Chapter – I

Hydromagnetic mixed convective
Heat and Mass Transfer Through a
Porous Medium in a non-uniformly
heated vertical
channel with heat sources and
dissipation
INTRODUCTION

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy and chemical species, have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. For example, in controlling surface temperature by evaporation, cooling controlling polymerization reaction products by injecting suitable reactants along the porous wall of the reactor, distillation of volatile components from a mixture with non-volatiles, are a few technological process in which mass transfer accompanies the transfer of heat. Likewise such combined heat and mass transfer occurs in natural environmental vapourisation of mist and fog.
evaporation in terrestrial bodies of water viz. the oceans, rivers and the ponds, drying of food grains, simultaneous diffusion of metabolic heat and perspiration to control body temperature. Obviously the understanding of this transport process is desirable in order to effectively control the overall transport characteristics. The combined effect of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors (17,24,31,41,47,46,52,54).

Flow through porous media is very prevalent in nature, and therefore the study of flow through porous medium has become of principal interest in many scientific and engineering applications. In the theory of flow through a porous medium, the role of momentum equation or force balance is occupied by the numerous experimental observations summarized mathematically as the Darcy's law. It is observed that the Darcy's law is applicable as long as the Reynolds number based on average grain(pore) diameter does not exceed a value between 1 and 10. But in general the speed of specific discharge increases, the convective forces get developed and the internal stress generated in the fluid due to its viscous nature produces distortions in the velocity field. Also in the case of highly porous media such as fiber glass, papus of dendilion etc., the viscous stress at the surface is able to penetrate into medium and produce gradient. Thus between the specific discharge and hydraulic gradient is inadequate in describing high sped flows or flows near
surfaces which may be either permeable or not. Hence consideration for non-Darcian
description for the viscous flow through porous media is warranted. Saffman (49)
employing statistical method derived a general governing equation for the flow in a
porous medium which takes into account the viscous stress. Later another
modification has been suggested by Brinkman (9).

\[ 0 = -\nabla P - \frac{\mu}{k} \ddot{v} + \mu \nabla^2 \ddot{v} \]

in which \( \mu \nabla^2 \ddot{v} \) is intended to account for the distortions of the velocity profiles
near the boundary. The same equation was derived analytically by Tam (56) to
describe the viscous flow at low Reynolds number past a swarm of small particles.

This problem of combined buoyancy driven thermal and mass diffusion has
been studied in parallel plate geometries by a few authors in the recent times, notably
Gebhart (21), Lai (33,33a), Chen and Mout Soglo (14), Poulikakos (44), Trevisan
and Bejan (57), Mehta and Nanda Kumar (40) and Angiras et al (1).

In the risk assessment of nuclear power plants, the possibility and the
consequences of a melt down of the reactor core are usually considered. During the
course of such an accident molten fuel and coolant may interact. Violent thermal
reaction can dispose the molten fuel into fine particles. These small particles quickly
solidify in the coolant and settle on internal structures of the reactor pressure vessel
forming a saturated porous bed. The question arises, under what conditions the nuclear decay heat can be removed from the particle bed to the ambient coolant by natural convection. Thus the problem of natural convection in saturated porous layers. This analysis of heat transfer in a viscous heat generating fluid also important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids (34,36). The Volumetric heat generation has been assumed to be constant (2,6,7,7a,8,15,42,43) or a function of space variable (12,14,25,26,29,37). For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate (20). The heat losses from the geothermal system in some cases can be treated as if the heat comes from the heat generating sources (27). Keeping this in view, porous medium with internal heat source has been discussed by several authors (11,20,27,28,43).

The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of Magnetohydro dynamic heat transfer. This MHD heat transfer has gained significance owing to recent advancement of space technology. The MHD heat transfer can be divided into two sections. One contains
problems in which the heating is an incidental by product of the electromagnetic fields as in MHD generators and pumps etc, and the second consist of problems in which the primary use of electromagnetic fields is to control the heat transfer(13). With the fuel crisis impending all over the world, there is a great concern to utilize the enormous power beneath the earths crust in the geothermal region(39). Liquid in the geothermal region is an electrically conducting liquid because of high temperature and that they undergo the influence of a magnetic field.

Magneto thermal dynamics phenomenon in a porous medium results from the mutual effect of a magnetic filed and conducting fluid flowing through the porous medium. Examination of the flow model reveals the combined influence of porosity and magnetic field on the velocity, temperature profiles and the local heat transfer etc.

In the above mentioned investigations the bounding walls are maintained at constant temperature. However, there are a few physical situations which warrant the boundary temperature to be maintained non-uniform. It is evident that in forced or free convection flow in a channel (pipe) a secondary flow can be created either by corrugating the boundaries or by maintaining non-uniform wall temperature Such a secondary flow may be of interest in a few technological process. For example in
drawing optical glass fibres of extremely low loss and wide bandwidth, the process of modified chemical vapour deposition (MCVD) [32,50] has been suggested in recent times. Performs from which these fibres are drawn are made by passing a gaseous mixture into a fused – silica tube which is heated locally by an oxy-hydrogen flame. Particulates of SiO$_2$- GeO$_2$ composition are formed from the mixture and collect on the interior of the tube. Subsequently these are fined to form a vitreous deposit as the flame traversed along the tube. The deposition is carried out in the radial direction as the flame traversed along the tube. The deposition is carried out in the radial direction through the secondary flow created due to non-uniform.

All the above mentioned studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to be relevant for fluids with high values of the dynamic viscous flows. Moreover Gehart(22), Gebhart and Mollen dorf(23) have shown that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravitational filed. On the other hand Barletta(3) has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the Nusselt numbers may occur in the fully developed forced convection in tubes. In view of this several authors notably, Soundalgekar and Pop(53) Raptis etc al (48),
Ramana Murthy and Soundelgekar et al (51), Barletta (3,4), Sreevani(55). El-hakein (18), Bulent Yesilata(10). Rossidi schio(48a) and Israel et al (30) have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for some different cases including the natural convection from horizontal cylinder. The natural convection from horizontal cylinder embedded in a porous media has been studied by Fand and Brucker(19). They reported that the viscous dissipation may not be neglected in all cases of natural convection from horizontal cylinders and further, that the inclusion of a viscous dissipation term in porous medium may lead to more accurate correlation equations.

The effect of viscous dissipation has been studied by Nakayama and Pop(38) for steady free convection boundary layer over a non-isothermal bodies of arbitrary shape embedded in porous media. They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. This observation has been pointed out also by Murthy and Singh(*) for the natural convection flow along an isothermal wall embedded in porous medium. They concluded that the effect of viscous dissipation increases as we move from non-Darcy regime to Darcy regime. Costa(16) has analysed a natural convection in enclosures with viscous dissipation. Recently Jambal et al have discussed the effects
enclosures with viscous dissipation. Recently Jambal et al have discussed the effects of viscous dissipation and fluid axial heat conduction heat transfer for non-Newtonian fluids in ducts with uniform wall temperature.

**AUTHOR'S CONTRIBUTION**

We discuss the combined effect of convective heat and mass transfer on hydromagnetic electrically conducting, viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. A uniform magnetic field of strength $H_0$ is applied transverse to the bounding walls. Assuming the magnetic Reynolds to be small, we neglect the induced magnetic field in comparison to the applied field. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken into account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, the temperature, the shear stress and the rate of heat transfer have been analysed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.
We analyse the steady motion of viscous, electrically conducting incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source and the concentration on these walls are taken to be constant. A uniform magnetic of strength $H_0$ is applied transverse to the walls. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosity $\nu$, the thermal conductivity $k$ are treated as constants. We choose a rectangular Cartesian system $\mathbb{R}(x,y)$ with $x$-axis in the vertical direction and $y$-axis normal to the walls. The walls of the channel are at $y = \pm L$. The equations governing the steady hydromagnetic flow, heat and mass transfer are

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (2.1)

Equation of linear momentum:
\[
\rho_e \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - (\sigma_e \mu^2 H_0^2)u - \left( \frac{\mu}{k} \right)u
\] (2.2)

\[
\rho_e \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left( \frac{\mu}{k} \right)v
\] (2.3)

Equation of Energy:

\[
\rho_e C_e \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q + \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \left( \frac{\mu}{\lambda k} + \sigma_e \mu^2 H_0^2 \right) (u^2 + v^2)
\] (2.4)

Equation Diffusion:

\[
\left( u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_c \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)
\] (2.5)

Equation of State:

\[
\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta \rho_e (C - C_e)
\] (2.6)

where \( \rho_e \) is the density of the fluid in the equilibrium state, \( T_e, C_e \) are the temperature and Concentration in the equilibrium state, \( u, v \) are the velocity components along \( O(x, y) \) directions, \( p \) is the pressure, \( T, C \) are the temperature and Concentration in the flow region, \( \rho \) is the density of the fluid, \( \mu \) is the constant coefficient of viscosity, \( C_p \) is the specific heat at constant pressure, \( \lambda \) is the coefficient of thermal conductivity, \( k \) is the is the magnetic permeability of the
porous medium \( \sigma \) is the electrically conductivity and \( \mu \) permeability of the fluid, \( \beta \) is the coefficient of thermal expansion, \( \beta^* \) is the coefficient of expansion with mass fraction, \( D_1 \) is the molecular diffusivity and \( Q \) is the strength of the constant internal heat source.

