Chapter VII
CHAPTER VII

A study of load capacity of finite slider bearings
lubricated with couple stress fluids considering thermal effects

7.1. Introduction:

Different Newtonian and non-Newtonian fluids have been considered to explain the lubrication behaviours by many researchers. In modern industrial technology, the use of non-Newtonian fluids as lubricants is more important than that of Newtonian fluids. The lubricants with stable suspensions of fine particles of insoluble solids, which have different material characters and can be used as the viscosity-index improver in order to prevent the viscosity variation with temperature, exhibit non-Newtonian behaviour. The interactions of such suspended particles in the flow of fluid have been referred to as couple stresses. A number of theories of micro-continuum have been developed to explain the behaviours of these non-Newtonian polymeric fluids [Ariman et al. (1973, 1974)]. A special theory of fluids proposed by Stokes (1966) has been applied to explain the effect of couple stresses in fluids, known as couple stress theory of fluids.

Presented in National Conference on Recent Developments in Mathematics and Applications in Assam University, Silchar, 2001.
Many papers have been published considering this couple stress theory to explain theoretically the effects of couple stresses in various journals and slider bearings. Bujurke and Jayaraman (1982) used the couple stress theory to explain the effects of couple stresses in a squeeze film synovial-joint. Ramanaiah and Sarkar (1978) and Ramanaiah (1979) have also considered the couple stress fluid model to explain the effect of couple stresses on squeeze film thrust bearing. A similar analysis dealing with load capacity, frictional force etc. has been considered by Sinha and Singh (1981), Bujurke and Naduvinami (1990) for rolling bearing and Lin (1997a) for finite journal bearings. A comparative study of various lubricating factors for different forms of slider bearings lubricated with couple stress fluid has been considered by Das and Bhattacharjee (1993). All such analyses have been considered under isothermal consideration.

For an understanding of the frictional behaviour on different lubricating factors such as viscosity, load capacity etc., the effect of temperature produced by the flow of viscous fluids as well as bearing motion is to be considered. So, the lubricating factors in lubrication technology are assumed to be functions of temperature. Some papers presenting the effect of temperature in some special cases have been published. Sadeghi et al. (1987) considered a study of thermal effects in two-dimensional rolling / sliding contacts lubricated with incompressible Newtonian fluids. Johnson and Greenwood (1980) have considered the Eyring fluid model for analysis the thermal effects in EHD traction. Considering power law fluid model the
thermal effects have been analyzed by Prasad et al. for rollers without cavitation (1987) and with cavitation (1993) and by Shukla and Isa (1978) for externally pressurized bearing with squeeze films.

An analysis of the maximum load capacity has been made by Das (1999), considering power law fluids as lubricants in the absence of any body force. A similar analysis in presence of magnetic field has been presented by Das (1998), considering couple stress fluid model and side-flow. The present analysis deals with the thermal effects in finite slider bearings lubricated with couple stress fluid model proposed by Stokes' in order to present in the joint effects of temperature and side-flow. Variation of viscosity with temperature raised by the frictional heat produced by the flow of fluid has also been considered. The variations of lubricating factors are analyzed numerically by using an iterative technique in a finite difference method of partial differential equations. A comparative study of load capacities for various values of couple stress and bearing parameters and maximum load capacities of a finite and an infinite slider bearing has been presented, considering the non-Newtonian fluid and Newtonian fluid models as lubricants in both thermal and isothermal conditions.

7.2. Equation of motion:

The physical configuration in our problem is a three-dimensional finite slider bearing shown in Fig. 7.1. It has two spaced surfaces, the lower
one is a straight rigid surface moving with uniform sliding velocity $u_0$ and the upper one is rigid in general form with finite length $d$ and width $a$, squeezing down with velocity $v_0$. A steady and incompressible flow of couple-stress fluid is considered between the surfaces in the form film of thickness $h(x)$. We assume the thermal flow of the lubricants and non-thermal property of the surfaces. The basic equations under above assumptions in the simplified form [Das (1999) and Appendix A] are:

Continuity equation: $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) = 0 \quad (7.1)$

Momentum equation: $\frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial y^2} \right) - \eta \left( \frac{\partial^4 u}{\partial y^4} \right) \quad (7.2)$

