Chapter VI
Chapter VI

_Thermal analysis of slider bearings lubricated with couple stress fluid in presence of magnetic field_

6.1. Introduction:

For the importance in many industrial applications, the magneto-hydrodynamic lubricate phenomenon has received considerable attention in recent years. A number of theoretical and experimental investigations into the magneto-hydrodynamic effects in lubrication have been published considering different fluid models. Hamza (1988) has considered the flow of an electrically conducting fluid between two squeezing parallel discs in presence of magnetic field in order to analyze the electromagnetic effects in load bearing capacity. Kuzma et al. (1964) have presented both the theoretical and experimental extended results to include fluid-inertia effects and buoyant forces for a hydro-magnetic squeeze film using mercury between two circular flat plates. Days et al. (1967) have also analyzed both theoretical and experimental results for magneto-hydrostatic thrust bearing. Ghosh and Debnath (1986, 1988) used a dusty fluid model proposed by Saffman (1962) to study the flow of dusty fluids in rotating and oscillating cases, in presence of magnetic fields. Mitra and Bhattacharyya also used the same dusty fluid model to analyze the flow of dusty fluid in presence of magnetic field between two oscillating plates (1981a) and two parallel plates starting impulsively from rest (1981b). Stanisic et al. (1962) have considered
the hydro-magnetic fluid to analyze the flow between two oscillating flat plates.

The fluid with stable suspensions of fine particles of insoluble solids, is referred to a class of complex fluids and the particles have different material characters like body force, body couple, couple stress etc. Applying this couple stress theory developed by Stokes (1966), many papers have been published considering different lubricating flows in absence of any magnetic field [cited in Chapter IV]. The effects of couple stresses have also been analyzed in presence of magnetic field. Stokes (1968) proposed the effect of couple stresses in a hydro-magnetic channel flow. Das and Bhattacharjee (1994), Das (1998) have considered the couple stress fluid with magnetic field to analyze respectively the effects of couple stresses and the study of optimum load bearing capacity, for different slider bearing.

Different lubricating factors such as viscosity, couple stress parameter, load bearing capacity, frictional loss etc., are also affected by temperature due to the flow of fluids. Some investigators have considered the thermal effects on lubricating factors in absence of magnetic field [as considered in Chapter IV].

Since we have not noticed any work considering the thermal effects on couple stress fluid in lubricating problems, in presence of magnetic field,
so in our present problem, we have considered the thermal effects on couple stress fluid model in presence of magnetic field applied perpendicular to the bearing. We have also considered the variation of viscosity and couple stress parameter with temperature due to the frictional heat developed by the flow of fluid. The present analysis reveals a significant effect of temperature on viscosity, couple stress parameter and other viscosity dependent factors like load bearing capacity etc., in the flow region of lubricants in presence of magnetic field.

6.2. Equations of motion:

The geometry in our problem is a two-dimensional, an infinitely long slider bearing in general form, shown in Fig. 6.1. It has two rigid surfaces, the lower one is a straight with uniform sliding velocity $u_0$ and the upper one is in arbitrary shape with finite length $a$, and is squeezing down with velocity $v_0$. An incompressible and steady flow of a lubricant with couple stress fluid is considered between the surfaces with film thickness $h(x)$ in presence of a uniform magnetic field intensity $B$ in the $y$-direction, which induces an electric field $E_z$ along $z$-direction. We assume the thermal flow of the lubricants and non-thermal, non-magnetic properties of the surfaces. Neglecting the side flow and under the above assumptions the simplified form of equations are:

Continuity equation: \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \) \hspace{1cm} (6.1)
Momentum equation:
\[ \frac{dp}{dx} + \sigma E_B = \mu \left( \frac{\partial^2 u}{\partial y^2} \right) - \eta \left( \frac{\partial^4 u}{\partial y^4} \right) - \sigma u B^2 \] (6.2)

Energy equation: 
\[ k \left( \frac{\partial^2 T}{\partial y^2} \right) = -\mu \left( \frac{\partial u}{\partial y} \right)^2 + \eta \left( \frac{\partial u}{\partial y} \right) \left( \frac{\partial^3 u}{\partial y^3} \right) \] (6.3)

(ef. Appendix A)

where \( u, v \) are the velocity components of the fluid, \( \mu, \eta, \rho, k, \sigma \) and \( \tau \) are respectively viscosity coefficient, material constant of couple stress, pressure, thermal conductivity, electric conductivity and temperature of the fluid.

