CHAPTER 6

ANALYSIS OF THE OPTICAL PULSES

6.1. Grand Pulse Height Distribution of Optical Pulses:

The data obtained for all the optical pulses collected simultaneously with radio pulses of various frequencies are plotted as shown in fig. 6.1. In total, 696 optical pulses were collected during the time of experiment. Out of these, 220 pulses were collected along with 60 MHz radio pulses, 269 pulses were collected with 80 MHz radio pulses and 207 pulses were collected with 110 MHz radio pulses. Thus, fig. 6.1 gives a histogram of all the optical pulses given in fig. 5.3, fig. 5.8 and fig. 5.10.

6.2. Theoretical Pulse Height Distribution of Optical Cerenkov Pulses from EAS:

The general equation for frequency distribution of the optical Cerenkov pulses from EAS, which was developed by earlier workers (Goswami and Pathak, 1977), was used to test the data obtained in the present experiment. The theoretical deduction is presented here.
Fig. 6.1: Grand pulse height distribution of all the optical pulses.
The detection probability of an EAS detecting system like the one used in the present experiment depends on the location of the shower axis as well as on the primary energy. Again, for a particular position of the Čerenkov detector, the pulse height depends on the location of the shower axis and the energy of the primary. From these two considerations, one can eliminate the energy term and can integrate over all the locations of the shower axes (vertical) to arrive at a relation between the number of showers (vertical) detected and pulse heights.

Let a shower of primary energy $E$ be incident at $P(x,y)$ (Fig. 4.2) for which the densities at the detectors (trays $A_1$, $A_2$, $A_3$) be $A_1$, $A_2$, and $A_3$ respectively. Then the probability that the shower is detected is given by Galbraith (1958) as,

$$ P = \prod_{i=1,2,3} \left[ 1 - \exp (A_i s_i) \right] \quad \ldots (6.1) $$

where $s_i = \text{area of the } i\text{-th tray}$.

Now from the number spectrum, the frequency of showers having total number of particles between $n$ and $n + \text{d}n$ is,

$$ K(n) \text{ d}n = A^n \gamma^{-1} \text{ d}n $$

where $\gamma = 1.55 + 0.0195 \ln (n/10^6)$ and $A = \text{constant}$ for $10^{16} < E < 10^{19}$ eV, $\gamma = 1.55$, so that,
\[ K(n) \, dN = A n^{-2.55} \, dN \quad \ldots \quad (6.2) \]

From Olbert's calculations (1957), the relation between the total number of particles at sea level and the primary energy is,

\[ N = 5 \times 10^{-13} \, E^{1.16} \quad \ldots \quad (6.3) \]

where \( E \) is in eV.

So, the frequency of EAS having primary energy between \( E \) and \( E + dE \) is,

\[ n(E) \, dE = 3 \, E^{-2.80} \, dE \quad \ldots \quad (6.4) \]

\((3 = \text{Constant})\)

The number per unit time of each shower having primary energy \( E \) and \( E + dE \) falling on an area \( dx \, dy \) at \((x, y)\) is,

\[ C \, dE \, dx \, dy = \prod_{i=1,2,3} \left[ 1 - \exp\left(-\Delta_i S_i\right) \right] \, B E^{-2.80} \, dE \, dx \, dy \quad \ldots \quad (6.5) \]

The density of particles at a distance \( R \) (metres) from the axis is,

\[ \Delta = Nf(R) = 5 \times 10^{-13} \, E^{1.16} \, f(R) \quad \ldots \quad (6.6) \]

where \( f(R) \) is taken as,

\[ f(R) = 1.75 \times 10^{-3} \, \frac{1}{R} \, \exp\left(-\frac{R}{80}\right) \]
\[
A_i = 8.75 \times 10^{-16} \Sigma i \, \frac{1}{R_i} \exp \left(-\frac{R_i}{80} \right) \quad \ldots \quad (6.7)
\]

Again, on the assumption that the pulse height (H) is directly proportional to the intensity (I), i.e., \( H = K I \) (K = Constant) and also from theoretical calculation of Zatsepin and Chudakov (1962) one gets,

\[
I = D r^{-1} E \quad \ldots \quad (D = \text{Constant})
\]

So that,

\[
H = D_1 r^{-1} E \quad (D_1 = \text{Constant}) \quad \ldots \quad (6.8)
\]

On eliminating \( E \) from equations 6.5, 6.7, and 6.8 and integrating for all positions of the shower axis, one gets,

\[
C dH = \int \int \left[ \prod_{i=1}^{3} \left[ 1 - \exp \left\{ -8.75 \times 10^{-16} \left( \frac{r}{D_1} H \right)^{1.16} \frac{1}{R_i} \exp \left( -\frac{R_i}{80} \right) \right\} s_i \right] \right] \frac{B \left( \frac{r}{D_1} H \right)^{-2.80}}{r \frac{1}{D_1}} dH \, dx \, dy
\]