$\frac{\partial p}{\partial y} = 0, \quad (7.3)$

$\frac{\partial p}{\partial z} = \mu \left( \frac{\partial^2 w}{\partial y^2} \right) - \eta \left( \frac{\partial^4 w}{\partial y^4} \right) \quad (7.4)$

Energy equation: $k \left( \frac{\partial^2 u}{\partial y^2} \right)^2 = -\mu \left( (\partial u/\partial y)^2 + (\partial w/\partial y)^2 \right) + \eta \left[ (\partial u/\partial y)(\partial^3 u/\partial y^3) + (\partial w/\partial y)(\partial^3 w/\partial y^3) \right] \quad (7.5)$

where $u$, $v$, $w$ are the velocity components of the fluid, $\mu$, $\eta$, $p(x,z)$, $k$ and $t$ are respectively viscosity coefficient, material constant of couple stress, pressure, thermal conductivity and temperature of the fluid.
We have considered the variation of viscosity $\mu$ with temperature due to the frictional heat generated by the flow of fluid. Further, the relation for temperature-dependent $\mu$ (pressure independent) is defined as follows [Prasad et al. (1987)]:

$$\mu = \mu_0 \exp\left[-\beta(t_m - t_0)\right]$$  \hspace{1cm} (7.6)

where $\mu_0$ and $t_0$ are respectively the viscosity coefficient and temperature when $p = 0$ (ambient pressure). $\beta$ is the temperature viscosity coefficient and $t_m$, the mean temperature across the film thickness $h$, is defined as

$$t_m = \frac{1}{h} \int_0^h t \, dy$$  \hspace{1cm} (7.7)

The boundary conditions for velocity components, pressure and temperature in the lubricant region are:

$$u = u_0, \quad v = 0, \quad w = 0 \text{ at } y = 0;$$

$$u = 0, \quad v = v_0, \quad w = 0 \text{ at } y = h$$  \hspace{1cm} (7.8)

$$p = 0 \text{ at } x = 0, a \text{ and } z = 0, d$$  \hspace{1cm} (7.9)

$$t = t_0 \text{ at } y = 0 \text{ and } y = h$$  \hspace{1cm} (7.10)
No-couple stress conditions:

\[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial y^2} = 0 \text{ at } y = 0 \text{ and } y = h(x) \quad (7.11) \]

7.3. Solutions of equation:

Since \( \mu \) defined in equation (7.6) is function of \( x \) and \( y \) only, the solution of \( u \) and \( w \) can be obtained from equations (7.2) and (7.4) with the boundary conditions (7.8) and (7.9) and no-couple stress condition (7.11) in the forms:

\[
\begin{align*}
    u &= u_o \left( 1 - \frac{y}{h} \right) + \left[ \frac{\partial p}{\partial x} \left( \frac{1}{2\mu} \right) \right] \left[ y - h + \frac{2\eta}{\mu} \right](1) \\
    &\quad - \cosh \left\{ \frac{2(y - h)}{2(\mu/\eta)} \right\} \cosh \left\{ \frac{h}{2(\mu/\eta)} \right\} \\
    w &= \left[ \frac{\partial p}{\partial z} \left( \frac{1}{2\mu} \right) \right] \left[ y - h + \frac{2\eta}{\mu} \right](1) \\
    &\quad - \cosh \left\{ \frac{2(y - h)}{2(\mu/\eta)} \right\} \cosh \left\{ \frac{h}{2(\mu/\eta)} \right\} \\
\end{align*}
\]

\[ (7.12) \]

and

\[
\begin{align*}
    w &= \left[ \frac{\partial p}{\partial z} \left( \frac{1}{2\mu} \right) \right] \left[ y - h + \frac{2\eta}{\mu} \right](1) \\
    &\quad - \cosh \left\{ \frac{2(y - h)}{2(\mu/\eta)} \right\} \cosh \left\{ \frac{h}{2(\mu/\eta)} \right\} \\
\end{align*}
\]

\[ (7.13) \]