In the equilibrium state

\[
0 = -\frac{\partial \rho_x}{\partial x} - \rho_x g \tag{2.7}
\]

Where \( p = p_e + p_d, p_d \) being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

\[
Q = \frac{1}{2L} \int_{-L}^{L} u \, dy. \tag{2.8}
\]

The boundary conditions for the velocity and temperature fields are

\[
\begin{align*}
    u &= 0, \quad v = 0 \quad \text{on } y = \pm L, \\
    T - T_e &= \gamma (\delta x / L) \quad \text{on } y = \pm L, \\
    C &= C_1 \quad \text{on } y = -L, \\
    C &= C_2 \quad \text{on } y = +L
\end{align*} \tag{2.9}
\]

\( \gamma \) is chosen to be twice differentiable function, \( \delta \) is a small parameter characterizing the slope of the temperature variation on the boundary.

In view of the continuity equation we define the stream function \( \psi \) as
\[ u = -\psi_y, \quad \nu = \psi_x \]  

(2.9)

the equation governing the flow in terms of \( \psi \) are

\[
\begin{align*}
\left[ \frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial (\nabla^2 \psi)}{\partial x} \right] &= \nu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)^2 - \beta g \frac{\partial T}{\partial y} - \\
- \beta' g \frac{\partial C}{\partial y} - M^2 \frac{\partial^2 \psi}{\partial y^2} &= \lambda \nabla^2 \theta + Q + \mu \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\mu}{k} \right) \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2
\end{align*}
\]  

(2.10)

\[
\frac{\partial \psi}{\partial y} \left( \frac{\partial C}{\partial x} - \frac{\partial C}{\partial y} \right) = D_T \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)
\]  

(2.11)

\[
\frac{\partial \psi}{\partial x} \left( \frac{\partial C}{\partial x} - \frac{\partial C}{\partial y} \right) = D_T \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)
\]  

(2.12)

Introducing the non-dimensional variables in (2.10)- (2.12) as

\[
(x', y') = (x, y)/L, \quad (u', v') = (u, v)/U, \quad \theta = \frac{T - T_e}{\Delta T_e}, \quad C^* = \frac{C - C_1}{C_2 - C_1}
\]

\[
p' = \frac{p}{\rho U^2}, \quad \gamma' = \frac{\gamma}{\Delta T_e}
\]  

(2.13)

(under the equilibrium state \( \Delta T_e = T_e(L) - T_e(-L) = \frac{QT^2_e}{\lambda} \))

the governing equations in the non-dimensional form (after dropping the dashes) are
\[
R \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, y)} = \nabla^4 \psi + \left( \frac{G}{R} (\theta_x + NCy) - D^{-1} \nabla^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2} \right) 
\]

(2.14)

and the energy diffusion equations in the non-dimensional form are

\[
PR \left( \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \right) = \nabla^2 \theta + 1 + \left( \frac{PR^2 E_c}{G} \right) \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} \right)^2 + D^{-1} \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 
\]

(2.15)

\[
RSc \left( \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \right) = \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) 
\]

(2.16)

where

\[
R = \frac{UL}{\nu} \quad \text{(Reynolds number)}
\]

\[
G = \frac{\beta g \Delta T L^3}{\nu^2} \quad \text{(Grashof number)}
\]

\[
P = \frac{\mu c_p}{k_1} \quad \text{(Prandtl number),}
\]

\[
D^{-1} = \frac{L^2}{k} \quad \text{(Darcy parameter),}
\]

\[
E_c = \frac{\beta g L^3}{C_p} \quad \text{(Eckert number)}
\]

\[
M^2 = \frac{\sigma \mu \beta H_0^2 L^2}{\rho \nu} \quad \text{(Hartmann Number)}
\]

\[
N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad \text{(Buoyancy Number)}
\]
The corresponding boundary conditions are

\[ \psi(+) - \psi(-) = 1 \]

\[ \frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \]  \hfill (2.17a)

\[ \theta(x, y) = f(\delta x) \quad \text{on } y = \pm 1 \]  \hfill (2.17b)

\[ C = 0 \quad \text{on } y = -1 \]  \hfill (2.17c)

\[ C = 1 \quad \text{on } y = 1 \]  \hfill (2.17c)

\[ \frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \]  \hfill (2.18)

The value of \( \psi \) on the boundary assumes the constant volumetric flow in consistent with the hypothesis (2.8). Also, the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function \( \gamma(x) \).

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

\[ \bar{x} = \delta x \]

With this transformation the equations (2.14) - (2.16) reduce to
and the energy & diffusion equations in the non-dimensional form are

\[
PR\delta \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = F^2 \theta + 1 + \left( \frac{PR^2 E_s}{G} \right) \left( \delta^2 \psi^2 \right) + \delta^2 \left( \frac{\partial^2 \psi}{\partial x^2} \right) + (D^{-1} + M^2) \left( \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right)
\]

(3.2)

\[
\delta \text{Re} \left( \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} \right) = F^2 C
\]

(3.3)

where

\[
F^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]

for small values of the slope $\delta$, the flow develops slowly with axial gradient of order $\delta$ and hence we take

\[
\frac{\partial}{\partial x} \approx O(1)
\]

We follow the perturbation scheme and analyse through first order as a regular perturbation problem at finite values of $R, G, P, Sc$ and $D^{-1}$.

Introducing the asymptotic expansions

\[
\psi(x, y) = \psi_0(x, y) + \delta \psi_1(x, y) + \delta^2 \psi_2(x, y) + \ldots
\]

\[
\theta(x, y) = \theta_0(x, y) + \delta \theta_1(x, y) + \delta^2 \theta_2(x, y) + \ldots
\]

\[
C(x, y) = C_0(x, y) + \delta C_1(x, y) + \delta^2 C_2(x, y) + \ldots
\]

(3.4)
On substituting (3.4) in (3.1) – (3.3) and separating the like powers of \( \delta \) the equations and respective conditions to the zeroth order are

\[
\psi_{0,yyyyy} - M_1^2 \psi_{0,yy} = \frac{G}{R} (\theta_{0,y} + NC_{0,y})
\]

\[
\theta_{0,yy} = -\frac{PR^2 Ec}{G} \psi_{0,yy}^2 - \frac{PM_1^2 Ec}{G} \psi_{0,y}^2
\]

\[
C_{0,yy} = 0
\]

with

\[
\psi_0^{(+1)} \psi^{(-1)} = 1,
\]

\[
\psi_{0,y} = 0, \quad \psi_{0,x} = 0 \quad \text{at} \quad y = \pm 1
\]

\[
\theta_0(\pm 1) = \gamma(x) \quad \text{at} \quad y = \pm 1
\]

\[
C_0(-1) = 0 \quad C_0(+1) = 1
\]

and to the first order are

\[
\psi_{1,yyyyy} - M_1^2 \psi_{1,yy} = \frac{G}{R} (\theta_{1,y} + NC_{1,y}) + R(\psi_{0,y} \psi_{0,xy} - \psi_{0,x} \psi_{0,yy})
\]

\[
\theta_{1,yy} = PR(\psi_{0,x} \theta_{0,y} - \psi_{0,y} \theta_{0,x}) - \frac{PEc}{G} (R^2 \psi_{1,yy}^2 + M_1^2 \psi_{1,y}^2)
\]

\[
C_{1,yy} = RSc (\psi_{0,y} C_{0,x} - \psi_{0,x} C_{0,y})
\]

\[
\psi_{1(+1)} \psi_{1(-1)} = 0
\]

\[
\psi_{1,y} = 0, \quad \psi_{1,x} = 0 \quad \text{at} \quad y = \pm 1
\]

\[
\theta_1(\pm 1) = 0 \quad \text{at} \quad y = \pm 1
\]
\[ C_0(-1) = 0, C_0(+1) = 0 \]  \hspace{1cm} (3.12)

