1.1. Fluid dynamics: Stratified fluid and its flows.

Fluid dynamics is the study of the variety and complexity of the flow patterns of liquids and gases (collectively called fluids) under the action of forces of various kinds. It involves a comparison between the theoretical and conceptual structure built on the basic laws with the experimental observations. In fluid motions we usually deal matter in bulk (macroscopic) so that every portion of the fluid is regarded as continuous and as having the same property, however small the portion may be. Since different fluids have different properties, we propose here to study the flow of ideal fluids (having no tangential stress between adjoining portions in relative motion) with stable density variation. We give a brief account of the occurrence of density variation in fluids in nature and the behaviour of the flows.

Fluids such as salt solutions of variable concentrations or homogeneous liquids of variable temperature occur in nature and are found in estuaries, lakes and reservoirs. Even though sea water on the surface in tropical areas may be quite warm, deep down it will be only a few degrees above the freezing point.
Thus in oceans, solar heating produces a warm upper layer (the epilimnion) followed by a transition layer (the metalimnion or the thermocline) where the density varies significantly and then the lowermost cold region (the hypolimnion). Atmospheric observations also indicate the occurrence of cold currents in the atmosphere, formation of air waves in the lee of mountains and stagnation of air in the neighbourhood of a mountain range whenever there is a strong inversion. All these are characterised by the non-homogeneity in density or entropy of the fluid and by the important role played by gravity. Such fluids are examples of stratified fluids and their flows fall in the category of stratified flows.

It is known that the physical cause of the oscillatory nature of all waves is the restoring force acting on a material particle as the particle is displaced from its mean position. If the restoring force is gravitational in nature, then the density variation (statically stable) is essential for the restoring force to exist. Thus in a gravitational field of force, a non-homogeneous fluid has the characteristic property of sustaining internal gravity waves and therefore its study has a wide application to many geophysical phenomena as explained in the next section.

1.2. Geophysical applications:

One of the common occurrences in nature is the intrusion of a heavy fluid into a lighter one. It occurs when a cold air current advances in a warm atmosphere, a muddy stream enters...
reservoir of clear water or tides cause wedges of salt to progress in estuaries. The density difference as well as the difference in the specific weights are responsible for such phenomena. Water waves generated by wind blowing over its surface is also a geophysical phenomenon. In this connection, the works of Ursell (1956), Miles (1957, '59 a-b, '60), Helmholtz (1868), Jeffrey (1924, '25a) and Lighthill (1962) on water waves may be cited.

Atmospheric observations have shown the existence of alternately ascending and descending air currents above a mountain range. It has also indicated the existence of stationary waves at various heights on the lee side of it. These waves are called lee-waves and their existence is not surprising since the atmosphere is stratified in entropy (or potential density or temperature). The theories due to Lyra (1943), Queeny (1947, '48) and scorer (1949) on the two-dimensional waves on the lee of mountains may be mentioned. Scorer and jinkinson (1956), Crapper (1959, '62) and Brazin (1961) studied the three-dimensional lee-waves particularly on the internal wave resistance encountered by submerged bodies moving in a stratified fluid.

Internal waves also known as boundary waves are generated along a discontinuity or sharp vertical gradient of density within the sea. Since the density difference between two portions of sea water are much less than that between water and air, these waves become much higher and steeper than surface waves and also move very slowly. These waves have some dangerous effects on sailors in sea. Internal waves are also used to propagate sound
waves along water-mass boundaries in a layer called SOFAR. It also helps in replenishment and renewal of water in many basins.

All these geophysical phenomena are due to non-uniformity in fluid density and the gravitational force acting on it. Thus the study of stratified flows under the gravitational field of force help much to explain various problems of geophysical phenomena.

