analogous to the act of predicing. It is, for Frege, passing from the sense to the reference of the sentence.