Assuming \( Ec \ll 1 \) to be small we take the asymptotic expansions as

\[ \psi_0(x, y) = \psi_{00}(x, y) + Ec\psi_{01}(x, y) + \ldots \]

\[ \psi_1(x, y) = \psi_{10}(x, y) + Ec\psi_{11}(x, y) + \ldots \]

\[ \theta_0(x, y) = \theta_{00}(x, y) + Ec\theta_{01}(x, y) + \ldots \]

\[ \theta_1(x, y) = \theta_{10}(x, y) + Ec\theta_{11}(x, y) + \ldots \]

\[ C_0(x, y) = C_{00}(x, y) + EcC_{01}(x, y) + \ldots \]

\[ C_1(x, y) = C_{10}(x, y) + EcC_{11}(x, y) + \ldots \]  \hspace{1cm} (3.13)

Substituting the expansions (3.13) in equations (3.5)-(3.12) and separating the like powers of \( Ec \) we get the following equations

\[ \theta_{00,yy} = -1 \quad , \quad \theta_{00}(\pm 1) = f(x) \]  \hspace{1cm} (3.14)

\[ C_{00,yy} = 0 \quad , \quad C_{00}(-1) = 0, C_{00}(+1) = 1 \]  \hspace{1cm} (3.15)

\[ \psi_{00,yyyy} - M_1^2 \psi_{00,yy} = -\frac{G}{R} (\theta_{00,yy} + NC_{00,yy}) \]

\[ \psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,y} = 0, \psi_{00,x} = 0 \quad \text{at} \; y = \pm 1 \]

\[ \theta_{01,yy} = -\frac{PM_1^2}{G} \psi_{00,yy}^2 - \frac{PR^2}{G} \psi_{00,yy}^2, \quad \theta_{01}(\pm 1) = 0 \]  \hspace{1cm} (3.17)
\[ C_{01,yy} = 0 \quad , \quad C_{01}(\pm 1) = 0 \quad (3.18) \]

\[ \psi_{01,yyyy} - M_1^2 \psi_{01,yy} = -\frac{G}{R} (\theta_{01,y} + NC_{01,y}) , \]

\[ \psi_{01}(+1) - \psi_{01}(-1) = 0, \quad (3.19) \]

\[ \psi_{01,y} = 0, \psi_{01,x} = 0 \quad at \quad y = \pm 1 \]

\[ \theta_{10,yy} = RP(\psi_{00,y} \theta_{00,x} - \psi_{00,x} \theta_{00,y}) \quad , \quad \theta_{10}(\pm 1) = 0 \quad (3.20) \]

\[ C_{10,yy} = RP(\psi_{00,y} C_{00,x} - \psi_{00,x} C_{00,y}) \quad , \quad C_{10}(\pm 1) = 0 \quad (3.21) \]

\[ \psi_{10,yyyy} - M_1^2 \psi_{10,yy} = -\frac{G}{R} (\theta_{10,y} + NC_{10,y}) + 
\]

\[ + R(\psi_{00,y} \psi_{00,yy} - \psi_{00,x} \psi_{00,yy}), \quad (3.22) \]

\[ \psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,y} = 0, \psi_{10,x} = 0 \quad at \quad y = \pm 1 \]

\[ \theta_{11,yy} = RP(\psi_{00,y} \theta_{11,x} - \psi_{11,x} \theta_{00,y}) \quad , \quad \theta_{1}(\pm 1) = 0 \quad (3.23) \]

\[ C_{11,yy} = RP(\psi_{00,y} C_{00,x} - \psi_{11,x} C_{00,y}) \quad , \quad C_{11}(\pm 1) = 0 \quad (3.23) \]

\[ \psi_{11,yyyy} - M_1^2 \psi_{11,yy} = -\frac{G}{R} (\theta_{11,y} + NC_{11,y}) + R(\psi_{00,y} \psi_{11,yy} - 
\]

\[ - \psi_{00,x} \psi_{01,yy} + \psi_{01,y} \psi_{00,yy} - \psi_{01,x} \psi_{00,yy}), \quad (3.24) \]

\[ \psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \quad at \quad y = \pm 1 \]
Solving the equations (3.14)- (3.22) subject to the relevant boundary conditions we obtain

\[ \theta_{\infty} = 0.5(1 - y^2) + \gamma(x) \]

\[ C_{00} = 0.5(y + 1) \]

\[ \psi_{00} = a_4 Ch(M_1, y) + a_5 Sh(M_1, y) + a_6 y + a_7 + a_8 y^3 + a_9 y^2 \]

\[ \theta_{01} = 0.5 PM_1^2 (y^2 - 1) \]

\[ C_{01} = a_6 (y^2 - 1) \]

\[ \psi_{01} = a_{10} Sh(M_1, y) + a_{11} y + a_{12} + a_4 y^3 \]

\[ \theta_{10} = a_{24} y^2 + a_{25} y^3 + a_{26} y^4 + a_{27} y^5 + a_{28} y^6 + \]

\[ + (a_{29} + y a_{22}) Ch(M_1, y) + (a_{21} + y a_{23}) Sh(M_1, y) \]

\[ C_{10} = a_{31} (y^2 - 1) + a_{32} (y^3 - y) + a_{33} (y^4 - 1) + \]

\[ + a_{34} (y^5 - y) + a_{35} (y^6 - 1) + (a_{36} + y a_{34}) Ch(M_1, y) - \]

\[ - Ch(M_1)) + a_{37} (Sh(M_1, y) - y ShM_1) + \]

\[ + a_{39} (y Sh(M_1, y) - Sh(M_1)) \]

\[ \psi_{10} = b_8 Ch(M_1, y) + b_9 Sh(M_1, y) + b_{10} y + b_{11} + \phi(y) \]
\[ \phi(y) = a_{70}y^2 + a_{71}y^3 + a_{72}y^4 + a_{73}y^5 + a_{74}y^6 + \\
+ a_{75}y^7 + (b_1y + b_3y^2 + b_3y^3)Ch(M_1y) + \\
+ (b_2y + b_4y^2 + b_6y^4)Sh(M_1y) + b_7y^4Sh(M_1y) \\
\]

\[ \theta_{11} = b_{15}y^2 + b_{16}y^3 + b_7Ch(2M_1y) + b_{18}Sh(2M_1y) + b_{19}y^4 + b_{20}y^6 + \\
+ b_{21}y^8 + b_{22}y^{10} + b_{23}y^{11} + b_{24}y^{12} + b_{25}y^8Sh(2M_1y) + \\
+ b_{26}y^7Ch(2M_1y) + b_{27}y^6Sh(2M_1y) + b_{28}y^4Ch(2M_1y) + \\
+ b_{29}y^6Sh(2M_1y)b_{30}y^3Ch(2M_1y) + b_{31}y^2Sh(2M_1y) + \\
+ b_{32}yCh(2M_1y) + b_{33}Sh(2M_1y) + b_{34}y + b_{35} \\
\]

\[ \psi_{11} = b_{34} + b_{35}y + b_{36}Ch(M_1y) + b_{37}Sh(M_1y) + \phi_1(y) \\
\phi(y) = b_{34}y^{14} + b_{35}y^{13} + b_{36}y^{12} + b_{37}y^{11} + b_{38}y^{10} + b_{39}y^9 + b_{40}y^6 + b_{41}y^7 + \\
+ b_{42}y^4 + b_{43}y^3 + b_{44}Ch(2M_1y) + b_{45}Sh(2M_1y) + b_{46}y^8Ch(2M_1y) + \\
+ b_{47}y^7Sh(2M_1y) + b_{48}y^6Ch(2M_1y) + b_{49}y^5Sh(2M_1y) + \\
+ b_{50}y^4Ch(2M_1y) + b_{51}y^3Sh(2M_1y) + b_{52}y^2Ch(2M_1y) + \\
+ b_{53}ySh(2M_1y) \\
\]

where \( a_1, a_2, \ldots, a_{75}, b_1, b_2, \ldots, b_{53} \) are constants given in the appendix
5. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by

\[ \tau = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=\pm L} \]

which in the non-dimensional form reduces to

\[ \tau = \left( \frac{\mu U}{a} \right) = (\psi_{yy} - \delta^2 \psi_{xx}) \]

\[ = [\psi_{00,yy} + Ec\psi_{01,yy} + \delta(\psi_{10,yy} + Ec\psi_{11,yy} + O(\delta^2))]_{y=\pm 1} \]

and the corresponding expressions are

\[ (\tau)_{y=1} = d_4 + Ec d_4 + \delta d_4 + O(\delta^2) \]

\[ (\tau)_{y=-1} = d_6 + Ec d_7 + \delta d_6 + O(\delta^2) \]