Or

\[
C dH = \int \int \left[ \prod_{i=1}^{3} A_1 \left[ 1 - \exp \left\{ -r^{1.16} (A_2 H)^{1.16} \frac{1}{R_i} \exp \left( -\frac{R_i}{80} \right) \right\} \right] \right] \frac{r^{-1.80}}{(A_2 H)^{-2.80}} dH \, dx \, dy \quad \ldots \quad \ldots \quad (6.9)
\]

where \( A_1 \) and \( A_2 \) are constants and
\[ R_1 = \sqrt{(x - 35.3)^2 + (y - 40.2)^2} \]
\[ R_2 = \sqrt{(x - 35.3)^2 + (y + 40.2)^2} \]
\[ R_3 = \sqrt{(x + 53.5)^2 + (y)^2} \]
and
\[ r = \sqrt{(x - 26.5)^2 + (y - 5)^2} \]

Equation (6.9) which was computed in IBM 360 Model 44 computer at Physical Research Laboratory, Ahmedabad gives the pulse height distribution where both C and H are relative. The fact that the equation (6.9) is independent of the energy of the primary particle makes it more useful because primary energy is really a difficult parameter to measure precisely. It must be pointed out here that the approximations (rather over simplified) that lead to equation (6.9) take into account only of vertical showers, and this procedure leaves a slight bias in favour of large showers.

6.2.4. Comparison between Theoretical Prediction and Experimental Results:

The theoretical differential pulse height distribution \( C(H)/H \) vs \( H \) as calculated from equation (6.9) is presented in table 6.1. The observed values of \( C(H)/H \) from the present
Theoretical differential pulse height distribution of optical Cerenkov pulses along with experimental data, normalised at an arbitrary point $H = 2.0$.

<table>
<thead>
<tr>
<th>Pulse heights</th>
<th>Differential pulse heights $S(m)/n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td>1 - 2</td>
<td>26.5</td>
</tr>
<tr>
<td>2 - 3</td>
<td>6.9</td>
</tr>
<tr>
<td>3 - 4</td>
<td>2.4</td>
</tr>
<tr>
<td>4 - 5</td>
<td>1.0</td>
</tr>
<tr>
<td>5 - 6</td>
<td>0.51</td>
</tr>
<tr>
<td>6 - 7</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Fig. 6.2: Theoretical differential pulse height distribution of Cerenkov pulses along with experimental data (present experiment (G), Goswami et al. (A) Galbraith & Jelley (•)) normalised at an arbitrary point $H = 2.5$. 
experiment as well as those from Galbraith and Jelley (1965) and Goswami and Pathak (1977) are also shown in the table. The experimental data are normalised at an arbitrary point $H = 2.5$. The theoretical data are then plotted to obtain a graph (fig. 6.2) giving differential pulse height distribution $G(H)/H$ vs $H$. The data obtained from the present experiment and those of Galbraith and Jelley, and Goswami and Pathak are also plotted on this graph.

The differential pulse height distribution obtained from the data of the present experiment agrees reasonably well with the theoretical deduction. The number of pulses decreases steadily with increasing height showing that pulses of higher heights are progressively fewer. As the Čerenkov light pulses are a cumulative effect of individual particles, the pulse height is a function of particles in a shower. This fact is supported by Jelley (1965) who concluded that Čerenkov light intensity from EAS would be just proportional to the number of particles in a shower. Therefore the larger pulse heights are caused by larger showers, i.e., showers with higher primary energies. A study of differential frequency distribution of primary energy against intensity reveals that there is a rapid fall off in the number of showers as shower size increases. Consequently number of large Čerenkov pulses should also fall off. This conclusion seems to be supported by the present theoretical as well as experimental data. As
the frequency of shower decreases from higher to lower side with increasing energy, so also the number of Cerenkov pulses from EAS. The reported investigation of the primary spectral shape based on the study of Cerenkov light component of the EAS (Gerdes et al., 1973, 1975) also places weight on the above conclusion. These workers have shown that in the energy range $10^{12}$ to $10^{15}$ eV, the pulse height distribution of the Cerenkov pulses should reproduce the primary spectral shape directly. However, the effect of fluctuations which is a function of energy is an important parameter and the assumed one to one correspondence between the measured light pulse exponent and the primary energy exponent is over simplified (Hartman et al., 1977).

Further, we can compare the present results with those of Galbraith and Jelley (1953), who also conducted a somewhat similar experiment. They studied only the Cerenkov light pulses from EAS unlike the present experiment where a correlation between simultaneously optical as well as radio pulses from EAS is studied. Accordingly data obtained by those workers are plotted in fig. 6.2. The agreement between their results and the theoretical distribution except at large pulse heights is clearly established. Thus it can be safely concluded that the observed optical pulses are due to direct contribution from Cerenkov radiation from shower particles.
One interesting fact observed in the present experiment is the absence of the earlier discrepancy between theory and experiment (Goswami and Pathak, 1977) at large pulse heights. This may possibly be due to better discrimination against noise by the present optical detector system (developed by the author) than by the earlier one. The electronics of the existing system is considerably improved, the system bandwidth being increased threefold. Thus the overall resolving time of the detector is sufficiently improved.

Thus, within certain limitations, the equation 6.9 may be accepted to give the pulse height distribution of optical Cerenkov pulses from EAS (Wolfendale, 1975). The comparison between theory and the experiment is also seen to give good agreement. Thus the identity of the observed optical pulses as Cerenkov pulses from EAS of primary energy $E_p > 10^{16}$ eV is reasonably established.

In the next chapter, the correlation between simultaneously observed optical Cerenkov pulses and radio pulses will be analysed.