Chapter 7

Single Image Super-Resolution using directionlets on Noisy images

As in the case of color images, the single image super resolution method using directionlets can be extended to noisy low resolution images also. Since the learning based method involves mapping between the low resolution image and the training set, pre-processing using bilateral filter is done here. Advantage is that the same training set which has been used in grey image super-resolution can be used here. The new super resolution method is found to be effective for all types noises used.
7.1 Introduction

The super-resolution methods described in the previous chapters considered low resolution images free of noise. But this is not the actual case. An image is often corrupted by noise in its acquisition, recording and transmission. The performance of sensors is affected by a variety of factors, such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. For instance, while acquiring images with a CCD camera, light levels and sensor temperatures are major factors affecting the amount of noise in the resulting image. Images are corrupted during transmission principally due to interference in the channel used for transmission. For example, an image transmitted using a wireless network might be corrupted as a result of lightening or other atmospheric disturbances [76]. These random distortions make it difficult to perform the required picture processing.

With the exception of spatially periodic noise, noise is independent of spatial coordinates and is uncorrelated with respect to the image itself. That is, there is no correlation between pixel values and values of noise components. Image denoising methods are used to remove the additive noise while retaining as much as possible the important signal features.

The effects of directionlet-based single image super resolution method on noisy images is presented here. The training set contains directionlet transform coefficients of high resolution images and their low resolution images. The idea used is that the noisy input low resolution image is decomposed into different frequency bands using directionlet transform. These coefficients are compared with the corresponding training set coefficients to select the most similar ones. The higher bands of high resolution images are learned from the training set. The low bands are obtained from the low resolution noisy image. The inverse directionlet transform of all these bands gives the super resolved image of the noisy image.
7.2 Overview: Noises

Unlike analog signals which are more prone to noises, discreteness nature of digital signals offers some built-in tolerance to noise. The three most common types of random noise likely to be encountered in images are: Gaussian noise, Salt and Pepper noise, and Speckle noise.

Gaussian noise

Because of its mathematical traceability in both the spatial and frequency domains, Gaussian noise models are used frequently in practice. In fact, this traceability is so convenient that it often results in Gaussian models being used in situations where they are marginally applicable at best. The Probability Distribution Function (PDF) of a Gaussian random variable \( z \), is given by

\[
P(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\]

where \( z \) represents gray level, \( \mu \) is the mean or average value of \( z \), and \( \sigma \) is its standard deviation. The standard deviation squared, \( \sigma^2 \), is called the variance of \( z \).

Salt and Pepper Noise (Impulse noise)

The PDF of impulse noise is given by equation

\[
P(z) = \begin{cases} 
P_a; & \text{for } z = a \\ P_b; & \text{for } z = b \\ 0; & \text{otherwise} \end{cases}
\]

(7.2)

If \( b > a \), gray level \( b \) will appear as a light dot in the image and level \( a \) will appear like a dark dot. Impulse noise will resemble salt and pepper granules randomly distributed over the image. For this reason bipolar noise is also called Salt and Pepper noise.

“Spike” or impulse noise that drives the intensity values of random pixels to either their maximum or minimum values. It is simple to handle. They occur when
pixel’s intensities are either driven to minimum or maximum values. An effective noise reduction method for this type of noise involves the usage of a median filter, morphological filter or harmonic mean filter \[^3\]. Salt and pepper noise creeps into images in situations where quick transients, such as faulty switching, take place.

**Speckle Noise**

It is a form of multiplicative noise in which the intensity values of the pixels in the image are multiplied by random values.

Figure 7.1 shows images with these noises.

![Figure 7.1: Images with different noises](image)

(a) original  
(b) Gaussian  
(c) salt and pepper  
(d) speckle

7.3 **Denoising Methods**

Many denoising methods have been developed over the years. Multi resolution analysis has been proven to be an important tool for eliminating noise in signals. Using multi resolution analysis, it is possible to distinguish between noise and
image information better at one resolution level than another. The problem of image de-noising can be summarized as follows. Let \( A(i,j) \) be the noise-free image and \( B(i,j) \) the image corrupted with independent Gaussian noise \( Z(i,j) \), then

\[
B(i,j) = A(i,j) + Z(i,j)
\]  

(7.3)

The problem is to estimate the desired signal as accurately as possible according to some criteria. There exist many denoising algorithms which uses averaging filter, median filter etc. Among many denoising methods, wavelet thresholding is one of the most popular approaches.