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

\[ Nu = \frac{1}{\theta_m - \theta_w} \left( \frac{\partial \theta}{\partial y} \right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (Nu)_{y=1} = \frac{(d_{10} + \delta(d_{11} + d_{12}))}{(d_8 - \gamma(x) + \delta d_9)} \]

\[ (Nu)_{y=-1} = \frac{(-d_{10} + \delta(d_{12} - d_{11}))}{(d_8 - \gamma(x) + \delta d_9)} \]
where $d_1, d_4, \ldots, d_{10}$ are constants given in the appendix.

6. DISCUSSION OF THE NUMERICAL RESULTS

The aim of the analysis is to discuss the effect of dissipation on the convective heat and mass transfer through a porous medium confined in a vertical channel bounded by vertical channel on whose walls a non uniform temperature is maintained and we take the Prandtl number $P = 0.71$. We note that the channel walls are heated or cooled according the Grashof number $G$ is positive or negative. The non-linear coupled governing the flow have been solved by using the perturbation technique. The effect of different parameters governing the flow, heat and mass transfer on the velocity, temperature distributions is analysed and exhibited in figures (1)-(19). The axial velocity is in the vertically downward direction and hence $-u$ is actual velocity and $u > 0$ represents the reversal flow. The axial velocity $u$ is shown in fig1-5. It is found from fig1 that for $|G| \leq 10^3$, there is no reversal flow. For $G=3 \times 10^3$, we find reversal flow in the region $-0.2 \leq y \leq 0.4$. With increase in the Grashof number $G$, the region of reversal flow enlarges. For smaller and higher values of $G$, the maximum $u$ occurs in the mid region, while for $|G|=3\times10^3$, maximum $u$ occurs at $y = 0.4$ in the case of heating while it occurs at $y = -0.8$ in the case of cooling of the walls. In both heating and cooling of the channel, walls we find that the magnitude of
Fig. 1 Profiles for axial velocity $u$ with $G$

\[ D^1 = 2 \times 10^3, \text{Sc}=1.3, N=1 \]

<table>
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<td>$-3 \times 10^3$</td>
<td>$-5 \times 10^3$</td>
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</tr>
</tbody>
</table>

Fig. 1a $u$ with $D^{-1}$

\[ G=3 \times 10^3, N=1, \text{Sc}=1.3 \]

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</tr>
</tbody>
</table>
u reduces with $|G| \leq 3 \times 10^3$ and later enhances with $|G| \geq 5 \times 10^3$. The effect of porosity of the medium is exhibited in fig. 1a. For $D' \leq 10^3$, we find a reversal flow in the fluid region and the size of the reversal flow in the fluid region with increase in $D'$. This reversal flow disappears for $D' \geq 3 \times 10^3$. It is found that, the axial velocity ‘u’ reduces in magnitude for $D' \geq 3 \times 10^3$. Thus lesser the permeability of the porous medium, smaller the magnitude of the velocity ‘u’ and when the permeability is still lowered, we find an remarkable enhancement in u in the entire fluid region.

From fig 2, it is found that, the reversal flow which appears in the vicinity of the lower boundary for Reynolds number $R=35$ and $M=2$ disappears for higher values of R and M. Also the magnitude of ‘u’ increases with increase in R and M. The effect of the buoyancy ratio ‘N’ on ‘u’ (fig 3) shows that when the molecular buoyancy force dominates over the thermal buoyancy force, the region of reversal flow enlarges when the buoyancy forces act in the same direction while no such reversal flow appears when the buoyancy forces act in opposite directions. Also the magnitude of ‘u’ enhances with ‘N’ irrespective of the direction of the buoyancy forces. The effect of the amplitude of the non-uniform boundary temperature is exhibited in fig 4. It is found that in a given porous medium, reversal flow occurs in the region abutting the left boundary and the size of the region increases marginally
Fig. 6  Variation of transverse velocity ($v$) with $G$

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<td>-10^3</td>
<td>-3x10^3</td>
<td>-5x10^3</td>
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</tbody>
</table>

Fig. 7  $v$ with $R$ & $M$

$G = 3x10^3$, $N = 1$, $Sc = 1.3$

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with increase in ‘\(\alpha\)’. Also \(|u|\) reduces marginally with ‘\(\alpha\)’. Fig.5 shows that as we move along the axial direction, we find a reversal flow in the region adjacent to 
\(y = -1\) and in this region \(|u|\) reduces with \(x \leq \pi\) and enhances for \(x \geq 2\pi\). In the remaining region \(u\) experiences a marginal enhancement with an increase in \(x \leq \pi\) and decreases with \(x \leq 2\pi\)(fig.5).

A secondary velocity ‘\(v\)’ which is due to non-uniform temperature on the boundaries is analysed for different variations of \(G,D^{-1},R,M,N\) and shown in Fig.6-11. From fig.6, it is found that, in the case of heating of the channel the fluid in the left region \(-0.8 \leq y \leq 0.0\) is directed towards the mid region while the fluid in the right region \(0.2 \leq y \leq 0.8\) is directed towards the boundary. We find a reversed effect in the case of the cooling of the channel walls \(|v|\) increases with an increase in \(|G|\) with a maximum at \(y = -0.6\). An increase in the Reynolds number \(R\) decreases \(|u|\) in the left region and enhances it in the right region. For an increase in the Hartman number, \(|u|\) experiences an enhancement in the entire fluid region (Fig7). From fig8, it is noticed that, lesser the permeability of the porous medium, larger the magnitude of \(v\) in the entire fluid region. The variation of \(v\) with buoyancy ratio \(N\) shows that when the buoyancy forces act in the same direction the transverse velocity in the left region is directional towards the midregion while the reversal effect is noticed when they act in opposite directions. Also \(|v|\) increase with \(N\) when they act in the same direction.
Fig. 8: $v$ with $D^{-1}$

- $D^{-1}$:
  - I: $5 \times 10^3$
  - II: $10^3$
  - III: $2 \times 10^3$
  - IV: $3 \times 10^3$
  - V: $5 \times 10^3$

Fig. 9: $v$ with $N$

- $N$:
  - I: 1
  - II: 2
  - III: -0.5
  - IV: -0.8
Fig. 12  Variation of Resultant velocity (Rt) with G
D^{-1} = 2 \times 10^3, R = 35, N = 1
I  II  III  IV  V  VI
G  10^3  3 \times 10^3  5 \times 10^3  -10^3  -3 \times 10^3  -5 \times 10^3

Fig. 13  Rt with D^{-1}
G = 3 \times 10^3, R = 35, N = 1, Sc = 1.3
I  II  III  IV  V
D^{-1}  5 \times 10^3  10^3  2 \times 10^3  3 \times 10^3  5 \times 10^3
Fig. 14  Variation of $R_t$ with $N$, $M$ & $R$

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<td>-0.5</td>
<td>-0.8</td>
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<td>1</td>
</tr>
<tr>
<td>$M$</td>
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<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$R$</td>
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<td>35</td>
<td>35</td>
<td>35</td>
<td>70</td>
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Fig. 15  Variation of temperature($\theta$) with $G$

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<td>$-10^3$</td>
<td>$-3\times10^3$</td>
<td>$5\times10^3$</td>
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</table>
and enhances with $|N|$ when they act in opposite directions (fig. 9). An increase in the amplitude of the non-uniform boundary temperature enhances $|v|$ in the left region and reduces it in the right region (fig. 10). The fluid near the boundaries moves with greater velocities than in the midregion. The magnitude of velocity enhances marginally in the left region and decreases in the region with increase in the axial direction $x \leq \pi/2$ and for $x \geq \pi$ we notice a marginal enhancement in the entire fluid region (fig. 11).