1.3. Industrial applications:

The study of stratified fluids also find applications in industries. As an example, the concept of solar pond and ocean thermal energy conversion (OTEC) may be mentioned. Other examples worth mentioning are as follows:

The intrusion of a heavy fluid into a lighter one occurs in the process of manufacturing glass. Idea of fluid flow in porous media is applicable to hydrology and is of vital interest to petroleum industry, and also to paper industry. Axisymmetric flow of non-homogeneous fluids have also a bearing on such engineering devices as centrifuges, and such meteorological phenomena as tornados.

1.4. Aim of the thesis:

Because of the common occurrences of stratified flows in nature and their importances in geophysical and industrial applications, we propose in this thesis to study the stratified
fluid flow. We consider in the next article the general equation of motion for a stratified fluid in three dimensions and then consider the two-dimensional case. Linear forms of the equations are deduced and these will be applied to the problems considered in the subsequent chapters.

1.5. General equations of motion:

(i) The motion of a Newtonian fluid (homogeneous or heterogeneous) which does not involve temperature differences is governed by

(1.5.1)

The above basic hydrodynamic equations are to be supplemented by the equation of state. Since we are dealing only with the non-diffusive fluid without any application of heat, the equation of state is written as

(1.5.2)

In the above equations, \( \bar{q} \) denotes velocity vector, \( \bar{F} \) the external force other than the gravitational force, \( \bar{g} \) the acceleration due to gravity, \( p \) and \( \rho \) the pressure and density respectively.
We consider here an incompressible ideal fluid with stable density variation (i.e. with density decreasing upwards), and the flow will also be assumed to be steady. If there be no other external force than the gravitational force, then the equation of motion (1.5.1) reduces in the steady state to

\[ \rho \left( \nabla \cdot \mathbf{v} \right) \mathbf{v} = - \nabla p + \rho \mathbf{g} \]  \hspace{1cm} (1.5.4)

and the equation of continuity will be used in the form

\[ \nabla \cdot \mathbf{q} = 0 \] \hspace{1cm} (1.5.5)

as the incompressibility of the fluid demands

\[ \left( \nabla \cdot \mathbf{v} \right) \rho = 0 \] \hspace{1cm} (1.5.6)

the equation of state then takes, in view of steady motion and stable stratification, the form

\[ \rho = \rho \left( 
\frac{d}{dx} \right) \] \hspace{1cm} (1.5.7)

where \( \frac{d}{dz} < 0 \), \( z \) being measured vertically upwards.

The equations (1.5.4) to (1.5.7) are the fundamental equations for an incompressible, ideal, stratified fluid in the steady motion under gravity.

Considering the equation (1.5.4), the relative importance of the inertia term to the gravity term is given by the ratio

\[ \frac{\text{inertia term}}{\text{gravity term}} = \frac{\frac{U_0^2}{g d}}{g d} \]

where \( U_0 \) stands for a representative velocity, \( d \) for a
representative length and $g$ for the constant magnitude of the acceleration due to gravity. We put this ratio as $F_o^2$, so that

$$F_o = \left(\frac{U_o^2}{gd}\right)$$

(1.5.8)

It may be noted that the above ratio does not involve the density $\rho$.

The quantity $F_o$ is a pure number and is the usual Froude number; we shall call it the ordinary Froude number. If $F_o \gg 1$, the gravity effect is not significant and the flow appears to take place under the pressure gradient only as if the gravity is absent. If, on the other hand, $F_o \ll 1$, the gravity plays a dominant role just tending to maintain the static stable stratification, and the pressure almost balances the hydrostatic pressure. When $F_o \sim 1$, i.e. when the inertia and gravity effects are equally important then it is under this regime that formation of lee-waves and other complicated forms of the flow can be visualised as evidenced by the works of Long (1953, '53), Yih (1958, '61), Miles (1968a, '69), Drizin and Moore (1967).

