Chapter 7

Dielectric Properties of Urinary Crystals at Microwave Frequencies

7.1 Introduction

Microwaves play a large role in the treatment and diagnosis of many diseases. Biological effects of microwaves and the application of microwaves in medicine is a developing area of research. The application and effects of microwaves on tissues, microwave prostatic hyperthermia, heating off-centre tumours etc are reported [1-4]. It is essential to know the dielectric properties of the biological materials to detect the disease by microwave method. Recent advances in microwave imaging, have paved way for the tracing of urinary calculi in human systems. A thorough knowledge of the dielectric properties of the urinary stone materials like complex permittivity and conductivity will help to treat the patients for the particular stone they procure [5-6].

7.2 Review of Cavity Perturbation Method

The theory for cavity perturbation was suggested initially by Bethe and Schwinger [7]. The fundamental idea of cavity perturbation is that the change in the geometrical configuration of the electromagnetic fields on the introduction of the sample must be small.
The method of measurement of complex permittivity based on cavity perturbation technique was first developed by Bimbaum and Franeu [8] under the assumption that the electric field in the perturbing sample is equal to the electric field of the empty cavity. In these measurements, a rectangular cavity operating in the TE_{106} mode was used and the dielectric parameters of a cylindrical shaped BaTiO_3 single crystal was measured. In the theoretical analysis suggested by Prakash et al. [9], the sample did not have contact with the cavity wall. A set of formulae for calculating the dielectric constant 'ε' and conductivity 'σ' was introduced, based on the assumption that the sample acts like a dipole with an effective depolarising factor. For the measurements, small holes can be drilled in the cavity walls and the sample can then be inserted into the sample holder. Many researchers suggested various theoretical analyses for cavity perturbation techniques. The measurements of 'ε' and 'σ' are performed by inserting a small, appropriately shaped sample into a cavity and determining the properties of the sample from the resultant change in the resonant frequency and loaded Q-factor.

7.3 Design and Fabrication of Cavity Resonators

Generally rectangular or cylindrical wave-guide resonators are employed in the cavity perturbation techniques. The availability of resonator cavity makes it possible to measure the dielectric parameters at a number of frequencies in single band using sweep oscillators and network analysers.

Cavity resonators are constructed from sections of brass or copper wave-guides. If a hollow rectangular waveguide is scaled with conductive walls perpendicular to the direction of propagation, the incident and reflected waves are superimposed to generate a standing wave. The tangential electric and normal magnetic field components are zero at this wall and at distances of integral half wavelengths from it. In such a nodal plane, a second conductive wall can be placed without disturbing the field distribution in the waveguide, and thereby a cavity
resonator is obtained. If the resonator is excited through a coupling mechanism, the field intensity building up within it becomes maximum when the length of the resonator in the direction of propagation is equal to an integral multiple of the half wavelength. Because of the different field modes possibly existing in the waveguide, a number of resonant frequencies can occur. In general for a resonator of length \( d \) and guided wavelength \( \lambda_r \),

\[
d = \frac{p\lambda_g}{2}
\]

where \( p \) is an integer.

\[
\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}
\]

\( \lambda_0 \) is the wavelength in free space and \( \lambda_c \) is the cut off wavelength for the given waveguide.

The resonant wavelength, \( \lambda_r \), is given by

\[
\lambda_r = \frac{1}{\sqrt{\left(\frac{p}{2d}\right)^2 + \left(\frac{1}{\lambda_c}\right)^2}}
\]

This expression is valid for resonators with rectangular cross section and of circular cross section. The unloaded Q-factor of the rectangular cavity resonator [10] is given by

\[
Q = \frac{\sigma_c \delta \pi f_{10p} \mu a b d \left( a^2 + d^2 \right)}{2a^3 b + a^2 d + ad^3 + 2bd^3}
\]

where \( \sigma_c \) = conductivity of the walls of the material of the cavity.

\( \delta \) = skin depth.

\( \mu \) = permeability of the medium inside the cavity.