The resultant velocity ($R_t$) is exhibited in figs (12)-(14). It is found that the profiles of the resultant velocity are almost bell shaped curves with maximum attained in the midregion. $R_t$ decreases with $|G| \leq 3 \times 10^3$ and increases for higher $G \geq 5 \times 10^3$ in the both heating and cooling cases (fig. 12). From fig. 13 it is noticed that lesser the permeability of the porous medium higher the resultant velocity near the boundaries with a dip in the midregion for $D^{-1} \leq 10^3$. When the permeability still lowers we find a reduction in $R_t$ in the entire fluid region. But for $D^{-1} \geq 3 \times 10^3$, a remarkable growth in $R_t$ is observed in the entire fluid region. When the molecular buoyancy force dominates over the thermal buoyancy the resultant velocity experiences an enhancement with $|N|(\geq 0)$ irrespective of the directions of the buoyancy forces. An increases in $M$ or $R$ leads to an increase in the resultant velocity (Fig. 14).
The temperature distribution ($\theta$) is exhibited in figs (15)-(19) for variations in $G, D^{-1}, R$ and $x$. The temperature is positive for all variations in the governing parameters. It is found that the temperature enhances in the heating of the channel walls and reduces in the cooling case. The temperature gradually enhances from its prescribed value on $y = -1$ to attain the maximum in the mid region and later falls to its prescribed value on $y = 1$. An increase in $R$ or $M$ depreciates the temperature in the entire fluid region (fig. 16). From fig. 17 we find that the temperature decreases with increase in $D^{-1}$. Thus lesser the permeability of the porous medium smaller the temperature in the fluid region. The variation of $\theta$ with $N$ shows that when the molecular buoyancy force dominates over the thermal buoyancy force $\theta$ enhances in the right region and reduces in the left region when the buoyancy forces are in the same directions. When the forces act in opposite directions we find a reversed effect in $\theta$ (fig. 18). The temperature experiences an increase with $x \leq \pi/2$ and reduction for $x \geq \pi$ (fig. 19).

The Shear stress and the rate of heat transfer on the boundaries $y = \pm 1$ have been evaluated for different variations in the governing parameters $G, R, D^{-1}, N, \alpha, M$ and $x$ and exhibited in tables (1)-(16). It is found that the stress at left boundary wall $y = -1$ is positive and that on the right boundary wall $y = 1$ is negative.
Table 1
Shear Stress (τ) at y = -1
P = 0.71

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
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<td>10^3</td>
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<td>1.4456</td>
<td>6.7401</td>
<td>5.9675</td>
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Table 2
Shear Stress (τ) at y = -1
P = 0.71

<table>
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### Table 3
Shear Stress(τ) at y = -1
\( P=0.71 \)

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### Table 4
Shear Stress(τ) at y = -1
\( P=0.71 \)

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### Table 5
Shear Stress ($\tau$) at $y = 1$
P = 0.71

<table>
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<th>G ( \times 10^3 )</th>
<th>I</th>
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<th>IV</th>
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<th>VI</th>
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<th>VIII</th>
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<tbody>
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<td>-1.09865</td>
<td>-1.0035</td>
<td>-3.6983</td>
<td>-3.4405</td>
<td>-3.6752</td>
<td>-2.6793</td>
<td>-2.6992</td>
</tr>
<tr>
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<td>0.3317</td>
<td>0.7215</td>
<td>-6.0174</td>
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<td>-1.7799</td>
<td>-1.5414</td>
<td>0.3061</td>
</tr>
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<td>1.3556</td>
<td>-47.1571</td>
<td>-37.5529</td>
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<td>0.8991</td>
<td>-127.1173</td>
<td>-102.4641</td>
<td>-63.3433</td>
<td>-49.8039</td>
<td>-22.2948</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
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<tbody>
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### Table 6
Shear Stress ($\tau$) at $y = 1$
P = 0.71

<table>
<thead>
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<th>I</th>
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<th>III</th>
<th>IV</th>
</tr>
</thead>
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<td>3.3154</td>
<td>3.1947</td>
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<tr>
<td>3x10^3</td>
<td>-30.6207</td>
<td>-56.3353</td>
<td>20.1886</td>
<td>25.9687</td>
</tr>
<tr>
<td>5x10^3</td>
<td>-85.1212</td>
<td>-162.0409</td>
<td>50.7059</td>
<td>70.5761</td>
</tr>
<tr>
<td>-10^4</td>
<td>-3.4353</td>
<td>-8.4885</td>
<td>0.0863</td>
<td>2.2542</td>
</tr>
<tr>
<td>-3x10^3</td>
<td>-30.7504</td>
<td>-66.3471</td>
<td>10.5012</td>
<td>23.1468</td>
</tr>
<tr>
<td>-5x10^3</td>
<td>-85.3375</td>
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<td>34.5604</td>
<td>65.8729</td>
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<table>
<thead>
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<th>II</th>
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<th>IV</th>
</tr>
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<tbody>
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<td>N</td>
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<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
</tr>
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for all variations. The stress on the wall $y = -1$ increases with increase in $|G| (>0)$. An increase in the Reynolds number $R$ leads to a depreciation in $|\tau|$. An increase in $D^{-1}$ decreases $\tau$ at $y = 1$. Thus lesser the permeability smaller the stress at $y = 1$. (table 1). From table 2 we observe that the stress on $y = 1$ is positive for $N > 0$ and negative for $N < 0$ for any given $|G|$ and $D^{-1}$. When the molecular buoyancy force dominates over the thermal buoyancy force the magnitude of $\tau$ enhances when the buoyancy forces either act in the same direction or in opposite directions. The variation of stress with the amplitude $\alpha$ of the boundary temperature indicates that for $\alpha \leq 1.0$, $\tau$ is positive and for higher $|\alpha| \geq 1.5$, $|\tau|$ is negative. $|\tau|$ decreases with increase in the amplitude $\alpha$ (table 3). From table 4 we find that the shear stress increases with increase in the axial distance $x \leq \pi/2$ and for further increase in $x$ it decreases and increases alternately with $x$. The stress on $y = -1$ is negative for all variations. $|\tau|$ enhances with increases in $|G| (>0)$ and decreases with $R$. An increase in $D^{-1}$ decreases $|\tau|$. Thus lesser the permeability of the porous medium smaller the magnitude of $\tau$. Also an increase in $M$ decrease $|\tau|$ (table 5). From table 6 we find that the stress is negative when the buoyancy forces act in the same direction and positive when they act in the opposite directions. Also we find that $|\tau|$ enhances with $|N| (>0)$ irrespective of the directions of the two buoyancy forces. The variation of $\tau$ with $\alpha$ indicates that for $\alpha \leq 1$, the stress is positive and negative for higher values of $\alpha$. 