### 7.3.1 Wavelet Methods

In wavelet thresholding, a signal is decomposed into its approximation (low-frequency) and detail (high-frequency) sub-bands. Since most of the image information is concentrated in a few large coefficients, the detail sub-bands are processed with hard or soft thresholding operations. In the wavelet domain, if an orthogonal wavelet transform is used, the problem can be formulated as

\[
Y(i,j) = W(i,j) + N(i,j)
\]  

(7.4)

where \( Y(i,j) \) is noisy wavelet coefficient, \( W(i,j) \) is true coefficient and \( N(i,j) \) noise [93]. De-noising of natural images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet transform yields a large number of small coefficients and a small number of large coefficients. Simple de-noising algorithms that use the wavelet transform consist of three steps.

1. Calculate the wavelet transform of the noisy signal.

2. Modify the noisy wavelet transform coefficients according to some rule. Coefficients that are supposed to be affected by noise are replaced by zero or by another suitable value, and also the other coefficients may be modified. This process is called thresholding.
3. Compute the inverse transform using the modified coefficients to obtain the noise free image.

7.3.2 Bilateral filter
The bilateral filter takes a weighted sum of pixels in a local neighborhood; the weights depend on both the spatial distance and the intensity distance. In this way, edges are preserved well while noise is averaged out. The bilateral filter is a nonlinear filter that does spatial averaging without smoothing edges; it has shown to be an effective image denoising technique [106], [56].

7.4 Single image super resolution in noisy images
Single image super resolution method described in previous chapters can easily be extended to noisy images. The noisy low resolution image is super resolved to a noiseless high resolution image. The method explained here is based on the paper [19] and single image super resolution method using directionlet transform.

Figure 7.2: Block diagram of noisy image super resolution using directionlet transform

To get the noise free high resolution image of an input noisy low resolution image, it’s high resolution frequency bands are learned from a training set. The training set contains directionlet coefficients of patches of noise free high resolution images and their low resolution images. To obtain the low frequency bands, up sample the low resolution noisy image patches and apply the directionlet transform on it. The bands AL and AH obtained by this process are used as the AL and AH
of the high resolution image. The Figure 7.2 shows block diagram of noisy image super resolution using directionlet transform.

**Implementation and Discussions**

**Implementation**

The same training set used in the earlier implementations with grey level images is used here. The training set contains directionlet transform coefficients corresponding to noiseless low resolution patches and high resolution patches. Experiments are conducted with different types of noises. Three types of noises are used: Gaussian, Speckle, Salt and Pepper. The low resolution noisy images are obtained by adding noises in low resolution images. A magnification factor of q=2 is used.

**Discussions**

The results are shown in Figure 7.3. Different types of noises with standard deviation 0.1 are added to the low resolution image to simulate different noisy images. The noisy input image itself is used for obtaining AL and AH components and matching during the learning process. Here the new method is compared with Sapan et al method because this method uses wavelet denoising method. Figures 7.3(a), (d), (g) are low resolution images with noises Gaussian, Speckle and Salt and Pepper (with standard deviation $\sigma=0.1$). Figures 7.3(b), (e), (h) are super resolved images using Sapan et al method and Figures 7.3(c), (f), (i) are super resolved images of Figures 7.3(a), (d), (g) using new directionlet method, without using any preprocessing for noise removal. From results it is observed that smooth regions of super resolved image is still noisy and when compared with the existing method, quality of super resolved image is not good.
Figure 7.3: (a),(d),(g) Low resolution images with gaussian, speckle and salt and pepper noises \( \sigma = 0.1 \) (b),(e), (h) Super resolved images using Sapan et al method (c), (f), (i) super resolved images of (a), (d), (g) respectively.

### 7.5 Single image super resolution in noisy images using Bilateral filter

Since super resolution involves mapping between input image patches and training set patches, the noisy input image is preprocessed before the process of super resolution. The above mentioned method is modified by applying a bilateral filter on low resolution input image. It’s block diagram is shown in Figure 7.4. The
input noisy image is pre processed using bilateral filter. The preprocessed noisy image is divided into patches and the Directionlet transform is applied on input noisy image patches and the high frequency bands HL, HH, VL, VH, DL, DH are learned from the training set. The low frequency bands are obtained from the input image. The inverse directionlet transform gives the almost noise free high resolution image.

![Block diagram of noisy image super resolution using directionlet transform with Bilateral filter for preprocessing](image)

**Implementation and Discussions**

The experiments are done on images with Gaussian noise, Speckle noise and Salt and Pepper noise of different values of standard deviation (variance). The same training set used with grey images is used here.

