CHAPTER-1

INTRODUCTION

1.1 A brief history of the development of the subject

The problem of fluid flow, in most practical cases, are turbulent in nature. The present day knowledge of the turbulent flow is still quite limited. But the understanding of a laminar viscous flow provides a pre-requisite for the complete understanding of turbulent flow. In order to minimize the friction drag, as well as, the cooling requirements resulting from the aerodynamic heating, the maintenance of extensive laminar flow is highly desirable. Thus the study of the laminar flow of viscous fluids has remained one of the most interesting problems of aeronautical engineers for the last several decades. Secondly, the study of heat transfer has received the attention of numerous authors, in view of its importance in several fields of engineering and technology. The laws of heat transmission are important in the design and operation of many diverse forms of preheaters, exchangers and coolers, especially in the field of nuclear technology.

Magnetohydrodynamics (MHD) is the science of the motion of electrically conducting fluids under magnetic fields. This subject has attracted numerous scientists and engineers because of its fascination and importance in many technological devices and in understanding the diverse cosmic phenomena.
Another interesting field is that which deals with problems on viscous fluids with suction or injection at the walls. The problem of boundary-layer control with suction or injection has become important because of its application in aeronautical engineering, in the design of aircraft wings, transpiration, cooling of rocket engine and in diffusion technology.

Our investigations leading to the thesis for the award of the Ph.D. degree consists of some flow and heat transfer problems. In all cases, the fluid is considered viscous incompressible and the flow laminar. In the case of flows past flat plates, boundary layer flows are investigated. Both free and forced convection of flow and the effects of suction at the walls are considered. However, main stress is given on magnetohydrodynamics (MHD) or on magneto-convective flows. That is, a major part of our work comprises the flow and heat transfer problems of an electrically conducting viscous incompressible fluid. The works on MHD are restricted to small magnetic Reynolds number so that the induced magnetic fields may be neglected and in the momentum and the energy equations the magnetic field appears only as a constant (i.e. applied constant magnetic field).

In all the problems we have considered the fluid to be Newtonian except in one particular problem where the fluid is considered to be non-Newtonian. In most cases, we have encountered coupled non-linear equations to be solved. These equations have been linearised by adopting suitable perturbation
techniques. In one particular problem of dusty gas we have used the conventional Laplace transform technique.

Before going to actual problems we now give a brief description of boundary-layer theory, role of suction/injection in boundary-layer control, heat transfer in fluids and thermal boundary-layers, Magnetohydrodynamics, dusty gas, non-Newtonian fluids etc with some of the relevant literature. Some basic equations and a brief review of the flow along different geometries and an outline of the thesis are also given at the end of this chapter.

(a) Boundary-Layer theory - its development and applications:

Boundary-layer theory is the corner stone of our knowledge of the flow of air and other fluids of small viscosity under circumstances of interest in many engineering applications. Thus many complex problems in aerodynamics have been clarified by a study of the flow within the boundary-layer and its effect on the general flow around the body. Such problems include the variations of minimum drag and maximum lift of airplane wings with Reynolds number, wind-tunnel turbulence, and other parameters. Even in those cases where a complete mathematical analysis is at present impracticable, the boundary-layer concept has been extraordinarily fruitful and useful.

Towards the end of the 19th century the science of
fluid mechanics began to develop in two directions which had practically no points in common. On the one side there was the science of theoretical hydrodynamics which was evolved from Euler's equations of motion for a frictionless, non-viscous fluid and which achieved a high degree of completeness. Since, however, the results of the so called classical science of hydrodynamics stood in glaring contradiction to experimental results in particular as regards to the very important problem of pressure losses in pipes and channels, as well as with regard to the drag of a body which moves through a mass of fluid, it had little practical importance. For this reason, practical engineers, prompted by the need to solve the important problems arising from the rapid progress in technology, developed their own highly empirical science of hydraulics. The science of hydraulics was based on a large number of experimental data and differed greatly in its methods and in its objects from the science of theoretical hydrodynamics.

The equation of motion of a viscous real fluid was established in the first half of the nineteenth century by Navier (1), Poisson (2), Saint-venant (3) and Stokes (4) and its components are known as Navier-Stokes equations. But these being non-linear partial differential equations, there exist only a few exact solutions of these equations for the cases where either the non-linear terms vanish automatically, or when the equations can be reduced to ordinary differential equations by taking recourse to Laplace transform or some suitable similarity transformation. Stokes (5) investigated the case of parallel flow past a sphere for the limiting case when the viscous forces
are considerably greater than the inertia forces and so the nonlinear terms in the Navier-Stokes equations were neglected. Oseen (6) gave an improvement on the Stokes' solution by taking partly into account the inertia terms in these equations. However these types of solutions are valid for small Reynolds number which corresponds to slow motion. Such motion are often called creeping motion and do not occur often in practical applications. As a result, there was not much progress, till the beginning of the twentieth century in dealing with the flow problems of real fluids by considering the full Navier Stokes equations along with the no slip condition at a solid wall.