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### Table 7
Shear Stress(\( \tau \)) at \( y = 1 \)
\( P=0.71 \)

<table>
<thead>
<tr>
<th>( G \times 10^3 )</th>
<th>I</th>
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<th>III</th>
<th>IV</th>
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</thead>
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<td>-1.4547</td>
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<td>-30.6207</td>
<td>-23.8789</td>
<td>-13.1315</td>
<td>2.3086</td>
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<tr>
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<td>-85.1212</td>
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### Table 8
Shear Stress(\( \tau \)) at \( y = 1 \)
\( P=0.71 \)

<table>
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<td>-85.3375</td>
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<td>( \pi/2 )</td>
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Table 9
Average Nusselt Number (Nu) at y = -1
P = 0.71

<table>
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<th>G/τ</th>
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<th>V</th>
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<th>VII</th>
<th>VIII</th>
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<tr>
<td>10^3</td>
<td>0.1168</td>
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<td>0.5158</td>
<td>0.1101</td>
<td>0.1177</td>
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<td>0.0147</td>
<td>0.0836</td>
<td>0.1312</td>
<td>0.0142</td>
<td>0.0152</td>
<td>0.0072</td>
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<td>-0.0053</td>
</tr>
<tr>
<td>5x10^3</td>
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<td>0.0366</td>
<td>0.0652</td>
<td>0.0052</td>
<td>0.0053</td>
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<td>0.0033</td>
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<td>-0.2523</td>
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<td>0.0637</td>
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<td>1.2359</td>
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<td>35</td>
<td>35</td>
<td>35</td>
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<td>3x10^3</td>
<td>3x10^3</td>
<td>10^3</td>
<td>2x10^3</td>
<td>5x10^3</td>
<td>3x10^3</td>
<td>3x10^3</td>
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<td>2</td>
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Table 10
Average Nusselt Number (Nu) at y = -1
P = 0.71

<table>
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<td>-0.1552</td>
<td>-0.1414</td>
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<tr>
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<td>0.0059</td>
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<td>-0.0253</td>
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<tr>
<td>5x10^3</td>
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<td>0.0012</td>
<td>-0.0142</td>
<td>-0.0109</td>
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<tr>
<td>-10^3</td>
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<td>0.1973</td>
<td>0.7497</td>
<td>-1.0369</td>
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<td>-0.0318</td>
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<tr>
<td>-5x10^3</td>
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<td>0.0072</td>
<td>-0.0195</td>
<td>-0.0087</td>
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</table>

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<th>III</th>
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Table 11
Average Nusselt Number (Nu) at y = -1
\( P = 0.71 \)

<table>
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<tr>
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<th>III</th>
<th>IV</th>
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<td>0.1145</td>
<td>0.1114</td>
<td>0.1051</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>0.0147</td>
<td>0.0132</td>
<td>0.0112</td>
<td>0.0073</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>0.0047</td>
<td>0.0038</td>
<td>0.0023</td>
<td>-0.0012</td>
</tr>
<tr>
<td>( -10^3 )</td>
<td>0.6711</td>
<td>0.7245</td>
<td>0.8162</td>
<td>1.0102</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
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<td>0.0366</td>
<td>0.0403</td>
<td>0.0463</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>0.0123</td>
<td>0.0136</td>
<td>0.0154</td>
<td>0.0186</td>
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</table>

<table>
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<th>III</th>
<th>IV</th>
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<td>( 0.3 )</td>
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<td>1.0</td>
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</tr>
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</table>

Table 12
Average Nusselt Number (Nu) at y = -1
\( P = 0.71 \)

<table>
<thead>
<tr>
<th>( G' \times r )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
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<td>( 10^3 )</td>
<td>0.1168</td>
<td>0.1201</td>
<td>0.1267</td>
<td>0.1339</td>
<td>0.1105</td>
</tr>
<tr>
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<td>0.0147</td>
<td>0.0158</td>
<td>0.0185</td>
<td>0.0237</td>
<td>0.0315</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>0.0047</td>
<td>0.0055</td>
<td>0.0072</td>
<td>0.0106</td>
<td>0.0037</td>
</tr>
<tr>
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<td>0.6711</td>
<td>0.6779</td>
<td>0.6634</td>
<td>0.5048</td>
<td>0.6114</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>0.0343</td>
<td>0.0325</td>
<td>0.0284</td>
<td>0.0201</td>
<td>0.0364</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>0.0123</td>
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<td>0.0043</td>
<td>0.0135</td>
</tr>
</tbody>
</table>

<table>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
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<tbody>
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<td>( \pi/4 )</td>
<td>( \pi/3 )</td>
<td>( \pi/2 )</td>
<td>( \pi )</td>
<td>( 2\pi )</td>
<td></td>
</tr>
</tbody>
</table>
The average Nusselt number which measures the rate of heat transfer at the boundaries is exhibited in tables(9-15) for variation in G,R,D$^{-1}$,M,N and x. It is noticed that the Nusselt number Nu increases with increase in Reynolds number R when G>0 and decrease with R for G<0. An enhancement in |G(>0)| leads to a reduction in |t|. In the case of heating of the channel walls the shear stress enhances with D$^{-1}$ ≤ 2 x10$^3$ and for further increase in D$^{-1}$ ≥3x10$^3$ a reduction in Nu is observed. Thus lesser the permeability of the porous medium larger the magnitude of Nu and further lowering of the permeability leads to a reduction in |Nu|. For G≤3x10$^3$ and |G| = 10$^3$ we find a marginal decrease in Nu with increase in the Hartman number M. But for G ≥5x10$^3$ and |G|≤10$^3$ we notice an enhancement in |Nu| with increase in M(table.9). From table.10 it is found that Nu decreases in magnitude with increase in |N|(>0). Irrespective of the directions of the two buoyancy forces, the rate of heat transfer experiences a reduction with increase in the amplitude α of the boundary temperature (table.11). Also Nu exhibits a fluctuating nature as we move along the axial direction (table.12). The variation of rate of heat transfer at y = +1 shows that it reduces in magnitude with increases in |G| and enhances with R. An increase in D$^{-1}$ leads to an increase in |Nu|. Thus lesser
### Table 13
**Average Nusselt Number (Nu) at y = 1**
P = 0.71

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>-0.2636</td>
<td>-0.4119</td>
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<td>-0.0745</td>
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<td>-0.0077</td>
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<td>2.6592</td>
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<tr>
<td>-3x10^3</td>
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<td>0.2327</td>
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<td>-0.0144</td>
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<td>0.0034</td>
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</table>

<table>
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<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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<th>VIII</th>
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<td>35</td>
<td>35</td>
<td>35</td>
<td></td>
</tr>
<tr>
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<td>3x10^3</td>
<td>3x10^3</td>
<td>10^3</td>
<td>2x10^3</td>
<td>5x10^3</td>
<td>3x10^3</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

### Table 14
**Average Nusselt Number (Nu) at y = 1**
P = 0.71

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>-0.2636</td>
<td>-0.08227</td>
<td>0.14512</td>
<td>0.12698</td>
</tr>
<tr>
<td>3x10^3</td>
<td>-0.2217</td>
<td>-0.01384</td>
<td>0.02607</td>
<td>0.01832</td>
</tr>
<tr>
<td>5x10^3</td>
<td>-0.0093</td>
<td>-0.00632</td>
<td>0.01013</td>
<td>0.00631</td>
</tr>
<tr>
<td>-10^3</td>
<td>-0.5866</td>
<td>-0.15053</td>
<td>-0.8018</td>
<td>1.15479</td>
</tr>
<tr>
<td>-3x10^3</td>
<td>-0.227</td>
<td>-0.00768</td>
<td>0.10261</td>
<td>0.04411</td>
</tr>
<tr>
<td>-5x10^3</td>
<td>-0.0061</td>
<td>-0.00142</td>
<td>0.02731</td>
<td>0.01513</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
<td></td>
</tr>
</tbody>
</table>
### Table 15
Average Nusselt Number (Nu) at y = 1
\( P=0.71 \)

<table>
<thead>
<tr>
<th>( G \times \tau )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>-0.2636</td>
<td>-0.1383</td>
<td>-0.1456</td>
<td>-0.1582</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>-0.2217</td>
<td>-0.0236</td>
<td>-0.0259</td>
<td>-0.0297</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>-0.0093</td>
<td>-0.0104</td>
<td>-0.0119</td>
<td>-0.0143</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
<td>-0.5866</td>
<td>-0.5999</td>
<td>-0.6230</td>
<td>-0.6716</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>-0.227</td>
<td>-0.0204</td>
<td>-0.0171</td>
<td>-0.0114</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>-0.0061</td>
<td>-0.0048</td>
<td>-0.0030</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3</td>
<td>0.7</td>
<td>1.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### Table 16
Average Nusselt Number (Nu) at y = 1
\( P=0.71 \)