(ii) Deductions of the equations:

The equation (1.5.4) can be expressed as

$$\rho (\vec{w} \times \vec{q}) - \left(\frac{1}{2} q^2 + g y\right) \nabla \rho = - \nabla H$$

(1.5.9)

where,
\[ H = \rho + \frac{1}{2} \rho q^2 + \rho g y, \text{ and } \vec{w} = \text{curl } \vec{q} \text{ is the vorticity.} \]

Taking the scalar product of (1.5.9) with \( \vec{q} \) and using (1.5.5), it follows that

\[ \vec{q} \cdot \nabla H = 0 \quad (1.5.10) \]

The equations (1.5.5) and (1.5.10) just show that \( \rho \) and \( H \) are each constant along a streamline.

The term \(-\frac{1}{2} q^2 \nabla \rho\) on the left hand side of (1.5.9) can be got rid off if we put

\[ \vec{q} = \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \vec{q}' \quad (1.5.11) \]

where \( \rho_0 \) is a reference density.

From (1.5.11) one finds

\[ \vec{w} = \text{curl } \vec{q} = \text{curl } \left\{ \left( \frac{\rho}{\rho_0} \right)^{-\frac{1}{2}} \vec{q}' \right\} = \left( \frac{\rho}{\rho_0} \right)^{-\frac{1}{2}} \vec{w}' - \frac{1}{2} \left( \frac{\rho}{\rho_0} \right)^{-\frac{1}{2}} \rho^{-1} \nabla \rho \times \vec{q}' \]

where,

\[ \vec{w}' = \text{curl } \vec{q}' \]

and so,

\[ \rho (\vec{w} \times \vec{q}) = \rho \left( \frac{\rho}{\rho_0} \right)^{-1} \left[ \vec{w}' \times \vec{q}' - \frac{1}{2} \rho^{-1} (\nabla \rho \times \vec{q}') \times \vec{q}' \right] \]

\[ = \rho \left[ (\vec{w}' \times \vec{q}') + \frac{1}{2} \frac{1}{\rho} \left\{ \vec{q}'^2 \nabla \rho - \vec{q}' (\vec{q}', \nabla \rho) \right\} \right] \]
The last term is zero by virtue of (1.5.5). Hence, substitution in equation (1.5.9) gives

\[ \psi (\mathbf{w}' \times \mathbf{q}') - g \gamma \varphi = - \nabla \mathbf{H}. \]  

(1.5.12)

The quantity \( \mathbf{q}' \) defined in the equation (1.5.11) has the dimension of velocity. This is the velocity associated with the flow given by (1.5.12) (afterwards known as the pseudo-flow) and will be called the 'pseudo-velocity'. The equivalent equation of continuity to be satisfied by the pseudo-velocity \( \mathbf{q}' \), is (as can readily be verified)

\[ \nabla \cdot \mathbf{q}' = 0, \]  

(1.5.13)

and that for incompressibility is

\[ (\nabla \cdot \mathbf{q}') \rho = 0. \]  

(1.5.14)

The equations (1.5.12) to (1.5.14) together with the equation of state (1.5.7) can be regarded as constituting the basic equations of the pseudo-flow associated with the stratified flow under gravity.

The necessity of the transformation (1.5.11) will clearly be seen when we come to consider the two-dimensional case in the following articles 1.6 and 1.7.

1.6. Two-dimensional case:

In two dimensions (when the motion is parallel to a vertical plane), considering the actual flow, a stream function \( \psi \) can, in
view of (1.5.5), be defined such that
\[ \vec{q} = \text{curl} \left( \vec{R} \psi \right) = -\vec{R} \times \nabla \psi \]

where \( \vec{R} \) is the unit vector perpendicular to the plane of the motion. (This gives, \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \)).