The theory of laminar boundary-layers was initiated by a German Scientist Ludwig Prandtl (7) in the year 1904. Prandtl, in a paper on "Fluid motion with very small friction" made an hypothesis that for fluids with very small viscosity the flow about a solid body can be divided into two regions (i) a very thin layer in the neighbourhood of the body (known as the boundary-layer) in which the viscous effects may be considered to be confined and (ii) the region outside this layer where the viscous effects may be considered as negligible and the fluid is regarded as inviscid. With the aid of this hypothesis he simplified the Navier-Stokes equations to a mathematically tractable form which are then called the boundary-layer equations and thus succeeded in giving a physically penetrating explanation of the importance of viscosity in the assessment of frictional drag. In the beginning, the theory was developed mainly for laminar flow of viscous incompressible fluids but in the later years it was extended to include compressible fluids.
and turbulent flow.

Thus, in the case of motion of a real fluid past a solid body, the fluid adheres to the solid wall. It means that in the thin boundary layer adjacent to the solid body or the wall, the frictional force retards the motion of the fluid near the wall and the velocity of the fluid increases from its zero value at the solid wall to its full value which corresponds to the external potential flow at the periphery of the boundary-layer. The smaller the viscosity, thinner is the transition layer. But the steep velocity gradient near the rigid boundary surface inspite of the small viscosity, produces marked effects which are comparable in magnitude with those due to inertia force if the thickness of the transition layer is proportional to the square root of the kinematic viscosity.

The decelerated fluid particles in the boundary layer do not in all cases remain in the thin layer which adheres to the solid body along the whole wetted length of the wall. In some cases the boundary layer thickness increases considerably in the downstream direction and the flow in the boundary-layer becomes reversed. This causes the decelerated fluid particles to be forced outwards which means that the boundary-layer is separated from the wall. This is called boundary-layer separation. This phenomenon is always associated with the formation of vortices and with large energy losses in the wake of the body. It occurs primarily near blunt bodies, such as circular cylinders and spheres.
Thus, the concept of Prandtl's boundary-layer theory unified the two divergent branches of fluid dynamics, namely inviscid hydrodynamics and hydraulics and gave quite agreeable results for drag on the solid body around which the fluid moves as compared to the results obtained by Stokes' method of neglecting the non-linear inertia terms in the Navier-Stokes equations. This boundary-layer theory proved extremely fruitful in that it provided an effective tool for the development of fluid dynamics. Since the beginning of the current century the new theory has been developed at a very fast rate under the additional stimulus obtained from the recently founded science of aerodynamics. In a very short time it became one of the foundation stones of modern fluid dynamics together with the other very important developments in the aerofoil theory and the science of gas dynamics.

Modern investigations in the field of fluid dynamics in general, as well as in the field of boundary-layer research, are characterised by a very close relation between theory and experiment. A review of the development of boundary-layer theory which stresses the mutual cross-fertilization between theory and experiment is contained in an article written by A. Betz (8) in 1949.

The boundary-layer theory finds its applications in the calculation of the skin-friction drag which acts on a body as it is moved through a fluid. Problems of heat transfer between a solid body and a fluid (gas) flowing past it also belong to the class of problems in which boundary-layer phenomena play a
decisive part. The simplest example of the application of the boundary-layer equations is afforded by the flow along a very thin semi-infinite flat plate. Historically this was the first example illustrating the application of Prandtl's boundary-layer theory; it was discussed by Blasius (9) in 1908 and is often referred to as Blasius problem. Subsequently Bairstow (10) in 1925 and Goldstein (11) in 1930 solved the same equation with the aid of a slightly modified procedure. Some what earlier, Toepfer (12) in 1912 solved the Blasius equation numerically by Runge-Kutta method. The same equation was again solved by Howarth (13) in 1938 with increased accuracy.

(b) Role of Suction in Boundary-layer Control in laminar flow:

The problem of laminar flow control (LFC) has become very important in recent years particularly in the fields of aeronautical engineering owing to its application to reduce drag and hence to enhance the vehicle power by a substantial amount. Several methods have been developed for the purpose of artificially controlling the boundary-layer. The boundary-layer suction is one of the effective methods of laminar flow control (14).