We have considered the variation of viscosity \( \mu \) (stated in Chapter IV) and couple stress parameter \( \eta \) with temperature due to the frictional heat produced by the flow of the fluid and the relations for \( \mu \) [Shukla and Isa (1978)] and \( \eta \) as follows:

\[ \mu = \mu_0 \exp[-\beta_1(t_m - t_0)] \] (6.4)

\[ \eta = \eta_0 \exp[-\beta_2(t_m - t_0)] \] (6.5)

where \( \mu_0, \eta_0 \) and \( t_0 \) are respectively the viscosity coefficient, couple stress parameter and temperature when \( p=0 \) (ambient pressure). \( \beta_1, \beta_2 \) the temperature coefficients of viscosity and couple stress parameter. \( t_m \) the mean temperature across the film thickness \( h \), is defined as
Since the surfaces are insulators, then the electric field $E_z$ is defined [Das and Bhattacharjee (1994)] as

$$E_z = - \frac{B}{h} \int_0^h u \, dy$$  \hspace{1cm} (6.7)$$

and the Hartmann number at the least film thickness $h_o$, is

$$M_o = \frac{B h_o (\sigma / \mu_o)^{1/2}}{h}$$

The boundary conditions for velocity components, pressure and temperature in the lubricant region are:

$$u = u_o, \quad v = 0 \text{ at } y = 0; \quad u = 0, \quad v = v_o \text{ at } y = h$$  \hspace{1cm} (6.8)$$

$$p = 0 \text{ at } x = 0 \text{ and } x = a$$  \hspace{1cm} (6.9)$$

$$t = t_o \text{ at } y = 0 \text{ and } y = h$$  \hspace{1cm} (6.10)$$

No-couple stress condition at the surfaces gives

$$\frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = 0 \text{ and } y = h(x)$$  \hspace{1cm} (6.11)$$
6.3. Solutions of equations:

Making use the following dimensionless values

\[ X = \frac{x}{a}, \quad Y = \frac{y}{h_0}, \quad H = \frac{h}{h_0}, \quad M = \frac{\mu}{\mu_0}, \]

\[ L = \left( \frac{r_j}{\mu_0} \right)^{1/2} / h_0, \quad L_o = \left( \frac{r_j}{\mu_0} \right)^{1/2} / h_0, \quad U = \frac{u}{u_0}, \]

\[ V = \frac{v a}{h_0 u_0}, \quad V_o = \frac{v a}{h_0 u_0}, \quad P = \frac{p h_0^2}{(\mu_0 u_0 a)}, \]

\[ T' = \frac{t}{t_o}, \quad T_m = \frac{t_m}{t_o}, \quad Pr = \frac{\mu_0 c_p}{k}, \quad E = \frac{u_o^2}{(c_p t_o)}, \]

\[ B_a = \frac{\beta_1 t_o}{}, \quad B_b = \frac{\beta_2 t_o}{}, \quad E_z = \frac{E}{B u_o}, \]

where \( h_0 \) is the minimum film thickness, equations (6.1) and (6.3) to (6.7) can be rewritten in dimensionless form as follows:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6.12) \]

\[ \frac{\partial^2 T}{\partial Y^2} = Pr.E \left\{ - M(\partial U/\partial Y)^2 + L^2 (\partial U/\partial Y) (\partial^3 U/\partial Y^3) \right\} \quad (6.13) \]

\[ M = \exp\{- B_a (T_m - 1)\} \quad (6.14) \]

\[ L = L_o \exp\{- B_b (T_m - 1)/2\} \quad (6.15) \]

\[ T_m = (1/H)_o \int^H T \, dY \quad (6.16) \]

and \[ E_z = -(1/H)_o \int^H U \, dY \quad (6.17) \]
The boundary conditions reduce to:

\[
U = 1, \quad V = 0 \text{ at } Y = 0 \quad \text{and} \quad U = 0, \quad V = V_0 \text{ at } Y = H \quad (6.18)
\]

\[
P = 0 \text{ at } x = 0 \text{ and } X = 1 \quad (6.19)
\]

and \( T = 1 \text{ at } Y = 0 \text{ and } Y = H \quad (6.20) \)

and no-couple stress condition taken the form:

\[
\frac{d^2 U/dY^2}{2} = 0 \text{ at } Y = 0 \text{ and } Y = H \quad (6.21)
\]