\( a, b \) and \( d \) = breath, height and length of the cavity

\( f_{10p} \) = resonant Frequency for TE\(_{10p}\) mode
The skin depth at resonance frequency \( \delta = \frac{1}{\sqrt{\pi f_{10p} \mu \sigma_c}} \) \( \text{(7.5)} \)

Resonant frequency and Q-factor are the fundamental parameters of a resonator. These parameters are evaluated using the above equations.

7.4 Theory

7.4(a) Complex permittivity of materials

When a small sample is inserted in a cavity, which has the electric field \( E_0 \) and magnetic field \( H_0 \) in the unperturbed state, the fields in the interior of the object are \( E \) and \( H \). Beginning with Maxwell's equations, an expression for the resonant frequency shift can be deduced. For a lossless sample, the variation of resonant frequency is given as by Harrington [11] as

\[
\frac{\omega - \omega_0}{\omega} = \frac{\int (\Delta \varepsilon E_0^* + \Delta \mu H_0^*) \, d\tau}{\int (\varepsilon E_0^* + \mu H_0^*) \, d\tau} \quad \text{(7.6)}
\]

\( \varepsilon \) and \( \mu \) are the permittivity and permeability of the medium in the unperturbed cavity, \( d\tau \) is the elemental volume, \( \Delta \varepsilon \) and \( \Delta \mu \) are the changes in the above quantities due to the introduction of the sample in the cavity. Waldron [16] gave an expression for the shift due to a lossy sample in a cavity without affecting the generality of Maxwell's equation as

\[
\frac{\delta \Omega}{\Omega} = \frac{\int (\varepsilon_r - 1)E_0^* \, dV + (\mu_r - 1)H_0^* \, dV}{\int (D_0^*E_0 + B_0^*H_0) \, dV} \quad \text{(7.7)}
\]

where \( \delta \Omega \) is the complex frequency shift, \( B_0, H_0, D_0 \) and \( E_0 \) are the fields in the unperturbed cavity. \( E \) and \( H \) are the fields in the interior of the sample.
\[ \varepsilon_r = \varepsilon'_r - j\varepsilon''_r \]  
\[ \mu_r = \mu'_r - j\mu''_r \]  
\[ (7.8) \]
\[ (7.9) \]

\( V_c \) and \( V_s \) are the volumes of the cavity and sample respectively.

Two approximations are made in applying equation [7.7] based on the assumptions that the fields in the empty part of the cavity are negligibly changed by the insertion of the sample, and that the fields in the sample are uniform over its volume. Both these assumptions are valid if the object is sufficiently small relative to the resonant wavelength. The negative sign in equation [7.7] indicates that by introducing the sample the resonant frequency is lowered. Since the permittivity of a material is complex, the resonant frequency should also be considered as complex.

In terms of energy, the numerator of equation [7.7] represents the energy stored in the sample and the denominator represents the total energy stored in the cavity. When a dielectric sample is introduced at the position of maximum electric field as shown in figure 7.1, only the first term in the numerator of equation 7.7, is significant, since a small change in \( \varepsilon \), at a point of zero electric field or a small change in \( \mu \) at a point of zero magnetic field does not change the resonance frequency.

![Electric field distribution inside a rectangular cavity](image)

**Figure 7.1:** Electric field distribution inside a rectangular cavity
Thus equation [7.7] can be reduced to

$$\frac{\delta \Omega}{\Omega} = \frac{\left(\varepsilon_r - 1\right) \int E.E_0^* \max dV}{\frac{\psi_s}{v_s} \frac{2 \int |E_0|^2 dV}{v_c}}$$

[7.10]

Let $Q_0$ be the quality factor of the cavity in the unperturbed condition and $Q_s$ the quality factor of the cavity loaded with the object. The complex frequency shift is related to measurable quantities by [12]

$$\frac{\delta \Omega}{\Omega} = \frac{\delta \omega}{\omega} + \frac{j}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_0} \right]$$

[7.11]

Substituting equation [7.11] into [7.10] and equating the real and imaginary parts

$$\frac{f_s - f_0}{f_s} = \frac{\left(\varepsilon_r - 1\right) \int E.E_0^* \max dV}{\frac{\psi_s}{v_s} \frac{2 \int |E_0|^2 dV}{v_c}}$$