The effect of suction consists in the removal of decelerated fluid particles from the boundary-layer (through some slits or suction holes) before they are given a chance to cause separation. A new boundary-layer which is again capable of overcoming a certain adverse pressure gradient is allowed to
form in the region behind the slit. With a suitable arrangement of the slits and under favourable conditions separation can be prevented completely. Simultaneously, the amount of pressure drag is greatly reduced owing to the absence of separation. The application of suction, which was first tried by L. Prandtl (15) was later widely used in the design of aircraft wings. By applying suction, considerably greater pressure increases on the upper side of the aerofoil (i.e. lower absolute pressure) are obtained at large angles of incidence, and, consequently, much larger maximum lift values. Schrenk (16) investigated a large number of different arrangements of suction slits and their effect on maximum lift. In more recent times suction was also applied to reduce drag. By the use of suitable arrangements of suction slits it is possible to shift the point of transition in the boundary-layer in the downstream direction; this causes the drag co-efficient to decrease, because laminar drag is substantially smaller than turbulent drag.

(c) Heat transfer in fluids and thermal boundary-layer:

In fluids flowing past heated or cooled bodies the transfer of heat takes place by conduction and convection. Heat radiation is negligible unless the temperature is very high. When the conductivity of the fluid is small, which is true in ordinary fluids, the heat transfer due to conduction is comparable to that due to convection only across a thin layer near the surface of the body. This means that the temperature field which spreads from the body extends, essentially, over a narrow zone in the immediate vicinity of its surface whereas the
fluid at a large distance from the surface is not materially
effected by the heated body. This narrow region ( thin layer )
near the surface of the body is known as thermal boundary-layer
analogous to the concept of velocity boundary-layer.

Heat transfer considerations are often of crucial
importance in modern engineering design. Equipment size in power
production and chemical processing may be determined primarily
by the attainable heat transfer rates. A considerable fraction
by the cost of many devices—for example, air-condition and
refrigeration system is due to heat exchangers. In many types of
equipments a successful design is possible only if provision is
made to maintain reasonable temperatures by adequate heat
transfer. Among such modern devices are rocket nozzles, compact
electronic components, high speed aircraft, and atmosphere
re-entry vehicles.

There are three distinct modes of heat transmission,
namely, conduction, convection and radiation. Conduction is the
process in which heat is transferred from regions of higher
temperature to regions of lower temperature within a system or
between two systems which are in contact physically without any
relative motion of the different parts of system or systems.
Radiation is an energy transfer process from material into
surrounding space by electromagnetic waves. A heat ( or mass )
transfer process whose rate is directly influenced by fluid
motion is called a convective process.

It may be emphasized that in most of the situations
Occurring in nature, heat flows by more than one of these processes acting simultaneously. The phenomena of conduction and convection are affected primarily by temperature difference, and very little by temperature levels whereas radiation interchange increases rapidly with increase in the temperature levels. At low temperatures, conduction and convection are the major contributors to the total heat transfer whereas at very high temperatures, radiation is the controlling factor.

In a system of fluid motion heat is transferred by conduction and convection. At the surface, heat is first transferred by conduction to the adjacent fluid elements which in turn move to regions of lower temperature (thus convecting heat) and impart heat to the neighbouring fluid particles by conduction as well. Of course, it is virtually impossible to observe pure heat conduction in a fluid because, as soon as a temperature difference is imposed on a fluid, natural convection currents ensue due to resulting density differences. Thus convective process dominated a heat transfer phenomenon in fluid mechanics. As the convective heat transfer process and the motion of the fluid are inseparable, a study of hydrodynamic behaviour of the fluid is necessary in order to understand heat transfer taking place within a moving fluid.

Convection heat transfer is classified, according to the modes of motivating flow, into forced and free convections, if the flow is imposed by some external agency, such as a pressure gradient or a pump or blower the process is called
'forced convection'. In this case the velocity field is independent of the temperature field though the temperature field is dependent on the velocity field. Mathematically, the problem reduces to finding the temperature field due to heated or cooled boundaries in a given velocity field. On the other hand, when the mixing motion takes place merely as a result of the buoyancy forces, the process is called free or natural convection. This case occurs at a very small velocities of motion in presence of large temperature differences. The state of motion which accompanies natural convection is evoked by buoyancy forces in the gravitational field of earth, the latter being due to density differences and gradients. For example, the fluid motion which exists outside a vertical hot plate belongs to this class.