We integrate the continuity equation (7.1) with respect to \( y \) with the boundary condition of \( v \) in (7.8) and with the help of the principle of integration under differentiation. The modified Reynolds equation is obtained:

\[
\begin{align*}
    \left( \frac{\partial}{\partial x} \right) \left[ s \left( \frac{\partial p}{\partial x} \right) \right] + \left( \frac{\partial}{\partial z} \right) \left[ s \left( \frac{\partial p}{\partial z} \right) \right] &= 6\mu u_o \frac{dh}{dx} \\
\end{align*}
\]

\[ (7.14) \]
where \( s = h^3 - 12 \int [h - 2(\gamma / \mu)^{1/2} \tanh \{h/2(\gamma / \mu)^{1/2}\}]/\mu \) \hspace{1cm} (7.15)

We introduce the following dimensionless quantities:

\[
X = x/a, \quad Y = y/h_o, \quad Z = z/d, \quad H = h/h_o, \quad M = \mu / \mu_o, \\
L = (\gamma / \mu_o)^{1/2} / h_o, \quad U = u/u_o, \quad V_o = v_o a/(h_o u_o), \quad W = w/a u_o, \\
P = p h_o^2 / (u_o u_o), \quad T = t / t_o, \quad T_m = t_m / t_o, \quad Pr = \mu_o c_p / k, \\
E = u_o^2 / (c_p t_o), \quad B = \beta t_o, \quad S = s / h_o^3, \quad \lambda = d/a,
\]

where \( h_o \) is the minimum film thickness. The non-dimensional form of equations (7.5) to (7.7) and boundary conditions (7.9), (7.10) can respectively be rewritten as

\[
(\partial^2 T / \partial Y^2) = Pr E \{ - M (\partial U / \partial Y)^2 + L^2 (\partial^2 U / \partial Y^2) (\partial^3 U / \partial Y^3) \} \\
+ \lambda^2 \{ - M (\partial W / \partial Y)^2 + L^2 (\partial^2 W / \partial Y^2) (\partial^3 W / \partial Y^3) \} \hspace{1cm} (7.16)
\]

\[
M = \exp \{ - B (T_m - 1) \} \hspace{1cm} (7.17)
\]

\[
T_m = \frac{1}{H} \int_{H}^{1} T \, dY \hspace{1cm} (7.18)
\]

and boundary conditions

\[
P = 0 \text{ at } x = 0,1 \text{ and } Z = 0,1 \hspace{1cm} (7.19)
\]
\[ T = 1 \text{ at } Y = 0 \text{ and } Y = H \quad (7.20) \]

Equations (7.12) – (7.15) also reduce to

\[
U = (\partial P/\partial X) \left[ Y^2 - HY + (2L^2/M) \left\{ 1 - \cosh\left(2Y - \frac{H}{2M}\right) \right\} \right] / 2M + (1 - Y/H) \quad (7.21)
\]

\[
W = (1/\lambda^2) \left( \partial P/\partial Z \right) \left[ Y^2 - HY + (2L^2/M) \left\{ 1 - \cosh\left(2Y - \frac{H}{2M}\right) \right\} \right] / 2M \quad (7.22)
\]

\[
(\partial/\partial X)(S \partial P/\partial X) + (1/\lambda^2) (\partial/\partial Z)(S \partial P/\partial Z) = 6M (dH/dX) \quad (7.23)
\]

\[
S = H^3 - 12L^2 \left( H - (2L/\lambda V) \tanh\left( \frac{HM}{\lambda V}/2L \right) \right) / M \quad (7.24)
\]

Since the temperature-dependent viscosity coefficient \( M \) is a function of \( T \), so the solution for \( T \) can be obtained from equation (7.16) by using equations (7.21), (7.22) and boundary condition (7.20) for \( T \). Using this value of \( T \) in equation (7.18), we obtain the value of \( T_m \) in the form:

\[
T_m = 1 + \text{Pr.E} \left[ \frac{1}{(\partial p/\partial x)^2} + \frac{1}{(\partial p/\partial z)^2} \right] \left( \frac{H^4}{240M} \right) + \frac{L^2/M^3}{\Phi \tanh\left( \frac{HM}{\lambda V}/2L \right)} + \frac{M}{12} \quad (7.25)
\]

where \( \Phi = 6L - (12L^2 + MH^2) / (M^2H) \)
As a particular case, we have taken film thickness $H(X)$ of inclined slider bearing where,