<table>
<thead>
<tr>
<th>( G \times \tau )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>-0.2636</td>
<td>-0.1319</td>
<td>-0.12658</td>
<td>-0.1108</td>
<td>-0.1337</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>-0.2217</td>
<td>-0.0212</td>
<td>-0.01851</td>
<td>-0.0133</td>
<td>-0.0236</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>-0.0093</td>
<td>-0.0088</td>
<td>-0.0072</td>
<td>-0.0038</td>
<td>-0.0105</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
<td>-0.5866</td>
<td>-0.6165</td>
<td>-0.6641</td>
<td>-0.61098</td>
<td>-0.5053</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>-0.227</td>
<td>-0.0244</td>
<td>-0.0285</td>
<td>-0.0363</td>
<td>-0.0202</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>-0.0061</td>
<td>-0.0069</td>
<td>-0.0092</td>
<td>-0.01348</td>
<td>-0.0048</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \pi/4 )</th>
<th>( \pi/3 )</th>
<th>( \pi /2 )</th>
<th>( \pi )</th>
<th>( 2\pi )</th>
</tr>
</thead>
</table>

---

Table: 15
Average Nusselt Number (Nu) at \( y = 1 \)
\( P=0.71 \)

Table: 16
Average Nusselt Number (Nu) at \( y = 1 \)
\( P=0.71 \)
the permeability of the porous medium larger the magnitude of $\text{Nu}$. With increase in the strength of the Lorentz force ($M \leq 4$) we observe a reduction in $|\text{Nu}|$ for $G > 0$ and an enhancement for $|G| > 0$. While for further increase in $M \geq 6$, a reversed effect is noticed. When the molecular buoyancy force dominates over the thermal buoyancy force the magnitude of $\text{Nu}$ reduces with $N$ irrespective of the directions of the buoyancy forces (table 14). Also $|\text{Nu}|$ enhances with increase in the amplitude $\alpha$ of the boundary temperature (table 15). For $x \leq \pi$ the magnitude of $\text{Nu}$ reduces as we move along the axial directions while for further moving along the axial direction we observe an increase in $|\text{Nu}|$ (Table 16).
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\( M_1^2 = D^{-1} + M^2 \)

\( a_1 = G / N \)

\( a_2 = GN / 2R \)

\( a_3 = -a_i / 6M_1^2 \)

\[ a_4 = -\frac{2d_2}{M_1, ShM_1} \]

\( d_{15} = -3a_5 - M_1 a_5, chM_1 \)

\( a_6 = 0 \)

\( a_7 = G(2a_6, N - PM_1^2) / R \)

\( a_8 = -\frac{a_7}{6M_1^2} \)

\[ a_9 = -\frac{2a_s}{M_1, chM_1 - shM_1} \]

\( a_{10} = -\frac{2a_s}{M_1, ShM_1} \)

\( a_{11} = -3a_4, y^1 - M_1 a_4, chM_1 \)

\[ a_{12} = 2d_2, y^1 + chM_1 + d_2^1 - a_3^1 \]

\( a_{13} = 3a_5, y^1 - a_3^1 + a_3^1, chM_1 \)

\( a_{14} = -d_2^1 \)

\( a_{15} = -a_3^1 \)

\( a_{16} = M_1 a_5 y^1 \)

\( a_{17} = -2d_2, y^1 / shM_1 \)

\( a_{18} = -a_4^1 \)

\( a_{19} = -a_3^1 \)

\[ a_{20} = (M_1 a_4 - 2a_{19}) / M_1^3 \]

\( a_{21} = (M_1 a_{19} - 2a_{19}) / M_1^3 \)

\[ a_{22} = a_{18} / M_1^2 \]

\( a_{23} = a_{19} / M_1^2 \)

\( a_{24} = a_{11} / 2 \)

\( a_{25} = a_{12} / 6 \)

\( a_{26} = a_{13} / 12 \)

\( a_{27} = a_{14} / 20 \)

\( a_{28} = a_{15} / 30 \)

\( a_{29} = -(a_{23} + a_{21} shM_1 + a_{22} chM_1) \)
\[ a_{33} = 0 \]
\[ a_{35} = 0 \]
\[ a_{36} = 0 \]
\[ a_{37} = 0 \]
\[ a_{39} = 0 \]
\[ a_{40} = \frac{G}{R} (a_{25} + N(a_{32} + a_{34} + a_{35} + a_{36} + a_{27} + a_{28} + a_{23} + a_{23} + a_{35} + a_{36} + a_{37} + a_{39})) \]
\[ a_{41} = \frac{G}{R} (2a_{24} + 2Na_{32}) \]
\[ a_{42} = \frac{G}{R} (3a_{24} + 3Na_{32}) \]
\[ a_{43} = \frac{G}{R} (4a_{26} + 4Na_{33}) \]
\[ a_{44} = \frac{G}{R} (a_{27} + Na_{34}) \]
\[ a_{45} = -\frac{6G}{R} (a_{28} + Na_{35}) \]
\[ a_{46} = \frac{G}{R} (M_1a_{21} + a_{22}N(a_{38} + M_1a_{32})) \]
\[ a_{47} = \frac{G}{R} (M_1a_{23} + a_{33} + N(M_1a_{36} + a_{39})) \]
\[ a_{48} = -\frac{M_1G}{R} (a_{23} + Na_{36}) \]
\[ a_{49} = \frac{M_1G}{R} (a_{23} + Na_{39}) \]
\[ a_{50} = -12a_1^2a_3 - 2M_1a_1^2a_5 + 6a_3a_1^2 + 6M_1a_1^2a_5 \]
\[ a_{51} = -24a_1a_3^1 + 4d_2d_1^1 - 6M_1a_1^2a_5 \]
\[ a_{52} = 12d_2^2a_2^3 \]
\[ a_{53} = 12a_1^3 \]
\begin{align*}
a_{54} &= 2d^3_2M_xa_y - M_1^2a_y^1(3a_3 + M_1a_ychM_1) \\
a_{55} &= -\frac{4d_2d^2_2}{shM_1} - M_1^2a_y^1(3a_3 + M_1a_ychM_1) + 6a_3a_y^1 - \frac{2d_2M_1^2}{shM_1}(d^2_2 + a_y^1chM_1) \\
a_{56} &= 6M_1a_y^3a_3 + 2d_2M_1^2a_y^1 + M_1^3a_y^1a_y^1 \\
a_{57} &= -\frac{12d_2a_y^1(1 + M_1^2)}{shM_1} = 2d_2M_1^2a_y^1 \\
a_{58} &= 3M_1^2a_y^1a_y^1 \\
a_{60} &= -M_1^3a_y^1a_y^1 \\
a_{62} &= a_y^1a_y^1\left(\frac{M_1^2}{shM_1} - 1\right) / 2 \\
a_{64} &= 0 \\
a_{66} &= a_{62} + a_{63} + a_{64} \\
a_{68} &= a_{41} + a_{51} \\
a_{70} &= -\frac{(a_{40} + a_{63})}{2M_1^2} \\
a_{72} &= -\frac{(a_{42} + a_{52})}{12M_1^2} \\
a_{74} &= -\frac{a_{44}}{30M_1^2} \\
a_{76} &= \frac{(a_{46} + a_{54})}{2M_1^3} \\
a_{78} &= \frac{a_{48} + a_{56}}{2M_1^3} \\
a_{79} &= \frac{a_{49} + a_{57}}{2M_1^3} \\
a_{80} &= \frac{a_{59}}{2M_1^3} \\
a_{81} &= \frac{(a_{47} + a_{55})}{2M_1^3} \\
a_{85} &= a_{50} + \frac{1}{2}(a_{63} - a_{64}) \\
a_{65} &= a_{63} + a_{65} \\
a_{69} &= a_{62} + a_{52} \\
a_{71} &= -(a_{41} + a_{51}) \\
a_{73} &= -(a_{43} + a_{53}) \\
a_{75} &= -\frac{a_{45}}{M_1^2} \\
a_{77} &= \frac{(a_{47} + a_{55})}{2M_1^3} \\
a_{79} &= \frac{a_{49} + a_{57}}{2M_1^3} \\
a_{80} &= \frac{a_{59}}{2M_1^3}
\end{align*}
\[ b_1 = a_{76} + a_{74} + \frac{a_{59}}{M_1^5} \quad ; \quad b_2 = a_{77} + a_{79} + \frac{a_{59}}{M_1^5} \]

\[ b_3 = -\frac{5a_{59}}{4M_1^4} \quad ; \quad b_4 = -\frac{5a_{59}}{4M_1^4} + \frac{3a_{60}}{4M_1^3} \]

\[ b_5 = \frac{a_{59}}{6M_1^3} - \frac{5a_{60}}{4M_1^3} \quad ; \quad b_6 = \frac{a_{58}}{6M_1^3} \]