Since in free convection, temperature difference is the cause of the fluid motion which in turn changes the rate of heat transfer, the temperature and velocity fields are interdependent here. This coupling between heat transfer and fluid motion causes natural convection process more difficult to analyse than similar forced convection arrangements. Because, here the momentum equation and the energy equation become coupled for velocity and temperature fields. If both the convective processes are equally important in a flow system then it is called combined forced and free convective process.

(d) Magnetohydrodynamics (MHD):

Magnetohydrodynamics (or hydromagnetics) is the
combination of hydrodynamics and electromagnetic theory. It is concerned, in the broad sense, with the motion of an electrically conducting fluid in the presence of a magnetic field; the fluid may be either a liquid or an ionized gas. The motion of a conducting fluid across the magnetic field generates electric currents which modify the magnetic field, and at the same time the electric currents react with the magnetic field to produce a body force which in turn modifies the motion. Thus, magnetohydrodynamics (often referred to as MHD) is the branch of science which deals with the study of interaction between the fluid motion and electromagnetic phenomena. The study was primarily inspired by geophysical and astrophysical problems and by problems associated with the fusion reactor.

It was known from Faraday's (17) time that a solid or a fluid material moving in a magnetic field experiences an electromotive force (e.m.f.). If the material is electrically conducting and a current path is available, electric currents ensue. Also, currents may be induced by change of the magnetic field with time. There are two basic consequences:

I. An induced magnetic field associated with these currents appears, perturbing the original magnetic field.

II. An electromotive force due to the interaction of currents and field appears, perturbing the original motion.

These are the two basic effects of magnetohydrodynamics (MHD). Recently MHD has been the subject of intensive study.
and the importance of the study has been extended to many other kinds of associated problems - even in missile rockets. Technological problems like controlled thermonuclear fusion, thrust production for propulsive devices, pumping of liquid metals, high-temperature resistant coating, re-entry problem of ballistic missiles, liquid metal lubrication, power conversion (i.e. extraction of electrical energy directly from a hot plasma) etc. etc. need the study of the flow of an electrically conducting fluid.

In the interwar period the astrophysicists, notably Cowling (18) and Ferraro (19) began to explore the formal theory of MHD and its applications, while other scientists and engineers such as William (20) and Hartmann (21) performed simple experiments on the flow of conducting liquids in the laboratory. Alfven (22) first published the classical paper on MHD. He explained in his paper that if a high conducting fluid moves in a magnetic field, the induced currents will tend, in some sense, to inhibit relative motion of the fluid and field, so that the field is conducted by the fluid.

The continuum approximation is made in MHD, just as in ordinary hydrodynamics. One postulates that the fluid may be treated as continuous and describable in terms of local properties such as pressure and velocities.

MHD differs from ordinary hydrodynamics in that the fluid is electrically conducting. It is not magnetic; it affects a magnetic field not by its mere presence but only by
virtue of electric currents flowing in it. In magnetohydrodynamic heat transfer problems, the additional body force term, viz, the Lorentz force term comes into play in the momentum equation and the term corresponding to Joule heating appears in the energy equation. In a forced convection system, the energy equation remains uncoupled from Maxwell's equations and Navier-Stokes equations. Thus, the electromagnetic and velocity fields can be determined independently of the temperature field. However, when natural convection forces are present the Navier-Stokes equation becomes coupled with the energy equation and simultaneous solution is required.

(e) Dusty gas:

The observation that adding dust to air flowing in turbulent motion through a pipe can appreciably reduce the resistance coefficient has been reported by Sproull (23) in 1961. A similar report that the aerodynamic resistance of a dusty gas flowing through a system of pipes is less than that of a clean gas has also been made by Kazakevich and Karpivin (24) in 1958. These observation can be expressed as saying that the pressure difference required to maintain a given volume rate of flow is reduced by the addition of dust. It is to be noted that the increased density of a dusty gas should, all other things being equal, require a large pressure difference to maintain a given volume flow rate. Sproull interprets the phenomenon as implying a substantial reduction in the value of the viscosity of a dusty gas, by as much as 40%, compared with the clean gas. His explanation of this reduction is that the viscosity is
proportional to the mean free path of the gas molecules and this is substantially reduced by the gas molecules colliding with the dust particles. However, his calculation of this effect is difficult to follow and moreover, it contradicts the Einstein's formula for the viscosity of a suspension (see for example, Landau & Lifshitz 1959, (25)), according to which the viscosity of a dusty gas should be increased by a factor proportional to the concentration by volume of the dust particles.