$$H(X) = A - (A - 1)X, \quad 0 \leq X \leq 1 \quad (7.26)$$

7.4. Numerical calculations:

The dimensionless modified Reynolds equation is solved for pressure $P$ numerically using a finite difference method in partial differential equations [Strikwerda (1989)]. The film region is divided by the grid spacing as shown in Fig. 7.2, and equation (7.23) is discretized as:

$$\left[\frac{\{S_{i+1/2,j}(P_{i+1,j} - P_{i,j})/\Delta X\} - \{S_{i-1/2,j}(P_{i,j} - P_{i-1,j})/\Delta X\}}{\Delta X}\right] + \frac{1}{\lambda^2} \left[\frac{\{S_{i,j+1/2}(P_{i,j+1} - P_{i,j})/\Delta Z\} - \{S_{i,j-1/2}(P_{i,j} - P_{i,j-1})/\Delta Z\}}{\Delta Z}\right] = \frac{6M_{i+1/2,j}(H_{i+1/2,j} - H_{i-1/2,j})/\Delta X}{\Delta X} \quad (7.27)$$

Equation (7.27) results in $m \times n$ simultaneous equations for $m \times n$ points in the film region. At the boundaries, these equations have been solved numerically assuming $P = 0$. In this paper, 50 divisions (i.e., $\Delta X = \Delta Z = 0.02$) have been taken in each unit length in both $X$- and $Z$-directions. The film pressures have been solved numerically using the conjugate gradient method of iteration.
The initial values for first iteration is assumed as $T_m = 1$, which suggests $M = 1$ [from equation (7.17)]. Using these values of $T_m$ and $M$ in equation (7.24) we obtain $S$. Using $S$ in equation (7.27), we obtained $P$ at each nodal point. These values of $P$ are referred as values at zero iteration i.e., $(r) = 0$. We have discretized the relations for $T_m$, $M$, $S$ in equations (7.25), (7.17), (7.24) and expression in equation (7.27) in terms of iteration level in the following superscriptial notations:

$$
T_m^{(r+1)} = 1 + Pr.E \left\{ \frac{(P_{i+1,j}^{(r)} - P_{ij}^{(r)})}{\Delta X} \right\}^2 +
\left( \frac{1}{\lambda^2} \right) \left\{ \frac{(P_{ij+1}^{(r)} - P_{ij}^{(r)})}{\Delta Z} \right\}^2 \left( \frac{H^4}{240 M^{(0)}} \right) +
\Phi \left\{ \frac{2}{M^{(0)}} \right\} \tanh \left\{ \frac{H(M^{(0)})^{1/2}}{2L} \right\} + \frac{M^{(0)}}{12}
$$

(7.28)

$$
M^{(r+1)} = \exp \left\{ -B(T_m^{(r+1)} - 1) \right\}
$$

(7.29)

$$
S^{(r+1)} = H^3 - 12L^2 \left[ H - \{ 2L/(M^{(r+1)}) \} \tanh \left\{ H(M^{(r+1)})^{1/2}/2L \right\} \right]/M^{(r+1)}
$$

(7.30)

and

$$
\left\{ S_{i+1/2,j}^{(r+1)}(P_{i+1,j}^{(r+1)} - P_{ij}^{(r)})/\Delta X \right\} + \left\{ S_{i-1/2,j}^{(r+1)}(P_{ij}^{(r)} - P_{i,j+1}^{(r+1)})/\Delta X \right\} + (1/\lambda^2) \left\{ \frac{\left\{ S_{ij+1/2}^{(r+1)}(P_{ij+1}^{(r+1)} - P_{ij}^{(r)})}{\Delta Z} \right\} + \frac{P_{ij}^{(r)} - P_{ij-1}^{(r)}}{\Delta Z} \right\} \Delta Z
$$

$$
= 6M_{i+1/2,j}^{(r+1)}(H_{i+1/2,j} - H_{i-1/2,j})/\Delta X
$$

(7.31)
\[ \Phi = 6L - (12L^2 + M^0H^2) / \{(M^0)^0H\}, \]

\((r)\) and \((r+1)\) are respectively previous and current iteration levels. Using the initial values (i.e., when \((r) = 0\)) of \(M, T, S\) and \(P\) in equations (7.28) – (7.31), we have obtained \(P\) at different nodal points at various iteration levels. Reaching to the iteration levels \((r) = 4\) and \((r) = 5\), we have obtained that the values in both levels are almost identical. Thus \((r) = 5\) is the last iteration level in our consideration.