Integrating equation (6.12) over the film thickness and using the rule of differentiation under the integral sign with variable limits, we obtain the flow rate per unit width

\[
Q = \int_0^H U \, dY = -V_0 \, X + c \quad (6.22)
\]

where \( c \) is a constant to be determined by the boundary conditions (6.18) and (6.19). Using equation (6.22) in equation (6.17), we get

\[
E_z = -\frac{(c - V_0 \, X)}{H} \quad (6.23)
\]
Applying the values of $E_z$ and $M_o$, the non-dimensional form of momentum equation (6.2) is given by

$$\partial^4 U/\partial Y^4 - (M/L^2)(\partial^2 U/\partial Y^2) + U M_o^2/L^2 = S \quad (6.24)$$

where $$S = [-dP/dX + M_o^2(c - V_o X)/H]/L^2$$

Equations (6.14), (6.15) show that $M$ and $L$ are function of $X$ only, so, the solution of $U$ can be obtained from equation (6.24) with the help of boundary conditions (6.18) and (6.19) and no-couple stress condition (6.21) in three cases. Applying the values of $U$, in equation (6.22) for flow rate per unit width, we obtain

$$dP/dX = [H + 2(V_o X - c)]/\Delta r, \quad (6.25)$$

where $$\Delta r = H[(H/\Gamma_r) - 2]/M_o^2, \quad r = 1, 2, 3$$

$$\Gamma_1 = (\alpha^2/\beta) \coth(\beta H/2) - (\beta^2/\alpha) \coth(\alpha H/2),$$

$$\alpha^2 = M[1 + \{1 - (2M_o L/M)^2\}^{1/2}]/2L^2$$

and $$\beta^2 = M[1 - \{1 - (2M_o L/M)^2\}^{1/2}]/2L^2$$

$$\Gamma_2 = (1/2b)\{3 - bH \csc(\text{cosech}(bH))\} \coth(bH/2),$$
\[ b = \left( \frac{m}{2} \right)^{1/2} L \]

and
\[ \Gamma_3 = L \left[ \gamma \sinh(\gamma H) + \delta \sin(\delta H) - R \left\{ \gamma \sin(\delta H) - \delta \sinh(\gamma H) \right\} \right] \left[ \cosh(\gamma H) + \cos(\delta H) \right] M_0, \]

\[ \gamma = \left\{ M(2M_0L/M + 1) \right\}^{1/2}/2L, \]

\[ \delta = \left\{ M(2M_0L/M - 1) \right\}^{1/2}/2L, \]

\[ R = (\gamma / \delta - \delta / \gamma)/2, \]

\[ N = \cosh^2(\gamma H) - \cos^2(\delta H). \]

Integrating equation (6.25) with respect to \( X \) over the length of the bearing with the help of boundary conditions (6.18) and (6.19), the value of constant \( c \) can be obtained as

\[
\frac{c}{A} = \left[ \frac{\int_0^1 (H + 2V_0 X) \, dX}{\int_0^1 2 \, dX} \right]  \left( \frac{2}{\int_0^1 \frac{1}{\Delta r} \, dX} \right) (6.26)
\]

Using the values of \( u \) and boundary condition (6.20), we obtain the values of \( T \) from equation (6.13). Applying these values of \( T \) in equation (6.16), \( T_m \) can be calculated for three cases as follows:
case 1: when $2M_{oL}/M < 1$,

$$T_{m} = 1 + \text{Pr.E} \left[ \frac{H^2/12 + \{\sinh(2\beta H)/(2\beta H) - \{1 + \cosh(2\beta H)/2\}/(4\beta^2)\} \{A_3A_1^2 + A_2^2\}}{\left(\frac{\sinh(2\alpha H)/(2\alpha H) - \{1 + \cosh(2\alpha H)/2\}/(4\alpha^2)\} \{A_4(A_1^2 + A_2^2)\}\right)} \right]$$

$$- \left\{\frac{\sinh(\alpha + \beta) H/(\alpha + \beta) H - \{1 + \cosh(\alpha + \beta) H\}/2\}/(\alpha + \beta)^2 + \right\} \left\{\sinh(\alpha - \beta) H/(\alpha - \beta) H - \{1 + \cosh(\alpha - \beta) H\}/2\}/(\alpha - \beta)^2\right\}$$