Thus the transport of solid particles by fluids has, from historic times, been the object of scientific and engineering research. The problem has appeared in various forms such as sediment transport by water and by air, the collection of ice on buildings and aircraft structures, the centrifugal separation of particulate matter from fluids, fluid-droplet sprays, fluidized beds and other two-phase flow phenomena of interest in chemical processing, and the electrostatic precipitation of dust. Later on, the motion of solid particles in rocket motor exhaust has focused attention upon some new features of the problem and has increased scientific interest in the entire field.

In order to formulate the problem in a reasonably simple manner and to bring out the essential features Saffman (26) in 1962 first studied the motion of a dusty gas in terms of a large number density \( N(x,t) \) of very small particles having the same radius. It was assumed that the bulk concentration of the dust particles is small, but the density of the dust material is taken large compared with the fluid density, so that
the mass concentration of the dust is an appreciable fraction of unity. Using the formulation of Saffman many authors have studied a number of dusty gas problems and the results are well documented in a review by Marbel (27).

(f) Non-Newtonian fluids:

Newton (28) in 1687, measured the force $F$ experienced by a plate which moves in water with velocity $V$ parallel to itself in presence of another stationary plate at a distance $d$. He observed that this force $F$ is in the direction of the motion of the plate and is directly proportional to $V$ and inversely proportional to $d$ which can be written as $F = (\mu_1 V)/d$, $\mu_1$ being the constant of proportionality. The ratio ($V/d$) can be taken as shear-rate. This law can be written in cartesian tensor notation as:

$$\tau_{ij}' = 2\mu_1 e_{ij}$$

where

$$e_{ij} = 1/2 (v_i v_j + v_j v_i), \quad \tau_{ij}' = -p \delta_{ij} + \tau_{ij}'$$

where $v_i$ is the velocity component in the direction of $i$, $\tau_{ij}$ is the stress tensor, $p$ is the undeterminate hydrostatic pressure and $\delta_{ij}$ is kronecker delta. Here a comma denotes covariant differentiation. Any fluid governed by the linear constitutive equation (1.1.1) is known as Newtonian fluid. The classical linear theory of isotropic, viscous fluids based on constitutive equation (1.1.1) provides a good approximation to that of ordinary mobile liquids like water, air, glycerine,
honey and many thin oils. It is capable of explaining the phenomena of lift, skin friction, form drag, separation and secondary flow etc.

However, Newtonian fluids fail to explain a number of interesting phenomena like Merrington effect, Weissenberg effect and Reiner effect which are observed in several other fluids. These fluids show marked deviations from Newtonian behaviour and are referred to as non-Newtonian fluids. For such fluids the viscosity at given pressure and temperature is a function of the velocity gradient. Merrington (29) observed that when a solution of rubber in mineral oil is forced through a straight pipe, the fluid swells on emerging from the pipe. As pointed out by Merrington, this phenomena of swelling is due to the elastic recovery of the liquid compressed in the tube. Weissenberg (30,31) found that when a high polymer solution is sheared between two coaxial cylinder (keeping the inner cylinder at rest and the outer cylinder in rotation), the liquid climbs up the inner cylinder against the action of the centrifugal force. This phenomena of climbing of the liquid in a direction perpendicular to the plane of shearing is known as Weissenberg effect or normal stress effect. He attributed this effect to the elastic character present in the fluid. The fluid exhibiting these effects include many other industrial products like plastics, synthetic fibres and shurries. Generally, the fluids which do not obey the equation (1.1.1) are referred to as non-Newtonian fluids.
1.2 BASIC EQUATIONS:

In this section we shall discuss some of the basic equations mainly related to this thesis.

(a) Hydrodynamical equations:

If $\rho$ is the density of the fluid, $\mathbf{q}$ be velocity, then equation of continuity is

$$\frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{q}) = 0 \quad (1.2.1)$$

An alternate form of the equation (1.2.1) is

$$\frac{D\rho}{Dt} + \rho \text{div} (\mathbf{q}) = 0 \quad (1.2.2)$$

where $\frac{D}{Dt}$ is the mobile operator, given by the expression

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \quad (1.2.3)$$

For an incompressible fluid, equation (1.2.2) reduces to

$$\text{div} \mathbf{q} = 0 \quad (1.2.4)$$

For an incompressible fluid of variable density, the above equation of continuity is supplemented by

$$\frac{\partial \rho}{\partial t} + (\mathbf{q} \cdot \nabla) \rho = 0 \quad (1.2.5)$$

which means that density of an element remains same as it moves about. This is to be regarded as a fundamental equation when the density is not uniform and fluid is incompressible. In heat transfer problems this equation is replaced by a standard
In cartesian coordinates, the equation of continuity (1.2.2) is to be supplemented by the equation

\[ \frac{D\rho}{Dt} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \]  