For numerical calculation, the following values or wide ranges of values of dimensionless quantities have been used:

\[ B = 0.7, \quad V_o = 0, \quad 0 \leq Pr.E \leq 1, \quad 1 \leq A \leq 2, \]

\[ 0.5 \leq \lambda \leq \infty \quad \text{and} \quad 0 \leq L \leq 10. \]

Dimensionless load capacity \(W\) is calculated from the equation as follows:

\[ W = \int_0^1 \int_0^1 P \, dX \, dZ = \Delta X \Delta Z \sum_i \sum_j P_{ij} \]

(7.32)

7.5. Results and discussions:

The dimensionless numerical values of load capacity \(W\) for various values of inlet-outlet film ratio \(A\), dimensionless maximum load capacity \(W_{max}\) and inlet-outlet film ratio corresponding to maximum load \(A_{wmax}\) for
dimensionless couple stress parameter $L$, length-breath ratio $\lambda$ and thermal parameter $Pr.E$ have been presented graphically and numerically.

Fig. 7.3 presents $W$ vs. $A$ for two values of $Pr.E$ and three values of $\lambda$ (three forms of finite slider bearings). The figure shows that for each pair of $\lambda$ and $Pr.E$, $W$ first increases rapidly as $A$ increases from 1 and then reduces slowly, attaining a maximum value ($W_{\text{max}}$) at a particular value of $A$ ($A_{w_{\text{max}}}$). The figure also reveals that $W$ increases with the increase of $\lambda$ but decreases with the increase of $Pr.E$ i.e., the effect of temperature has noticeable influence on load capacity for long slider bearings. The figure also shows that the position of $A_{w_{\text{max}}}$ shifts towards the origin with the increase of $Pr.E$ i.e., due to thermal effect. $A_{w_{\text{max}}} = 1.503$ for curve (4) with $Pr.E = 0$ and $A_{w_{\text{max}}} = 1.492$ for curve (3) with $Pr.E = 3$, both for fixed $\lambda = 1$.

Fig. 7.4 represents a comparative study of load capacity $W$ against $L$ for two values of $Pr.E$ and three values of $\lambda$ ($\lambda \rightarrow \infty$, corresponds to infinitely load slider bearing and $\lambda = 0.8, 1.2$ correspond to finite slider bearings i.e., represent the consideration of side-flow). The figure shows that $W$ increases with the increase of both $L$ and $\lambda$, but $W$ decreases with the increase of thermal parameter $Pr.E$ for each value of $L$ and/or $\lambda$. The figure also reveals a realistic case that the decrease of load capacity $W$ due to thermal effect is more noticeable when the plate is longer and/or $L$ is higher.
In fig. 7.5, we have presented $W$ vs. $\lambda$ for two values of $Pr.E$ and three values of $L$. The figure indicates that $W$ increases with the increase of $\lambda$ for a given value of $Pr.E$ or $L$. But this increment is higher with larger value of $L$ and lower with that of $Pr.E$. The figure also reveals that the increasing rate of $W$ for fixed values of $L$ and $Pr.E$ decreases even for larger values of $\lambda$. In this case, $\lambda = 500$ is the consideration of $\lambda \to \infty$. The values of $W$ change significantly with the lower values of $\lambda$. The values in the Newtonian case, in the absence of thermal parameter ($L = Pr.E = 0$) are found to be identical with those given by Gross et al. (1980), under similar conditions for slider bearing with the variation of $\lambda$.

Fig. 7.6 presents $A_{\text{wmax}}$ vs. $L$ for two values of $Pr.E$ and three values of $\lambda$. The figure displays that $A_{\text{wmax}}$ decreases with the increase of $L$ and the decreasing rate is larger for lower values of $L$. The figure also reveals that $A_{\text{wmax}}$ is diminished with the increase of $Pr.E$ or $\lambda$. This variation is higher for higher values of $L$. The figure also indicates that the value of $A_{\text{wmax}}$ for finite inclined slider bearing ($\lambda = 1$) in the isothermal Newtonian case ($L = Pr.E = 0$) is 2.512 and under similar circumstances this value even for step slider bearing is 2.431 as mentioned in the work of Das (1998).