### 7.5.1 Single image super resolution on images with Gaussian noise

Table 7.1 shows the SNR values obtained for super resolving different low resolution images with Gaussian noise of standard deviation $\sigma = 0.1, 0.2, 0.3$. From the table it is clear that the new directionlet method gives better result than Sapan et al wavelet method. For example SNR value for Butterfly is 23.92dB while it is only 19.55dB for Sapan et al method. It is also clear that as noise increases the quality of super resolved image decreases.

![Figure 7.5](image)  Figure 7.5, Figure 7.6 show the results obtained with Gaussian noise. The results are compared with super resolved images using Sapan et al method. In the new super resolution method the missing high frequency bands are learned from the training set which are entirely free of noise. The low frequencies are obtained from
Table 7.1: SNR values for different low resolution images with Gaussian noise

<table>
<thead>
<tr>
<th>images</th>
<th>Methods</th>
<th>SNR in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>σ=0.1</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>23.9251</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>19.401</td>
</tr>
</tbody>
</table>

The directionlet transform of interpolated version of low resolution image. Thus the noise present in the low resolution image is almost removed by the averaging process of low pass filters in low frequency bands.

Figures 7.5(a), (d), (g) show low resolution images of 'Barbara' with Gaussian noise of standard deviation σ=0.1, 0.2, 0.3. Figures 7.5(b), (e), (h) show super resolved images using Sapan et al method and Figures 7.5(c), (f), (i) show super resolved images using new directionlet method.

Figures 7.6(a), (d), (g) show low resolution images of 'butterfly' with Gaussian noise σ=0.1, 0.2, 0.3. Figures 7.6(b), (e), (h) show super resolved images using Sapan et al method and Figures 7.6(c), (f), (i) show super resolved images using new method. It is clear that directionlet super resolved images are better than the images super resolved with existing wavelet transform based method.
Figure 7.5: (a),(d),(g) Low resolution images with Gaussian noise ($\sigma=0.1, 0.2, 0.3$) (b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method
Figure 7.6: contd......
Figure 7.6: (a),(d),(g) Low resolution images with Gaussian noise ($\sigma=0.1, 0.2, 0.3$)(b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method.
7.5.2 Single image super resolution on images with Speckle noise

Table 7.2 shows the SNR values obtained for super resolving different low resolution images with Speckle noise of standard deviation $\sigma=0.1, 0.2, 0.3$. The SNR value of Butterfly for $\sigma=0.1$ is 24.59dB for new directionlet method while it is 20.4610dB for existing Sapan et al method.

Table 7.2: SNR values for different low resolution images with speckle noise

<table>
<thead>
<tr>
<th>Images</th>
<th>Methods</th>
<th>SNR in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma=0.1$</td>
</tr>
<tr>
<td>Butterfly</td>
<td>using Sapan et al method</td>
<td>20.4610</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>24.59</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>20.31</td>
</tr>
</tbody>
</table>

Figures 7.7(a), (d), (g) show low resolution images of ’barbara’ with Speckle noise $\sigma=0.1, 0.2, 0.3$. Figures 7.7(b), (e), (h) show images with Sapan et al method and Figures 7.7(c), (f), (i) show super resolved images with new directionlet method.

Figures 7.8(a), (d), (g) show low resolution images of butterfly with Speckle noise $\sigma=0.1, 0.2, 0.3$. Figures 7.8(b), (e), (h) show images with Sapan et al method and Figures 7.8(c), (f), (i) show super resolved images with new directionlet method. It is clear that super resolved images using new method are better than images using existing super resolution method. The directionlet method is better for removing speckle noise also.
Figure 7.7: (a),(d),(g) Low resolution images with Speckle noise ($\sigma=0.1, 0.2, 0.3$) (b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method
Super resolution on noisy images

Figure 7.8: contd......
Figure 7.8: (a),(d),(g) Low resolution images with Gaussian noise ($\sigma=0.1, 0.2, 0.3$) (b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method.
7.5.3 Single image super resolution on images with Salt and Pepper noise

Table 7.3 shows the SNR values obtained for super resolving different low resolution images with Salt and Pepper noise of standard deviation $\sigma=0.1, 0.2, 0.3$. The SNR values for Butterfly is 24.39dB while it is 20.3163dB for Sapan et al method for $\sigma=0.1$.