$\times A_3(A_1^2 + A_2^2) + A_1A_2(A_3) - (H^2/12)\cosh(\beta H) + \{\sinh(\beta H)/(\beta H)\}$

$$- \cosh(\beta H)/(4\beta^2)\} + A_4\} - (H^2/12)\cosh(\alpha H) + \{\sinh(\alpha H)/(\alpha H)\}$$

$$- \cosh(\alpha H)/(4\alpha^2)\} - A_5\} \left\{2\{\sinh(\alpha H) - \sinh(\beta H)\}/\{(\alpha - \beta) H\} - \{\cosh(\alpha H) + \cosh(\beta H)\}/(\alpha - \beta)^2 - [2\{\sinh(\alpha H) + \sinh(\beta H)\}/(\alpha + \beta)\} - \{\cosh(\alpha H) + \cosh(\beta H)\}/(\alpha + \beta)^2\}/\right\}$$

$$= (6.27)$$

where $A_1 = \frac{(1 - S^2/M_o^2)(\alpha^2 - \beta^2)}{1}$,

$$A_2 = \frac{(S^2/M_o^2)(\alpha^2 - \beta^2)}{1}$$

$$A_3 = \frac{\alpha^4\beta^2(\beta^2L^2 - M)/(2\sinh^2(\beta H))}{1}$$
\[
\Lambda_4 = \alpha^2 \beta^4 \left(\alpha^2 L^2 - M\right)/\left\{2\sinh^2(\alpha H)\right\},
\]
\[
A_5 = \alpha^3 \beta^3 \left\{\left(\alpha^2 + \beta^2\right) L^2 - 2M\right\}/\left\{2\sinh(\alpha H) \sinh(\beta H)\right\}.
\]

**Case 2:** when \(2MJL/M = 1\),

\[
T_m = 1 + Pr.E \left\{\left(B_r + B_0\right) - H^2/12 + \left\{\sinh(2bH)/(2bH) - \left\{1 + \cosh(2bH)/2\right\}/\left(4b^3\right)\right\} + B_{10}\right\} H^4/40 + \left\{3\sinh(bH)/(8b^5H) - \{5 + 2b^2H^2 + 3\cosh(2bH))/(16b^4)\} + B_{11}\right\} H^4/40 + \{(1 + 3b^2H^2)\sinh(2bH))/(8b^5H) - \{3 + 4b^2(2b^2H^2 - 1)\cosh(2bH))/(8b^4) + 1/16b^4\} + B_{12}\right\} - \{(b^2H^2 + 3)\cosh(bH))/(12b^2) + \{\sinh(bH)/(4b^3H)\} - B_{13}\right\} \{(b^4H^4 + 3b^2H^2 - 3)\sinh(bH))/(24b^4H) + \{\cosh(bH)/(4b^3)\} + B_{14}\} \{(3\sinh(2bH))/(16b^4H) - \{\cosh(2bH))/(8b^3)\}
\]
\[
+ 1/(4b^3)\} + B_{15}\} \{(1 + b^2H^2)\sinh(2bH))/(4b^4H) - \{3\cosh(2bH))/(8b^3)\}
\]
\[
- 1/(8b^3)\} + B_{16}\} \{(b^4H^4 + 3b^2H^2 - 3)\sinh(bH))/(24b^4H)\} + B_{17}\} \{(3(2 + b^2H^3)\sinh(bH))/(8b^5H) + \{(b^4H^4 - 5b^2H^2 - 30)\cosh(bH))/(40b^4)\}\}
\]

(6.28)
where \( B_1 = \frac{SL^2}{M_o^2} \),
\( B_2 = 1 - B_1 \),
\( B_3 = b^2 + 2 \),
\( B_4 = bH \coth(bH) + B_3 \),
\( B_5 = bH \cosech(bH) \),
\( B_6 = bH \coth(bH) + B_5 \),
\( B_7 = b^2 /\{8\sinh^2(bH)\} \),
\( B_8 = b^2 \left[ b^2 L^2 - M \right] \),
\( B_9 = f \left( \{B_2 B_7 + B_1 B_4 B_6 + B_1 B_2 B_7(B_4 + B_6)\} b^2 L^2 - \{B_2 B_7 + B_1 B_4\}^2 M \right) \),
\( B_{10} = B_7^2 (b^2 L^2 - M) \),
\( B_{11} = b^2 \left( b^2 L^2 - M \right) \),
\( B_{12} = f B_2 \left[ \{B_2 B_7 + B_1 (B_3 B_6 + B_4 B_5) + B_2 B_7(B_3 + B_5)\} - 2B_3 (B_1 B_4 + B_2 B_7) M \right) \),
\( B_{13} = f B_2 \left[ \{B_2 \left( B_3 B_6 + B_4 B_5 + 2B_2 B_7\right)\} - 2(B_1 B_4 + B_2 B_7) M \right) \),
\( B_{14} = B_7^2 \left( B_3 + B_5 \right) - 2B_3 M \),
\( B_{15} = f B_2 \left[ \{B_2 \left( B_3 B_6 + B_4 B_5 + 2B_2 B_7\right)\} - 2(B_1 B_4 + B_2 B_7) M \right) \),
\( B_{16} = f B_2 \left[ \{B_2 \left( B_3 + B_5\right) - 2B_3 M \right) \),
\( B_{17} = 2f B_2 B_2 \left( b^2 L^2 - M \right) \),