[7.12]

$$\frac{1}{2} \left[ \frac{1}{Q_s} - \frac{1}{Q_0} \right] = \frac{\varepsilon_r^* \int E.E_0^* \max dV}{\frac{\psi_s}{v_s} \frac{2 \int |E_0|^2 dV}{v_c}}$$

[7.13]

It is assumed that $E \approx E_0$ and the value of $E_0$ in TE$_{10}$ mode is $E_0 = E_{0\max} \sin (m \pi x / a) \sin (p \pi x / d)$ where 'a' is the border dimension of the waveguide and 'd' is the length of the cavity. Integrating and re-arranging, we obtain

$$\varepsilon_r - 1 = \frac{f_0 - f_s}{2f_s} \left[ \frac{\psi_s}{v_s} \right]$$

[7.14]
$$\varepsilon'_r = \left[ \frac{V_c}{4V_s} \right] \left[ \frac{1}{Q_s} - \frac{1}{Q_0} \right]$$  \hspace{1cm} \text{[7.15]}

If the frequency shift is measured from the resonance frequency $f$ of the cavity loaded with empty capillary tube rather than that with the empty cavity alone the above equation becomes

$$\varepsilon'_r - 1 = \frac{f_1 - f_s}{2f_s} \left[ \frac{V_c}{V_s} \right]$$  \hspace{1cm} \text{[7.16]}

$$\varepsilon''_r = \left[ \frac{V_c}{4V_s} \right] \left[ \frac{1}{Q_s} - \frac{1}{Q_t} \right]$$  \hspace{1cm} \text{[7.17]}

$Q_t$ is the quality factor of the cavity loaded with empty tube. $f_s$ and $Q_s$ are the resonance frequency and quality factor of cavity loaded with capillary tube containing the sample material.

7.4(b) Conductivity of materials

For a dielectric material having non-zero conductivity, Ampere's law in phasor form is

$$\nabla \times H = (\sigma + j\varepsilon'')E = (\sigma + \varepsilon'')E + j\varepsilon'E$$  \hspace{1cm} \text{[7.18]}

where $\varepsilon = \varepsilon' - j\varepsilon''$

The loss tangent $\tan \delta = \frac{\varepsilon''}{\varepsilon'}$  \hspace{1cm} \text{[7.19]}

The effective conductivity of the medium $\sigma_e = \sigma + \varepsilon''$

But $\tan \delta = \frac{1}{Q_m} - \frac{1}{Q_s} - \frac{1}{Q_t}$  \hspace{1cm} \text{[7.20]}

$Q_m$ is the loaded Q-factor of the cavity with the sample alone

The effective conductivity $\sigma_e = \frac{\varepsilon' \varepsilon}{Q_m} = \frac{\varepsilon' \varepsilon_0}{Q_m}$  \hspace{1cm} \text{[7.21]}
When \( \sigma \) is very small,
\[
\sigma_e = \omega \varepsilon'' = 2\pi \varepsilon_0 \varepsilon''
\]

---

7.5 Dielectric Properties of Grown Urinary Crystals and Natural Samples at Microwave Frequencies

The dielectric properties of urinary crystals were studied using a transmission type \( S \)-band rectangular cavity resonator figure 7.2 and the analysis were done on an HP 8714 ET network analyser and an interfacing computer shown in figure 7.3.

![Figure 7.2: Schematic diagram of the transmission type cavity resonator](image)

![Figure 7.3: Experimental set up](image)

The cavity resonator was excited in the \( TE_{105} \) mode. A typical resonant frequency spectrum of the cavity resonator is shown in figure 7.4. Initially, the resonant frequency \( f_0 \) and the corresponding
quality factor $Q$ of each resonant peak of the empty cavity were determined. Samples were finely powdered and filled in small teflon cups and introduced into the cavity resonator through the non-radiating slot. One of the resonant frequencies of the loaded cavity was selected and the position of the sample was adjusted for maximum perturbation (i.e. maximum shift of resonant frequency with minimum amplitude for the peak). The new resonant frequency $f_r$, and 3dB bandwidth and hence the quality factor $Q$ are determined. The procedure was repeated for resonant frequencies of 2247.359, 2439.605, 2684.433 and 2970.641MHz.