Then the equations (1.5.6) and (1.5.10) imply that \( \rho \) and \( H \) are each functions of \( \psi \), so that
\[ \rho = \rho(\psi) \quad \text{and} \quad H = H(\psi). \quad (1.6.1) \]

Using these expressions and also using the fact that
\[ \vec{w} = \text{curl} \, \vec{q} = -\vec{R} \nabla^2 \psi, \]

the equation (1.5.9) yields the scalar equation
\[ \nabla^2 \psi + \frac{1}{\rho} \frac{d\rho}{d\psi} \left[ \frac{1}{2} \left\{ \left( \frac{d\psi}{dx} \right)^2 + \left( \frac{d\psi}{dy} \right)^2 \right\} + gy \right] = \frac{d\rho}{d\psi}. \quad (1.6.2) \]

This is the equation obtained by Madame Dubreil-Jacotine (1935) for considering the two-dimensional flow of a stratified fluid, and was rediscovered by Long (1953) in connection with the stratified flow past a barrier.

For the pseudo-flow, we write
\[ \vec{q}' = -\vec{R} \times \nabla \psi' \]
where \( \psi' \) may be called the pseudo-stream function. The stream function \( \psi \) of the actual flow and the pseudo-stream function \( \psi' \) of the pseudo-flow are related by
\[ \psi = \int \left( \frac{\rho}{\rho'} \right) \, d\psi' \quad (1.6.3) \]

(omitting a constant which is irrelevant); this follows from the equation (1.5.11).

The relation (1.6.3) is called Yih's transformation. This transformation or the transformation (1.5.11) provides one way by which the inertial effects of density variation can be directly taken into account in the equation of motion as will be seen later.

Again, since \( \rho \) and \( H \) are each constant on \( \psi = \text{constant} \) (cf. equation (1.6.1)), it follows that they are also each constant on \( \psi' = \text{constant} \), so that for the pseudo-flow the equation (1.6.1) will be replaced by

\[ \rho = \rho(\psi) \quad \text{and} \quad H = H(\psi'). \quad (1.6.4) \]

Hence the equation (1.5.12) yields the scalar equation

\[ \nabla^2 \psi' + \frac{1}{\rho} \frac{d\rho}{d\psi'} \, \rho \psi = \frac{1}{\rho} \frac{dH}{d\psi'}. \quad (1.6.5) \]

This is the equation obtained by Yih (1958, '60) in terms of the pseudo-stream function \( \psi' \).

The equation (1.6.5) can be written in the non-dimensional form as follows:

Define non-dimensional quantities as

\[ (x, y) = c(x', y'); \quad \rho = \rho_0 \rho'; \quad \psi' = \int_0^x \psi' \, d\psi'; \quad H = \rho_0 \int_0^x H'. \]
where \( d \) is a reference length and \( U_0 \) is a reference velocity; then the equation (1.6.5) becomes in terms of the new variables

\[
\nabla^2 \psi^* + \frac{1}{F_o^2} \frac{d}{d\psi^*} \frac{d}{d\psi^*} \psi^* \frac{d}{d\psi^*} = \frac{dH^*}{d\psi^*} ; \\
(1.6.6)
\]

where,

\[
\nabla^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}
\]

and \( F_0 \) is the ordinary Froude number equal to \( \sqrt{\frac{g}{d}} \) (cf. equation (1.5.8)).

The non-dimensional pressure and velocity (pseudo) are respectively given by

\[
p^* = \frac{1}{\rho_0 U_0^2} p, \text{ and } q^* = \frac{1}{U_0} q^* .
\]

Dropping the asterisk marks we get from (1.6.6)

\[
\nabla^2 \psi' + \frac{1}{F_o^2} \psi' \frac{d}{d\psi'} \frac{d}{d\psi'} \psi' = \frac{dH'}{d\psi'} ; \\
(1.6.7)
\]

this is the basic equation in non-dimensional form obtained by Yih (1958) for studying the two-dimensional flow of a stratified, incompressible, inviscid fluid under the influence of gravity.