\[ T_m = 1 - Pr.E \{c_{14}/4\} \left[ 2H^2/3 + (\delta^2 - 3\gamma^2)\sin(2\delta H)\cosh(2\gamma H) - \gamma^2 - 3\delta^2)\cos(2\delta H)\sinh(2\gamma H) + (\gamma^2 + \delta^2)^3 H - [(\delta^2 - \gamma^2)\{1 + \cos(2\delta H)\}\times \cosh(2\gamma H)] - 2\delta \sin(2\delta H)\sinh(2\gamma H) + (\gamma^2 + \delta^2)^3 H - \delta \sin(2\delta H)\cosh(2\gamma H) + \delta (\delta^2 - 3\gamma^2)\cos(2\delta H)\sinh(2\gamma H) \right] / (\gamma^2 + \delta^2)^3 H
\] + \[(\delta^2 - \gamma^2)\sinh(2\delta H)\sinh(2\gamma H) + 2\delta \{1 + \cos(2\delta H)\} \] / (\gamma^2 + \delta^2)^3 H
\]

\[ c_{15} = [\{\sinh(2\gamma H)\}/(2\gamma H) - \{1 + \cosh(2\gamma H)\}/2] / \gamma^2 - \left[ \{\sin(2\delta H)\}/(2\delta H) - \{1 + \cos(2\delta H)\}/2 \right] / \delta^2 \]

(6.29)
where \[ c_1 = \frac{SL^2}{M_o^2}, \quad c_2 = 1 - c_1, \]
\[ c_3 = c_2 \{R \sinh(2\gamma H) - \sin(2\delta H)\}/2N, \]
\[ c_4 = c_1 \{R \sin(2\delta H) + \sinh(2\gamma H)\}/2N, \]
\[ c_5 = c_2 \{R \cosh(\gamma H) \sin(\delta H) + \sinh(\gamma H) \cos(\delta H)\}/N, \]
\[ c_6 = c_1 \{\cosh(\gamma H) \sin(\delta H) - R \sinh(\gamma H) \cos(\delta H)\}/N, \]
\[ c_7 = (c_3 - c_6)\delta - (c_4 + c_5)\gamma, \]
\[ c_8 = (c_3 - c_6)\gamma + (c_4 + c_5)\delta, \]
\[ c_9 = c_7 (\gamma^2 - \delta^2) + 2c_4 c_5 \delta, \]
\[ c_{10} = 2c_7 \gamma \delta + c_6 (\gamma^2 - \delta^2), \]
\[ c_{11} = c_7 (c_9 L^2 - c_7 M)/4, \]
\[ c_{12} = -c_6 (c_{10} L^2 - c_8 M)/4, \]
\[ c_{13} = -(2c_7 c_8 M + (c_7 c_{10} - c_8 c_9) L^2)/8, \]
\[ c_{14} = (c_{11} - c_{12})/2, \]
\[ c_{15} = (c_{11} + c_{12})/4. \]

When the values of \( dP/dX \) are known, one can evaluate the load bearing capacity per unit width, \( W \) and the centre of pressure \( X_1 \) from the relations

\[ W = \sigma \int P \, dX = -\sigma \int X \, (dP/dX) \, dX \quad (6.30) \]

and

\[ X_1 = \{ \sigma \int X \, P \, dX \}/W = -(1/2W) \sigma \int X^2 (dP/dX) \, dX \quad (6.31) \]

The integrals in equations (6.30) and (6.31) can be solved only when \( H(X) \) is defined. In order to demonstrate the application of the above analysis to observe the effect of temperature on lubricating factors, we have considered
two particular types of slider bearings, namely (i) inclined slider bearing and (ii) parabolic-arc slider bearing.

For the inclined slider bearing, $H(X)$ is defined as

$$H(X) = A - (A - 1)X, \ 0 \leq X \leq 1 \quad (6.32)$$

For parabolic-arc slider bearing

$$H(X) = A + (A - 1)(X^2 - 2X), \ 0 \leq X \leq 1 \quad (6.33)$$

where $A$ is the ratio of the inlet and outlet film thickness of the bearing. Here it is to mention that $H(X)$, defined in the case of parabolic slider bearing assumes $dP/dX = 0$ at $X = 1$, the outlet region and consequently there is no cavitation at the outlet.