(1.2.6)

The general Navier-Stokes' equation of motion for Newtonian fluid in vector form is

\[ \rho \frac{D\mathbf{q}}{Dt} = \rho \mathbf{F} - \nabla p + 1/3 \mu \nabla \cdot \mathbf{q} + \mu \nabla^2 \mathbf{q} \]  

(1.2.7)

where \( p \) is the pressure, \( \rho \) is the density, \( \mathbf{F} \) is the external force per unit mass, \( \mu \) is the co-efficient of viscosity of the fluid and \( \Theta = \nabla \cdot \mathbf{q}, \nabla^2 \) is the Laplacian operator. For an incompressible fluid \( \Theta = \nabla \cdot \mathbf{q} = 0 \)

\[ \therefore \text{the equation (1.2.7) simplifies to} \]

\[ \rho \frac{D\mathbf{q}}{Dt} = \rho \mathbf{F} - \nabla p + \mu \nabla^2 \mathbf{q} \]  

(1.2.8)

For the problems involving heat transfer the above equations are to be replaced by the energy equation:

\[ \rho c_v \frac{DT}{Dt} + p \nabla \cdot \mathbf{q} = \kappa \nabla^2 T + \mu \phi \]  

(1.2.9)

where \( T \), is the temperature.

\( k \), is the thermal conductivity of the fluid.

\( c_v \), is the specific heat at constant volume.
\[ \phi = - \frac{2}{3} \left( \frac{3}{\partial x} u + \frac{3}{\partial y} v + \frac{3}{\partial z} w \right)^2 + 2 \left( \frac{3}{\partial x} v \right)^2 + \frac{2}{\partial y} \left( \frac{3}{\partial z} w \right)^2 \]

Another form of the energy equation (1.2.9) is

\[ \rho c_p \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} = k \nabla^2 T + \mu \phi \]  

(1.2.10)

where \( c_p \) is the specific heat at constant pressure.

For incompressible fluid, \( \text{div} \ q = 0 \)

\[ \therefore \] the equation (1.2.9) simplifies to

\[ \rho c_v \frac{\partial T}{\partial t} = k \nabla^2 T + \mu \phi \]

(1.2.11)

Again for incompressible fluid \( \frac{\partial p}{\partial t} \) is too small

\[ \therefore \] Equation (1.2.10) reduces to

\[ \rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T + \mu \phi \]

(1.2.12)

The first and second term on the right hand side of the equations (1.2.11) and (1.2.12) give the amount of heat per unit volume due to heat conduction and viscous diffusion respectively.

Equations (1.2.1), (1.2.7) and (1.2.9) together with the equation of state, constitute the basic equations of the fluid dynamics.
We have altogether six equations viz.

Equation of continuity : One equation given by (1.2.1)

Equation of motion : three equations given by (1.2.7)

Energy equation : one equation given by (1.2.9)

Equation of State : One equation given by

\[
\frac{\rho}{\rho T} = \text{Constant} = \frac{\bar{R}}{m}
\]  

(1.2.13)

where \(\bar{R}\) is the gas constant

\(m\) is the molecular weight of gas, and there are six variables \(u, v, w, \rho, T\) and \(p\) for the unique solution.

(b) Magnetohydrodynamical equations:

The general equations of Magnetohydrodynamics are the ordinary hydrodynamical and electromagnetic equations, modified to take into account of the interaction between the fluid motion and the magnetic field.

Electromagnetic part of the equations:

The classical Maxwell's equations with some characteristics approximations provide the electromagnetic part of the equations. In a large class of electromagnetic problems involving conductors but not concerned with rapid oscillations, low-frequency approximations are successfully used to treat them. Likewise, in the consideration of MHD problems not concerned with rapid oscillations the same type of approximations can be used, and these lead to the neglect of the displacement current in Maxwell's equations. We shall not be
concerned herewith rapid oscillations, Maxwell's displacement current will, therefore, be ignored. As a consequence of this neglect, the accumulation of electric charge is also neglected in the equation of continuity of charge. This leads to the result that electric currents flow in closed circuits. Thus, if \( \mathbf{J} \) is the current density

\[
\text{div} \mathbf{J} = 0 \quad (1.2.14)
\]
everywhere in the fluid. (In the equation of continuity of electric charge, the term representing the rate of change of charge is, following low frequency approximation, of order \( v^2 / c^2 \), where \( v \) is the material velocity and \( c \) is the velocity of light; its neglect is thus quite appropriate in most ordinary problems).