The equation (1.6.7) however deals with the pseudo-flow. But after determining the pseudo-flow, one can then have the actual
flow through the transformation (1.6.3). The equation (1.6.2) which defines the actual flow is very difficult to deal with analytically, whereas the equation (1.6.7) defining the pseudo-flow is simpler in appearance and can be dealt with less difficulty. This explains the necessity of the transformation (1.6.3) or (1.5.11).

1.7. Some discussions on the equation (1.6.7) and particular cases:

To deal with the equation (1.6.7) one should know the dependence of $p$ and $H$ on $\psi$ so that $\frac{df}{d\psi}$ and $\frac{dH}{d\psi}$ can be written in terms of $\psi$. For a liquid extending to infinity, these can be written from a pre-supposed upstream condition at infinity.

Once the forms of $\frac{df}{d\psi}$ and $\frac{dH}{d\psi}$ are known in terms of $\psi$, the equation (1.6.7) whether linear or non-linear fully describes the flow; but linear equations may likely lead to analytical solutions with less difficulty. Though the exact forms of $p$ and $H$ at infinity are not known, yet it is not unrealistic to assume $p$ and $H$ in such a way that the equation (1.6.7) becomes linear. It is on this basis that Long (1953) and Yih (1958) linearized the equations (1.6.2) and (1.6.7) respectively.

(i) Long's linearization:

Let us recall the equation (1.6.2) viz.

$$\nabla^2 \psi + \frac{1}{\rho} \frac{df}{d\psi} \left[ \frac{1}{2} \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right] + my = \frac{dH}{d\psi}$$  (1.7.1)
This refers to the actual physical flow of the stratified fluid.

As mentioned earlier ρ and H are each functions of ψ,

and so, \( \frac{d\rho}{d\psi} \) and \( \frac{dH}{d\psi} \) are functions of ψ so that they will be
constant along a streamline. Since the streamlines extend up to
infinity, the values of \( \frac{d\rho}{d\psi} \) and \( \frac{dH}{d\psi} \) at any point will be the
same as their corresponding values at infinity on the streamline
passing through that point. This means

\[
\frac{d\rho}{d\psi} = \left( \frac{d\rho}{d\psi} \right)_{\text{inf.}} \quad \text{and} \quad \frac{dH}{d\psi} = \left( \frac{dH}{d\psi} \right)_{\text{inf.}}
\]

Again, at infinity ψ depends only on y. So, if \( y_0(\psi) \) is
the height of the streamline \( \psi = \text{constant far upstream} \) and \( \zeta(\psi) \)
is the vorticity thereat, then the equation (1.7.1) can be written
as

\[
\nabla^2 \psi + \frac{1}{\rho} \frac{d\rho}{d\psi} \left[ \frac{1}{2} (\nabla \psi)^2 + \gamma y \right] = \zeta(\psi) + \frac{1}{\rho} \left( \frac{d\rho}{d\psi} \right)_{\text{inf.}} \left\{ \frac{1}{2} U^2 + \gamma y_0 \right\}
\]

where \( U (\neq 0) \) is the horizontal velocity at infinity.

The above equation can be put as

\[
\nabla^2 \psi - \zeta(\psi) + \frac{1}{2 \rho} \frac{d\rho}{d\psi} \left[ (\nabla \psi)^2 - U^2 \right] = \frac{\xi}{\rho} \frac{d\rho}{d\psi} (y_0 - y). \quad (1.7.2)
\]

If we write,

\[
\frac{d}{d\psi} = \frac{dy_0}{d\psi} \quad \frac{d}{dy_0} = \frac{1}{U} \frac{d}{dy_0}
\]
then,
\[ \nabla^2 \psi = U \nabla^2 y_0 + \nabla U \cdot \nabla y_0, \]

\[ (\nabla \psi)^2 = U^2 (\nabla y_0)^2, \]

\[ \xi(\psi) = (\nabla^2 \psi)_{\text{inf.}} = \frac{dU}{dy_0}, \]

\[ \nabla U = \frac{dU}{dy_0} \nabla y_0. \]