6.4. Numerical consideration and parametric values:

For numerical calculation, we have considered 21 nodes, which divide the non-dimensional bearing length into 20 equal intervals, each of 0.05. To execute the calculation by an iterative method, we first assume $T_m = 1$, which suggests $M = 1$ from equation (6.14) and $L = L_o$ (a given value) from equation (6.15). Using the values of $M$ and $L$, the initial value of $c$ and the initial values of $dP/dX$ at each nodal point are calculated from the relations (6.26) and (6.25) respectively. These initial values of $M$, $L$ and $dP/dX$ subsequently are used to get the values of $T_{in}$, $M$ and $L$ from the
relations (6.27) – (6.29), (6.14) and (6.15) respectively for the first iteration. Values of M and L are then used to find \( dP/dX \) at each nodal point for the first iteration and then the values of \( dP/dX \) are used to calculate \( c \) for second iteration. The above procedure is repeated for higher number of iteration.

It is observed that within the range of parametric values considered in this problem, a good convergence is attained before we reach to the fifth iteration. In almost all cases considered in this investigation, the values obtained for \( T_m, M, L, c \) and \( dP/dX \) at each nodal point in fourth and fifth iterations are approximately identical. Thus fifth iteration is the final iteration in our consideration.

For numerical calculation, the following values and ranges of values of dimensionless quantities have been used:

\[
B_a = 0.7, \quad B_b = 0.75, \quad V_o = 0, \quad 0 \leq Pr.E \leq 0.6,
\]

\[
1.2 \leq A \leq 2.2, \quad 0 \leq M_o \leq 8 \quad \text{and} \quad 1 \leq L_o \leq 8.5.
\]

The effects of \( T_m \) on \( M, L, W \) and \( dP/dX \) have been analyzed and their variations have been considered. The analysis is thermal when \( Pr.E \neq 0 \) and isothermal when \( Pr.E = 0 \).
6.5. Results and discussion:

The numerical calculations of the effect of temperature on different lubricating factors in presence of magnetic field have been presented in graphically and in tabular form. In Fig. 6.2, we have presented the non-dimensional values of mean temperature $T_m$ at different positions along the length of the inclined slider bearing with the variation of magnetic parameter $M_o$ and temperature factor $Pr.E$. The figure reveals that $T_m$ increases with the increase of $Pr.E$ and decreases with the increase of $M_o$. This variation is maximum at the inlet region and minimum near the point where the pressure is maximum. The figure also reveals that the decreasing rate of $T_m$ increases with the increase of $M_o$.

Fig. 6.3 represents the variation of non-dimensional pressure $P$ at different positions along the length of the inclined slider bearing with the variation of magnetic parameter $M_o$ and temperature factor $Pr.E$. The figure also indicates that $P$ increases with the increase of $M_o$ and decreases with the increase of $Pr.E$. The rate of variation is more for larger values of $M_o$. It is also noticeable that the position of $X$, where the pressure is maximum, is slightly changed as the effect of temperature.

The variation of dimensional load capacity $W$ with the variation of inlet and outlet film thickness ratio $A$ for different values of magnetic parameter $M_o$ and temperature factor $Pr.E$ is shown in Fig. 6.4. It shows that $W$ increases first with the increase of $A$ and then decreases with the increase
of A. The figure also represents that \( W \) increases with the increase of \( M_0 \) and decreases with the increase of temperature factor \( \text{Pr.E} \). This increasing rate is greater for higher values of \( M_0 \).

In Fig. 6.5, we have presented the non-dimensional load capacity \( W \) against the non-dimensional couple stress parameter \( L_0 \) with the variation of magnetic parameter \( M_0 \) and temperature factor \( \text{Pr.E} \). The figure shows that \( W \) increases rapidly with the increase of \( L_0 \) and this increasing rate is accelerated by higher values of \( M_0 \). The figure also reveals that \( W \) increases with the increase of \( \text{Pr.E} \). This increasing rate due to \( \text{Pr.E} \) diminishes with the increase of \( M_0 \).

The Fig. 6.6 represents the values of non-dimensional load capacity \( W \) against the magnetic parameter for different values of non-dimensional couple stress parameter \( L_0 \) and temperature factor \( \text{Pr.E} \). The figure indicates that \( W \) increases with the increase of both \( L_0 \) and \( M_0 \) and decreases with the increase of \( \text{Pr.E} \). This increasing rate of \( W \) with the increase of \( M_0 \) is greater for higher values of \( L_0 \). The figure also reveals that the decreasing rate of \( W \) with the increase of \( \text{Pr.E} \) is less for higher values of \( M_0 \).