With the displacement currents ignored, the electromagnetic equations are

\[
\begin{align*}
\text{Curl} \quad \mathbf{B} & = 4\pi \mathbf{J} \quad (1.2.15) \\
\text{Curl} \quad \mathbf{E} & = -\mu \frac{\partial \mathbf{B}}{\partial t} \quad (1.2.16) \\
\text{div} \quad \mathbf{B} & = 0 \quad (1.2.17)
\end{align*}
\]

The electromagnetic variables are all measured in electromagnetic units (EMU); \( \mathbf{E} \) and \( \mathbf{B} \) are the intensities of the electric and the magnetic fields, \( \mathbf{J} \) is the current density and \( \mu \) is the magnetic permeability. In EMU the magnetic permeability is unity, however, \( \mu \) will be retained in the equations to identify the units. The equation (1.2.17) is only an initial condition since (1.2.16) implies that

\[
\frac{\partial}{\partial t} \text{div} \mathbf{B} = 0
\]

However, in steady problems, it is necessary as a governing equation.
In a stationary conductor Ohm's law states that
\[ \mathbf{J} = \sigma \mathbf{E}, \]
where \( \sigma \) is the electrical conductivity. In MHD, Ohm's law is modified to
\[ \mathbf{J} = \sigma ( \mathbf{E} + \mathbf{v} \times \mathbf{B} ) \]  \hspace{1cm} (1.2.18)
where \( \mathbf{v} \) is the material velocity. Through the occurrence of the velocity \( \mathbf{v} \) in the expression for \( \mathbf{J} \), the equations incorporate the effect of motion on the electromagnetic field.

The pondermotive force experienced by the matter (per unit volume) is
\[ \mathbf{f} = \mu \mathbf{J} \times \mathbf{B} ; \]  \hspace{1cm} (1.2.19)
this goes under the name Lorentz force, and this is the force responsible for the modification of the (otherwise purely hydrodynamical) motion, and this is to be taken into account in the ordinary hydrodynamical equation of motion.

Hydrodynamical part of the equations:

The equation of continuity remains the same as given earlier, namely,
\[ \frac{d\rho}{dt} + \text{div} (\rho \mathbf{v}) = 0 \]
and for incompressible fluid it reduces to
\[ \text{div} \mathbf{v} = 0 \]
In the equation of motion, there appears a body force
given by equation (1.2.19) of electromagnetic origin. If the only other body force is gravity with vector acceleration $\mathbf{g}$, the equation of motion is then

$$\rho \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{q} + \mu \mathbf{J} \times \mathbf{B} \quad (1.2.20)$$

For problems involving heat distribution the equation (1.2.12) is to be replaced by equation

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T + \phi + \frac{J^2}{\sigma} \quad (1.2.21)$$

In equation (1.2.21), the first, second and third terms on the right-hand side give the heating effect per unit volume due to heat conduction, viscosity and the flow of electric currents respectively.

Equations (1.2.14) to (1.2.18), (1.2.1), (1.2.20) and (1.2.21) constitute the basic equations of magnetohydrodynamics.

(c) Boundary-layer equations:

The fact that a boundary-layer is thin, compared with linear dimensions of the boundary, makes possible certain approximations in the equations of motion, also due to Prandtl, and thereby the flow in the boundary-layer may be determined in certain cases. For the purpose of explaining these approximation, we take the boundary to be a plane wall (at $y=0$) and the flow to be two dimensional. The boundary-layer thickness is supposed everywhere to be small compared with distance, parallel to the boundary over which the flow velocity changes,
appreciably.

The Navier-Stokes equations, for a viscous incompressible fluid, in a two dimensional flow along a plane wall (at \( y = 0 \)) are

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]

(1.2.22)

and the equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1.2.24)

where the \((x,y)\)-plane is the plane of motion, the \(x\)-axis is along the wall and \(y\)-axis perpendicular to it. \(u,v\) are the \(x\) and \(y\) components of velocity, \(p, \rho, \nu\) are respectively the pressure, the density and the kinematic viscosity of the fluid.