Using the above relations, the equation (1.7.2) takes the form

\[ \nabla^2 y_0 + \frac{1}{2} \left[ (\nabla y_0)^2 - 1 \right] \frac{d}{dy_0} (\ln U^2 \rho) = \frac{g}{U^2 \rho} \frac{d \rho}{dy_0} (y_0 - y). \quad (1.7.3) \]

with the further substitution \( \delta = y - y_0 \) implying

\[ \nabla^2 y_0 = \nabla^2 (-\delta + y) = -\nabla^2 \delta, \]

\[ (\nabla y_0)^2 - 1 = (\nabla \delta)^2 - 2 \frac{\partial \delta}{\partial y}, \]

the equation (1.7.3) can be thrown into the form

\[ \nabla^2 \delta - \frac{1}{2} \left[ (\nabla \delta)^2 - 2 \frac{\partial \delta}{\partial y} \right] \frac{d}{dy_0} (\ln U^2 \rho) = \frac{g}{U^2 \rho} \frac{d \rho}{dy_0}. \quad (1.7.4) \]

Here \( \delta \) represents the variation in height of the streamline \( y_0 = \) constant about its equilibrium height far upstream.

From the equation (1.7.4) one can obtain a linearized equation by assuming (1) the density variation is linear in \( y_0 \) and
(2) \( u^2 \rho = \text{constant upstream} \). These assumptions reduce the equation (1.7.4) to the linear form viz.

\[
(\nabla^2 + \sigma^2) \delta = 0
\]

where,

\[
\sigma^2 = \frac{g}{\epsilon} \frac{1}{\epsilon} \frac{d \rho}{dy_0} = \text{constant.}
\]

This is how Long obtained his model for linearization under the assumptions (1) and (2) mentioned above. The second assumption also known as Long's hypothesis means explicitly that the horizontal kinetic energy is constant far upstream.

Long (1953, '55) used the equation (1.7.5) in studying the flow over solid bodies.

(ii) Yih's linearization:

The equation (1.6.7) will be linear if the dependence of \( \rho \) and \( H \) on \( \psi \) be quadratic at most. In fact, this was used by Yih (1958, '60) in tackling the problem of the channel flow towards a sink and also of the flow over a barrier with applications to atmospheric waves.

For the channel flow, Yih assumed the pseudo-velocity to be constant at infinity (with no vertical component) and the density to be linearly decreasing with the upward vertical height. Thus at infinity he put

\[
\bar{q}', = \bar{I} \quad (\bar{I} \text{ is the unit vector along the } x\text{-axis}),
\]
and
\[ \rho = 1 - \alpha y, \quad (y \text{ is measured vertically upwards}), \quad (0 < \alpha < 1) \]
where,
\[ \alpha = 1 - \frac{\rho_0}{\rho_1} \quad (\rho_0 > \rho_1); \]
\[ \rho_0 \text{ and } \rho_1 \text{ being the densities of the uppermost and lowermost fluid layers respectively.} \]

These imply, at infinity
\[ \psi' = y, \quad \nabla^2 \psi' = 0 \quad \text{and} \quad \rho = 1 - \alpha \psi'. \]

So, at infinity \( \frac{d\rho}{d\psi'} = -\alpha \) (const.), that is, \( \frac{d\rho}{d\psi'} \mid_{\text{inf.}} = -\alpha. \)

Hence by the equation (1.6.7)
\[ \left( \frac{d\psi'}{d\psi} \right)_{\text{inf.}} = -F_0^{-2} \alpha y = -F_0^{-2} \alpha (\psi')_{\text{inf.}}. \]

As a particular streamline (i.e. \( \psi' \) fixed) has the same value throughout its entire length, one can write
\[ \left( \frac{d\psi'}{d\psi} \right)_{\text{inf.}} = -F_0^{-2} \alpha \psi'. \]