Fig. 7.7 represents $W_{\text{max}}$ vs. $L$ for two values of $Pr.E$ and three values of $\lambda$. In this figure, it is observed that $W_{\text{max}}$ increases with the increase of $L$. 
But the increasing rate is intensified with the increase of $\lambda$ but the increasing rate is diminished by the increase of Pr.E. In this figure, the value of $W_{\text{max}}$ is 0.352, when $L = 1$, Pr.E = 0 and $\lambda \to \infty$ and it is interesting to note that the value is almost identical with that given by Das (1998), under similar conditions.

In Table 7.1, we have presented $W_{\text{max}}$ vs. Pr.E for two values of L and three values of $\lambda$. In this table we observe that $W_{\text{max}}$ decreases with the increase of Pr.E, but increases with the increase of L and $\lambda$. Further the increasing rate is comparatively greater for higher values of L and $\lambda$. The close scrutiny signifies the thermal effect on maximum load capacity.

7.6. Conclusions:

Based on a special micro-continuum theory, the generalized Reynolds equation for thermal analysis is derived, and applied to investigate the performance of finite slider bearings lubricated with couple stress fluids. From this study the following conclusions have been drawn:
1. Load capacity, maximum load capacity and inlet-outlet film ratio corresponding to maximum load capacity depend on couple stress parameter, length-breath ratio of the slider plate and frictional temperature factor.

2. For both types of slider bearings (finite and infinite), maximum load capacity increase with the increase of couple stress parameter and decrease with the increase of temperature factor.

3. The value of inlet-outlet film ratio corresponding to maximum load capacity, decrease with the increase of couple stress parameter or, temperature parameter or length-breath ratio of the slider plate.

4. The maximum load capacity increases with the increase of length-breath ratio of the slider plate but decrease with the increase of frictional temperature factor. This decrement in load capacity due to temperature parameter is greater when the length-breath ratio of the slider plate is infinite.

The overall study on this analysis reveals that due to thermal effect of lubricants, a definite shape of a slider bearing for maximum load capacity can not be predicted. However, its construction can be predicted for maximum average load capacity during its period of operation, when thermal effects are considered.
Nomenclature:

\( a, A \) width of the plate, inlet-outlet film height ratio

\( A_{\text{wmax}} \) inlet-outlet film ratio corresponding to maximum load capacity

\( B \) non-dimensional coefficient of temperature

\( c_p \) specific heat

\( d \) length of the plate

\( E \) Eckert number

\( h, H, h_0 \) dimensional, non-dimensional film height, least film height

\( H_{ij} \) values of \( H \) at \( ij^{th} \) node on \( X-Z \) plane

\( k \) thermal conductivity

\( L \) non-dimensional couple stress parameter

\( M \) non-dimensional viscosity coefficient

\( p, P \) dimensional, non-dimensional fluid pressure

\( P_{ij}, Pr \) values of \( P \) at \( ij^{th} \) node on \( X-Z \) plane, Prandtl number