![Graph](image)

**Figure 7.4:** A typical resonant frequency spectrum
### Table 7.1: Dielectric constant and dielectric loss of grown and natural urinary crystals

<table>
<thead>
<tr>
<th>Samples</th>
<th>Frequency $f_0$(MHz)</th>
<th>Dielectric Constant $\varepsilon_r$</th>
<th>Dielectric Loss $\varepsilon_r''$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Grown Crystal</td>
<td>Natural Sample</td>
</tr>
<tr>
<td>Calcium oxalate monohydrate</td>
<td>2247.359</td>
<td>2.82</td>
<td>3.1</td>
</tr>
<tr>
<td>[Whewellite]</td>
<td>2439.605</td>
<td>2.9</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>2684.433</td>
<td>3.26</td>
<td>3.34</td>
</tr>
<tr>
<td></td>
<td>2970.641</td>
<td>2.61</td>
<td>3.03</td>
</tr>
<tr>
<td>Ammonium magnesium phosphate hexahydrate [Struvite]</td>
<td>2247.359</td>
<td>3.85</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>2439.605</td>
<td>4.35</td>
<td>3.19</td>
</tr>
<tr>
<td></td>
<td>2684.433</td>
<td>4.05</td>
<td>4.74</td>
</tr>
<tr>
<td></td>
<td>2970.641</td>
<td>3.94</td>
<td>4.75</td>
</tr>
<tr>
<td>Cystine</td>
<td>2247.359</td>
<td>3</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>2439.605</td>
<td>3.1</td>
<td>3.07</td>
</tr>
<tr>
<td></td>
<td>2684.433</td>
<td>2.72</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>2970.641</td>
<td>2.75</td>
<td>2.95</td>
</tr>
<tr>
<td>Brushite</td>
<td>2247.359</td>
<td>3.74</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>2439.605</td>
<td>3.72</td>
<td>4.23</td>
</tr>
<tr>
<td></td>
<td>2684.433</td>
<td>3.67</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>2970.641</td>
<td>3.33</td>
<td>4.22</td>
</tr>
<tr>
<td>Uric acid</td>
<td>2247.359</td>
<td>1.88</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>2439.605</td>
<td>2.32</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2684.433</td>
<td>2.13</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>2970.641</td>
<td>2.5</td>
<td>2.08</td>
</tr>
</tbody>
</table>

**Figure 7.5:** Variation of conductivity with frequency for grown and natural crystals
The dielectric constant ($\varepsilon'$) and dielectric loss ($\varepsilon''$) are determined for all the five crystals and are tabulated in Table 7.1. Struvite crystals have the highest value of dielectric constant and uric acid crystals have the least value of dielectric constant. Similar studies were carried out on the natural urinary stones. Natural urinary stones were procured from patients who had undergone surgery from the nearby hospital. The case histories of the patients were recorded and the stone samples were analysed to confirm the class to which they belong. Pre-treatment was done on these samples to remove the water of crystallization by drying. Still the samples obtained cannot be treated as hundred percent pure due to the inclusions present in the stone. However to the extend of comparison of dielectric parameters these samples were studied upon. Table 7.1 draws a comparative evaluation of the grown crystals with the natural samples. Although the dielectric parameters between these samples are nearly the same, the slight variations can be due to the biological inclusions trapped within the lattice and due to water molecules that still remain. The conductivity ($\sigma$) and loss tangent ($\tan \delta$) are evaluated and are represented graphically in Figures 7.5 and 7.6. In-vitro studies on the
dielectric properties of the urinary crystals and can pave way for the
detection of the urinary calculi in human systems by microwave
tomography.

References

1) Livesay D.E and Chen K.M; IEEE Trans. on Microwave Theory
   and Techniques. 1974, 44, 1273-1280.


3) David Despretz, Jean Christophe Camart et.al; IEEE Trans. on

4) Rapport C and Morgenthaler F; IEEE Trans. on Microwave

5) Murch R.D and Chann T.K.K; IEEE Trans. on Microwave Theory

6) Schwan H.P, Advances in Biological and Medical Physics. Vol. 5,

   University, March 1943.


   Microwave Theory Tech. 1979, 27, 791-794.

10) Clayton R.Paul and Syed A.Nazer, Introduction to Electromagnetic