Table 7.3: SNR values for different low resolution images with Salt and Pepper noise

<table>
<thead>
<tr>
<th>Images</th>
<th>Methods</th>
<th>SNR in DB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma=0.1$</td>
</tr>
<tr>
<td>Butterfly</td>
<td>using Sapan et al method</td>
<td>20.3163</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>24.39</td>
</tr>
<tr>
<td>barbara</td>
<td>using Sapan et al method</td>
<td>14.8965</td>
</tr>
<tr>
<td></td>
<td>Directionlet Method</td>
<td>20.03</td>
</tr>
</tbody>
</table>

Figures 7.9(a), (d), (g) show low resolution images of ’Barbara’ with Salt and Pepper noise $\sigma=0.1, 0.2, 0.3$. Figures 7.9(b), (e), (h) show images with Sapan et al method and Figures 7.9(c), (f), (i) show super resolved images with new directionlet method. It is clear that directionlet based super resolved image is better than images using Sapan et al super resolution method.

Figures 7.10(a), (d), (g) show low resolution images of ’butterfly’ with Salt and Pepper noise $\sigma=0.1, 0.2, 0.3$ etc. Figures 7.10(b), (e), (h) show images with Sapan et al method and Figures 7.10(c), (f), (i) show super resolved images with new method. It is clear that directionlet based super resolved image is better than images using Sapan et al super resolution method.
Figure 7.9: (a),(d),(g) Low resolution images with Salt and Pepper noise ($\sigma=0.1, 0.2, 0.3$) (b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method
Figure 7.10: contd...
Figure 7.10: (a),(d),(g) Low resolution images with Gaussian noise ($\sigma=0.1, 0.2, 0.3$) (b),(e),(h) super resolved images using Sapan et al wavelet method (c),(f),(i) super resolved images using directionlet method.
7.6 Conclusion

The single image super resolution method with directionlet transform is extended to noisy images. The low resolution noisy images are filtered using bilateral filter before undergoing super resolution process. The same training set used with grey image super resolution is used here. Experiments are conducted on different images with different types of noises like Gaussian, Speckle, Salt and Pepper with different variances. Results are compared with the existing wavelet transform method and it is observed that the new directionlet method is able to remove noise contents to a greater extent.
Chapter 8

Conclusion

A brief summary of the research work conducted and the important conclusions thereon are highlighted in this chapter. The scope for further work in this field as an extension of the present study has also been discussed.
8.1 Thesis summary and Conclusions

In this thesis, problem of single image super resolution method is addressed. Resolution beyond the limit of image capturing device is achieved by using directionlet transform to extract high frequency features from the high resolution images in the training set. The presented method is different from other conventional super resolution methods in the way that it is adaptive to local directional variations present in images.

Different methods presented in this thesis are summarized here. Initially a learning based super resolution method using learned wavelets is presented. It is obtained by modifying the method proposed by Jiji et al using patch based approach. Advantage of the new patch based wavelet method is that low resolution images of any size can be super resolved using a single training set. Artifacts are also reduced in the super resolved images using this wavelet method. But this method including traditional methods fail to remove artifacts like ringing effects and aliasing.

Next, a novel learning based super resolution method using directionlet transform is introduced. Advantage of this method is that for each patch, direction of transform is selected according to the information present in it. This method out performs the standard interpolation and wavelet methods. Artifacts like aliasing and ringing effect are also reduced by this method. This new method needs more computations and hence need more computation time. To speed up this process lifting based directionlet transform is used instead of conventional convolution based directionlet transform and this lifting based method is faster than the convolution based directionlet method.

A study is done to analyze the effect of different wavelets on the directionlet based super resolution method. Results show that though the wavelet basis 'rbior1.5' gives better super resolved image compared to other wavelets, it takes three or four times more time compared to 'db4' or 'bior3.3'.
The new single image super resolution method using directionlet transform is extended to color and noisy image also. It is found that the new method based on directionlet is effective in the case of colour and noisy images.

Thus a new directionally adaptive single image super resolution method is developed to super resolve grey, colour and noisy images. It super resolves images to their double size. It can be also used for super resolving to 4 times the size of the original image.

8.2 Suggestions for Future Work

These methods are computationally too expensive for real time applications. One should think of ways to speed up these algorithms. Training set needs large amount of memory and compression techniques can be used to reduce the size of memory. Here low resolution images which are free from blurring are used. The new super resolution method presented here can easily be extended to blurred observations also. Here five sets of directions are selected. By extending to more directions the quality of super resolved image can be increased further. Magnification factor two and four are used in the presented work. This can be extended to higher values.