\[ b_7 = \frac{a_{60}}{8M_1^3} \]

\[ b_8 = a_{71} + a_{72} + (b_1 + b_4)chM_1 + (b_4 + b_5)shM_1 \]

\[ b_{13} = 3a_{63} + 5a_{65} + 7a_{67} + (b_2 + 3b_5)chM_1 + (b_1 + b_3)shM_1 + \]

\[ + 2b_4 shM_1 + M_1 b_5 chM_1 + b_7 (4 shM_1 + M_1 chM_1) \]

\[ b_{14} = \frac{(b_{12} - b_{13})}{M_1 chM_1 - shM_1} \quad ; \quad d_{22} = 2a_3 + M_1 a_1 chM_1 \]

\[ d_1 = M_1 d_2 shM_1 + 2d_1^1 \quad ; \quad d_3 = d_1 + d_{22} \]

\[ d_4 = d_{22} - d_1 \]

\[ d = 2d_2 (1 - M_1 TanhM_1) + Ec(2a_{62} (1 - M_1 cothM_1 + 4a_{63} (3 - M_1 TanhM_1) + \]

\[ + 6a_{66} (5 - M_1 cothM_1) + 2b_3 (M_1 shM_1) + coth M_1 - \frac{M_1 \cosh^2 M_1}{\sinh M_1} + \]

\[ + 2(M_1 b_2 + 3b_5)shM_1 \]

\[ d_6 = 6a_{23} + M_1 a_5 shM_1 + Ec((a_{60} M_1^2 + b_5)shM_1 + 6a_5) \]

\[ d_7 = d_5 + d_6 \quad ; \quad d_8 = d_5 - d_6 \]
\[d_{23} = \frac{2}{3}(1 - PM_1^3)\]

\[d_9 = -\frac{4}{3}a_{24} + \frac{8}{5}a_{26} + \frac{12}{7}a_{28} + 2a_{30}\left(\frac{shM_1}{M_1} - chM_1\right) + \frac{2a_{32}}{M_1}\left(chM_1 - \frac{shM_1}{M_1}\right)\]

\[d_{10} = -2 + PEcM_1^2\]

\[d_{11} = 2a_{25} + 4a_{26} + 6a_{28} + M_1a_{29}shM_1 + a_{30}(shM_1 + M_1chM_1)\]

\[d_{12} = 2a_{25} + 4a_{27} + a_{21}(M_1chM_1 - shM_1) + a_{22}M_1shM_1\]

\[d_{13} = \frac{1}{3N} - 4ScS_0 + \frac{4a_{6}Ec}{3}\]

\[d_{14} = -\frac{4}{3}a_{31} - \frac{8}{5}a_{33} - \frac{12}{7}a_{35} + 2a_{36}\left(\frac{shM_1}{M_1}\right) - \frac{2a_{39}}{M_1}\left(chM_1\right)\]

\[d_{15} = \frac{2ScS_0}{N} - 2Eca_6\]

\[d_{16} = 2a_{31} + 4a_{33} + 6a_{35} + M_1a_{36}shM_1 + a_{39}(M_1chM_1 + shM_1)\]

\[d_{17} = 2a_{32} + 4a_{34} + M_1a_{36}shM_1 + a_{37}(M_1chM_1 - shM_1)\]

\[b_{16} = PM_1a_{16}ch(M_1) - 3a_8\]

\[b_{17} = \frac{PM_1^3(b_8^2 + b_9^2)}{8M_1^2}\]

\[b_{18} = \frac{PM_1^4b_8b_9}{4M_1^2}\]

\[b_{19} = (b_1^2 - b_5^2)/12\]

\[b_{20} = \frac{a_{20}^2 + b_3^2 - b_4^2}{30}\]

\[b_{21} = \frac{a_{21}^2 + b_5^2 - b_6^2}{56}\]

\[b_{22} = \frac{a_{22}^2 - b_7^2}{2}\]

\[b_{23} = \frac{a_{23}^2 + a_{24}^2}{11x12 + 2}\]

\[b_{24} = \frac{a_{24}^2}{13x14}\]
\[ b_{28} = \frac{9}{16M_1^3} - \frac{21b_7^2}{2M_1^2} \]
\[ b_{29} = \frac{b_3^2 + b_4^2}{2M_1} + \frac{15(b_5^2 + b_6^2)}{4M_1^2} + \frac{105b_7^2}{2M_1^4} \]
\[ b_{30} = \frac{(b_3^2 + b_4^2)}{M_1} - \frac{15(b_5^2 + b_6^2)}{2M_1^3} - \frac{105b_7^2}{2M_1^4} \]
\[ b_{31} = \frac{b_3^2 + b_4^2}{2M_1^2} + \frac{3(b_5^2 + b_6^2)}{2M_1^2} + \frac{15(b_5^2 + b_6^2)}{2M_1^3} + \frac{105b_7^2}{4M_1^6} \]
\[ b_{32} = \frac{(b_3^2 + b_4^2)}{2M_1^2} - \frac{3(b_5^2 + b_6^2)}{4M_1^3} - \frac{45(b_5^2 + b_6^2)}{8M_1^4} - \frac{315b_7^2}{8M_1^6} \]
\[ b_{33} = \frac{b_3^2 + b_4^2}{4M_1^3} + \frac{3(b_5^2 + b_6^2)}{8M_1^4} + \frac{45(b_5^2 + b_6^2)}{16M_1^5} + \frac{315b_7^2}{32M_1^8} \]
\[ b_{34} = \frac{b_{24}}{120M_1^2} \]
\[ b_{35} = \frac{b_{25}}{13x12M_1^2} \]
\[ b_{36} = \frac{b_{22}}{120M_1^2} + \frac{b_{24}}{M_1^4} \]
\[ b_{37} = \frac{b_{23}}{M_1^4} \]
\[ b_{38} = \frac{b_{22}}{M_1^4} + \frac{b_{21}}{90M_1^2} \]
\[ b_{39} = \frac{b_{20}}{56M_1^2} + \frac{b_{21}}{M_1^4} \]
\[ b_{40} = \frac{b_{19}}{M_1^2} + \frac{b_{20}}{M_1^4} \]
\[ b_{41} = \frac{b_{15}}{20M_1^2} \]
\[ b_{42} = \frac{b_{15}}{12M_1^2} \]
\[ b_{43} = \frac{b_{16}}{M_1^4} \]
\[ b_{44} = \frac{b_{17}}{6M_1^4} \quad ; \quad b_{45} = \frac{b_{18} + b_{33}}{6M_1^4} \]

\[ b_{46} = \frac{b_{25}}{6M_1^3} \quad ; \quad b_{47} = \frac{b_{26} - 4b_{25}}{6M_1^3} \]

\[ b_{48} = \frac{56b_{25}}{18M_1^4} \quad ; \quad b_{49} = \frac{b_{15}}{20M_1^2} \]

\[ b_{42} = \frac{b_{15}}{12M_1^2} \quad ; \quad b_{43} = \frac{b_{16}}{M_1^4} \]

\[ b_{44} = \frac{b_{17}}{6M_1^4} \quad ; \quad b_{45} = \frac{b_{18} + b_{33}}{6M_1^4} \]

\[ b_{46} = \frac{b_{25}}{6M_1^3} \quad ; \quad b_{47} = \frac{b_{26} - 4b_{25}}{6M_1^3} \]

\[ b_{48} = \frac{106b_{25} - 7b_{26} + 6M_1b_{27}}{36M_1^4} \quad ; \quad b_{49} = \frac{14b_{26} - 5b_{25} + M_1b_{28}}{6M_1^4} \]

\[ b_{50} = \frac{35b_{26} + 10M_1^2b_{27}}{6M_1^6} - 4.17M_1^2b_{28} + Mb_{29} \]

\[ b_{51} = \frac{-140b_{25} - 630b_{26} + 20b_7 + 20b_8 - 10b_{20} + b_{30}}{6M_1^7} \]

\[ b_{52} = \frac{-1260b_{26} - 5b_{27} + 5b_{28} + 4b_{29} - b_{30} + b_{31}}{144M_1^6} \]

\[ b_{53} = \frac{30b_{27} - 20b_{28} + 4b_{26} + b_{30} - 10b_{31} + b_{32}}{6M_1^8} \]