\( (r), (r+1) \) previous, current iteration levels

\( s, S \) dimensional, non-dimensional defined values

\( S_{ij} \) values of \( S \) at \( ij^{th} \) node on \( X-Z \) plane

\( t, T \) dimensional, non-dimensional fluid temperature

\( t_m, T_m \) dimensional, non-dimensional mean temperature

\( t_o \) temperature at \( p = 0 \) (i.e., ambient temperature)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$, $v$, $w$</td>
<td>dimensional fluid velocity components along $x$-, $y$-, $z$- directions</td>
</tr>
<tr>
<td>$U$, $V$, $W$</td>
<td>non-dimensional velocity components along $X$-, $Y$-, $Z$- directions</td>
</tr>
<tr>
<td>$u_0$</td>
<td>uniform sliding velocity of lower plate</td>
</tr>
<tr>
<td>$v_0$, $V_0$</td>
<td>dimensional, non-dimensional squeezing velocity of upper plate</td>
</tr>
<tr>
<td>$W$, $W_{\text{max}}$</td>
<td>non-dimensional load bearing capacity, maximum value of $W$</td>
</tr>
<tr>
<td>$x$, $y$, $z$</td>
<td>Cartesian coordinates in the flow region</td>
</tr>
<tr>
<td>$X$, $Y$, $Z$</td>
<td>non-dimensional Cartesian coordinates</td>
</tr>
<tr>
<td>$\Delta X$, $\Delta Z$</td>
<td>increments on $X$-, $Z$- directions in the fluid region</td>
</tr>
<tr>
<td>$\beta$</td>
<td>coefficient of temperature</td>
</tr>
<tr>
<td>$\eta$</td>
<td>couple stress parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>length-breath ratio of the plate</td>
</tr>
<tr>
<td>$\mu$, $\mu_0$</td>
<td>coefficient of viscosity, $\mu$ at $p = 0$</td>
</tr>
</tbody>
</table>
Table 7.1 \( W_{\text{max}} \) against Pr.E for two values of \( L \) and for three values of \( \lambda \).

<table>
<thead>
<tr>
<th>Pr.E</th>
<th>( L = 2 )</th>
<th>( L = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda = 0.8 ) ( \lambda = 1.0 ) ( \lambda = 1.2 )</td>
<td>( \lambda = 0.8 ) ( \lambda = 1.0 ) ( \lambda = 1.2 )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.2262 0.2517 0.3397</td>
<td>1.9434 2.1846 2.9339</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2078 0.2257 0.3136</td>
<td>1.9045 2.1492 2.8202</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1878 0.2012 0.2995</td>
<td>1.8621 2.1202 2.7315</td>
</tr>
<tr>
<td>0.6</td>
<td>0.1624 0.1815 0.2802</td>
<td>1.8102 1.9821 2.6233</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1488 0.1656 0.2636</td>
<td>1.7723 1.9386 2.5325</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1332 0.1539 0.2486</td>
<td>1.7332 1.9032 2.4133</td>
</tr>
</tbody>
</table>
Fig. 7.1 Physical configuration of finite slider bearings:

(a) Normal section,  (b) Three dimensional.
Fig. 7.2 Grid point notation for the film region in the X-Z plane.
Fig. 7.3  \( W \) against \( A \) for \( L = 3 \) and for

\[
\text{Pr.E} = 0.3, \ [ (1) \lambda = 0.8, \quad (3) \lambda = 1.0, \quad (5) \lambda = 1.2];
\]

\[
\text{Pr.E} = 0.0, \ [ (2) \lambda = 0.8, \quad (4) \lambda = 1.0, \quad (6) \lambda = 1.2].
\]
Fig. 7.4 \( W \) against \( L \) for \( A = 1.6 \) and for

\[
\begin{align*}
\text{Pr.E} &= 0.3, \ [ (1) \lambda = 0.8, \ (3) \lambda = 1.2, \ (5) \lambda \to \infty ]; \\
\text{Pr.E} &= 0.0, \ [ (2) \lambda = 0.8, \ (4) \lambda = 1.2, \ (6) \lambda \to \infty ].
\end{align*}
\]
Fig. 7.5 $W$ against $\lambda$ for $A = 1.4$ and for

$Pr.E = 0.3$, $(1) L = 0, (3) L = 3, (5) L = 6$;

$Pr.E = 0.0$, $(2) L = 0, (4) L = 3, (6) L = 6$. 
Fig. 7.6 $A_{\text{wmax}}$ against $L$ for

Pr.$E = 0.3$, [ (1) $\lambda = 1.2$, (3) $\lambda = 1.0$, (5) $\lambda = 0.8$ ];

Pr.$E = 0.0$, [ (2) $\lambda = 1.2$, (4) $\lambda = 1.0$, (6) $\lambda = 0.8$ ].
Fig. 7.7 $W_{\text{max}}$ against $L$ for

Pr.E = 0.3, [(1) $\lambda = 0.8$, (3) $\lambda = 1.0$, (5) $\lambda \to \infty$];

Pr.E = 0.0, [(2) $\lambda = 0.8$, (4) $\lambda = 1.0$, (6) $\lambda \to \infty$].