For close scrutiny, Table 6.1 presents some values of non-dimensional load capacity \( W \) and centre of pressure \( X_1 \) with the variation of temperature factor \( \text{Pr.E} \), for different values of magnetic parameter \( M_0 \) and
non-dimensional squeezing velocity \( V_n \) at the two slider bearings (inclined and parabolic-arc; \( \text{Pr.E} = 0 \) represents non-thermal case while \( \text{Pr.E} \neq 0 \) represents thermal case). The table shows that \( W \) increases and \( X_i \) decreases with the increase of \( \text{Pr.E} \). The table also reveals that \( W \) for parabolic-arc slider bearing is greater than that of inclined slider bearing corresponding to other identical parameters, particularly the value of \( A \) under consideration. Further, the effect of temperature (i.e., \( \text{Pr.E} \)) is more sensitive in the case of parabolic slider bearing.

In Table 6.2, we have represented the values of inlet and outlet film height ratio at the point where \( W \) is maximum (i.e., \( A_{\text{wmax}} \)) and maximum load capacity (i.e., \( W_{\text{max}} \)) with the variation of non-dimensional couple stress parameter \( L_0 \) for different values of magnetic parameter \( M_0 \) and temperature factor \( \text{Pr.E} \). The table shows that \( W_{\text{max}} \) decreases with the increase of both \( L_0 \) and \( \text{Pr.E} \) and increases with the increase of \( M_0 \). The decreasing rate is higher for lower values of \( L_0 \). The table also indicates that \( W_{\text{max}} \) increases with the increase of both \( L_0 \) and \( M_0 \) and decreases with the increase of \( \text{Pr.E} \). This increasing rate is higher for higher values of \( L_0 \) and \( M_0 \).
6.6. Conclusions:

We conclude the following remarks:

1. The effect of temperature is more significant with higher values of couple stress parameter of the polymeric fluids used as lubricant. The increase of temperature decreases load capacity much more from higher values of couple stress parameter.

2. The effect of magnetic field increases the load bearing capacity but this enhancement is lowered in presence of temperature effect. This decreasing rate of load bearing capacity with the increase of the effect of temperature is less for higher values of magnetic field.

3. The effect of temperature is more sensitive in both inlet and outlet region of the bearings under consideration.

4. The effect of magnetic field decreases the mean temperature but this decreasing rate is greater for higher values of magnetic field parameter.
Nomenclature:

a  length of the upper surface
A  inlet-outlet film height ratio
$A_{1,\ldots,A_3}$ defined values
$A_{\text{wmax}}$ inlet-outlet film ratio corresponding to maximum load capacity
B  uniform magnetic field in y-direction
$b, B_{1,\ldots,B_{17}}$ defined values
$B_n, B_i$ non-dimensional coefficient of temperature
$c$ integrating constant
$c_{1,\ldots,c_{15}}$ defined values
$c_p$ specific heat
E  Eckert number
$E_{Z_1}, E_Z$ dimensional, non-dimensional induced electric field in z-direction
$f$ a defined value
$h, H$ dimensional, non-dimensional film height
$h(x)$ function of $x$
$h_o$ minimum film thickness
$k$ thermal conductivity
$L, L_0$ non-dimensional couple stress parameter, $L$ at $P = 0$
$M$ non-dimensional viscosity coefficient
$M_o$ Hartmann number at $h = h_o$
$N$ a defined value
p, P  dimensional, non-dimensional fluid pressure
Pr  Prandtl number
Q  non-dimensional volume flow rate per unit width
R, S  defined values
t, T  dimensional, non-dimensional fluid temperature
t_m, T_m  dimensional, non-dimensional mean temperature
t_o  temperature at p = 0 (i.e., ambient temperature)
u, v  dimensional fluid velocity components along x-, y-directions
U, V  non-dimensional velocity components along x-, y-directions
u_o  uniform sliding velocity of lower surface
v_o, V_o  dimensional, non-dimensional squeezing velocity of upper surface
W, W_{max}  non-dimensional load bearing capacity, maximum value of W
x, y  Cartesian coordinates in the flow region
X, Y  non-dimensional Cartesian coordinates
X_1  centre of pressure
Z  Cartesian coordinate along the length of the surface
\alpha, \beta  defined values
\beta_1, \beta_2  coefficients of temperature
\gamma, \delta, \Delta r, \Gamma_r  defined values (r = 1,2,3)
\eta, \eta_o  couple stress parameter, \eta at p = 0
\mu, \mu_o  coefficient of viscosity, \mu at p = 0
\sigma  electrical conductivity
Table 6.1 Values of $W$ and $X_1$ against $Pr.E$ for two values of both $V_o$ and $M_o$ and for two slider bearings with $A = 1.6$ and $L_o = 4$.