Since \( \frac{d\rho}{d\psi'} \) and \( \frac{d\psi'}{d\psi} \) are constant along a streamline, it therefore follows that
\[ \frac{d\rho}{d\psi'} = \left( \frac{d\rho}{d\psi'} \right)_{\text{inf.}} = -\alpha. \]
and
\[ \frac{dH}{d\psi} = \left( \frac{dH}{d\psi} \right)_{\text{inf}} = -F_0^{-2} \alpha \psi'. \]

Note that under the assumptions mentioned above, \( \alpha \) will at most be a linear function of \( \psi' \), whereas \( H \) will at most be a quadratic function of \( \psi' \).

Substituting the above values of \( \frac{d\alpha}{d\psi} \) and \( \frac{dH}{d\psi} \) in equation (1.6.7), one gets the linear equation
\[ \int \left( \psi^2 + F^{-2} \right) \psi' = \frac{1}{2} \psi^2, \quad (1.7.6) \]
where,
\[ F_0 = \left( \frac{F_0}{a} \right) = \left( \frac{y_0^2}{ag} \right). \quad (1.7.7) \]

The number \( F \) defined above is called the internal Froude number, and it is through this number that the density stratification is now taken into account in the equation of motion. Note that \( F \) is now the parameter of the pseudo-flow; by studying thoroughly this flow, one can determine the actual flow. It is from this point of view that Yih remarks that the stratified flow depends not on the Froude number but on a number called the internal Froude number.

The equation (1.7.6) is the linear equation used by Yih in tackling the problem of the channel flow from \( x = -\infty \) towards a sink placed at the origin; the solution he obtained is
\[ \psi' = y + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin n \pi y \cdot \exp \left[ x(n^2 \pi^2 - F^2)^{\frac{1}{2}} \right], \quad (1.7.8) \]

and this is valid for \( F > \frac{1}{\pi} \).

This solution is for the pseudo-flow. From this the actual flow can be determined through the transformation given by the equation (1.6.3).

It is easy to see that the true velocity at infinity is not constant and also the true vorticity is not zero thereat. These happenings are less realistic and so Yih's linearity assumptions are somewhat artificial. However the flow pattern obtained can be taken as closely approximating the true pattern.

It may be observed that Yih's transformation viz.

\[ \overline{q} = (\frac{\rho}{\rho_0})^{-\frac{1}{2}} q', \]

leads to the Long's hypothesis. For, the above transformation gives

\[ \epsilon \overline{q}^{-2} = \rho_0 \overline{q'}^{-2}, \]

and so at infinity

\[ \epsilon \overline{q}^{-2} = \rho_0 \overline{U_0}^{-2} = \text{const. (as } \overline{q'} = U_0 \text{ at infinity by Yih's assumption)} \]

1.8. A brief account of the problems to be studied in this thesis:

In this thesis we shall use the linearized equation (1.7.6) and apply it to the stratified flow through a horizontal channel with prescribed boundary conditions. In chapter 2, we extend the
channel flow studied by Yih (1938) by assuming the flow from 
\( x = \pm \infty \) towards a sink place at the origin lying at the bottom of the channel. In chapter 3, we study the channel flow from 
\( x = -\infty \) over a dipole placed at the bottom of the channel; the axis of the dipole being parallel to the \( x \)-axis and directed against the flow. We also determine the form of the dividing curve line between the stratified flow and the irrotational dipole flow. Chapter 4 deals with the same channel flow over a solid body placed at the bottom of the channel. In chapters 5 we study the same channel flow when the solid body is placed at the middle of the channel.

1.3. Notations used in this thesis:

Here in this thesis a decimal system is used to indicate the chapter, section and number of equations. For example, the equation (1.5.3) (marked at the right hand side of an equation) indicates the third equation of the fifth section of the first chapter. References are cited at the end of the thesis by giving the names of the authors, years of publications, names of the journals / books respectively in the order of english alphabeticals of the authors.