<table>
<thead>
<tr>
<th>$Pr.E = 0.0$</th>
<th>$Pr.E = 0.3$</th>
<th>$Pr.E = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$ $X_1$</td>
<td>$W$ $X_1$</td>
<td>$W$ $X_1$</td>
</tr>
<tr>
<td>Inclined slider bearing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_o$</td>
<td>$M_o$</td>
<td>$W$</td>
</tr>
<tr>
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<td>3</td>
<td>11.6450</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>11.7347</td>
</tr>
<tr>
<td>-0.01</td>
<td>3</td>
<td>12.0332</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>12.1259</td>
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<tr>
<td>Parabolic-arc slider bearing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_o$</td>
<td>$M_o$</td>
<td>$W$</td>
</tr>
<tr>
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<td>12.8370</td>
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<tr>
<td></td>
<td>5</td>
<td>12.9243</td>
</tr>
<tr>
<td>-0.01</td>
<td>3</td>
<td>13.3485</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>13.4389</td>
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</tbody>
</table>
Table 6.2 Values of $A_{\text{Wmax}}$ and $W_{\text{max}}$ against $L_0$ for two values of both $\text{Pr.E}$ and $M_0$.

<table>
<thead>
<tr>
<th>$L_0$</th>
<th>$A_{\text{Wmax}}$</th>
<th>$W_{\text{max}}$</th>
<th>$A_{\text{Wmax}}$</th>
<th>$W_{\text{max}}$</th>
<th>$A_{\text{Wmax}}$</th>
<th>$W_{\text{max}}$</th>
<th>$A_{\text{Wmax}}$</th>
<th>$W_{\text{max}}$</th>
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</thead>
<tbody>
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<td>2.2032</td>
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<td>2.1705</td>
<td>0.2731</td>
</tr>
<tr>
<td>1</td>
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<td>0.9399</td>
<td>1.6153</td>
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<td>5</td>
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<td>18.1699</td>
<td>1.5102</td>
<td>15.5681</td>
<td>1.5115</td>
<td>15.8683</td>
</tr>
<tr>
<td>7</td>
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<td>27.9405</td>
<td>1.4938</td>
<td>28.0231</td>
</tr>
</tbody>
</table>
Fig. 6.1 Geometry of two slider bearings:

(1) Inclined slider bearing,      (2) Parabolic slider bearing.
Fig. 6.2 $T_m$ against $X$ for $\Lambda = 1.4$, $L_o = 4$ and for

Pr.E = 0.5, [(1) $M_o = 0$, (2) $M_o = 3$, (3) $M_o = 5$];
Pr.E = 0.2, [(4) $M_o = 0$, (5) $M_o = 3$, (5) $M_o = 5$].
Fig. 6.3 P against X for $A = 1.4$, $L_o = 4$ and for

$Pr.E = 0.2$, [(1) $M_o = 5$, (2) $M_o = 3$, (3) $M_o = 0$];

$Pr.E = 0.5$, [(4) $M_o = 5$, (5) $M_o = 3$, (6) $M_o = 0$].
Fig. 6.4 W against A for $L_0 = 4$ and for

Pr.$E = 0.0$, [(1) $M_0 = 5$, (2) $M_0 = 3$, (3) $M_0 = 0$];
Pr.$E = 0.3$, [(4) $M_0 = 5$, (5) $M_0 = 3$, (6) $M_0 = 0$].
Fig. 6.5 $W$ against $L_o$ for $A = 1.6$ and for

Pr.$E = 0.0$, [(1) $M_o = 3$, (2) $M_o = 2$, (3) $M_o = 1$];

Pr.$E = 0.3$, [(4) $M_o = 3$, (5) $M_o = 2$, (6) $M_o = 1$].
Fig. 6.6 $W$ against $M_o$ for $A = 1.6$ and for

- $L_o = 4$, [(1) Pr.E = 0.0, (2) Pr.E = 0.3, (3) Pr.E = 0.6];
- $L_o = 2$, [(4) Pr.E = 0.0, (5) Pr.E = 0.3, (6) Pr.E = 0.6].