Thus due to no-slip condition and since the wall is solid (non porous) both \(u\) and \(v\) will vanish at \(y = 0\), We will now assess the order of magnitude, symbolically \(O(\cdot)\), of the terms in equations (1.2.22) to (1.2.24). The velocity component \(u\) parallel to the wall in the boundary-layer rises rapidly from a value zero at the wall to a value \(U\) in the main stream within a short distance, \(\delta\) (say) the thickness of the boundary-layer, from the wall. Taking \(t, x, u\) as quantities of \(O(1)\) and \(y\) of \(O(\delta)\), where \(\delta < < 1\), we find that
\[ \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2} \] are each of \( O(1) \),
\[ \frac{\partial u}{\partial y} \] is of \( O(\delta^{-1}) \) and \[ \frac{\partial^2 u}{\partial y^2} \] is of \( O(\delta^{-2}) \)
in the boundary-layer.

It then follows from the equation of continuity that \( \frac{\partial v}{\partial y} \) is of \( O(1) \) and since \( y \) is of \( O(\delta) \) the velocity component \( v \) should also be of \( O(\delta) \). Hence
\[ \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2} \] are each of \( O(\delta) \)
and \[ \frac{\partial^2 v}{\partial y^2} \] is of \( O(\delta^{-1}) \)
Thus in equation (1.2.22) \[ \frac{\partial^2 u}{\partial x^2} \] may be neglected in comparison with \[ \frac{\partial^2 u}{\partial y^2} \] and then the equation becomes
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} \] \quad (1.2.5)

It is now supposed that the viscous term is of the same order as the inertia terms, i.e. of order unity, so that \( v \) is of \( O(\delta^2) \) or in other words \( \delta \) is of \( O(\sqrt{v}) \). This fact is already confirmed by some exact solutions. For references we may see Jeffery-Hamel flow (32, 33), Hiemenz flow (34) and Karman flow (35). The equation (1.2.23) then gives
\[ -\frac{1}{\rho} \frac{\partial p}{\partial y} \] is of order \( O(\delta) \) \quad \ldots \ldots \quad (1.2.26)
Therefore, the pressure increase across the boundary-layer is of $0 (\delta^2)$ and may be neglected. Hence the pressure is taken practically, constant in a direction normal to the boundary-layer, and may be assumed equal to that at the outer edge of the boundary-layer where it is determined by the inviscid flow (Potential flow). We may, therefore, write

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \quad \text{.................. (1.2.27)}$$

where $U$ is the potential flow velocity.

Hence the Prandtl boundary-layer equations for a two dimensional unsteady incompressible flow are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad \text{(1.2.28)}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{........... (1.2.29)}$$

The boundary conditions, under which these equations are usually integrated are

$$y = 0 : \quad u = v = 0 ;$$

$$y \rightarrow \infty : \quad u = U (x,t) \quad \text{......... (1.2.30)}$$

in which the first is the no-slip condition and the condition of non-porous wall and the second is obtained from the consideration that the velocity $u$, in the boundary-layer, must join smoothly onto the main stream velocity.
Equations governing elastico-viscous boundary-layer flow
( Two-dimensional flow near a stagnation point )

Here we intend to give a brief outline of the equations governing elastico-viscous boundary-layer flows (Oldroyd's and Walter's elastico-viscous liquids) which are relevant to one of the chapters (Chapter 1) included in this thesis.

A detailed theoretical investigation has been done by Thomas and Walters, (36,37) and Walters (38) for the incompressible elastico-viscous prototype designated liquid B. The equations of state for liquid B can be written in the form : (due to Beard and Walters (39)).

\[
P_{ik} = -p g_{ik} + p'_{ik}
\]

\[
p'_{ik} (x,t) = 2 \int_{-\infty}^{t} \psi(t-t') \frac{\partial x^i}{\partial x^m} \frac{\partial x^k}{\partial x^r} e^{(1)mr} (x',\dot{t'}) dt'
\]

where co-variant suffixes are written below, contravariant suffixes above, and the usual summation convention for repeated suffixes is assumed and moreover

\[
\psi(t-t') = \int_{0}^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')/\tau} d\tau
\]

\( N(\tau) \) being the distribution function of relaxation time \( \tau \) (see for example, (40)). In these equations, \( p_{ik} \) is the stress tensor, \( p \) an arbitrary isotropic pressure, \( g_{ik} \) the metric tensor of a fixed coordinate system \( x^i, \dot{x}^i (=x^i(x,t,t')) \) the position at
time $t'$ of the element that is instantaneously at the point $x^i$ at time $t$, and $e^{(1)}_{ik}$ is the rate-of-strain tensor.

The liquid designated liquid B by Oldroyd (41), with equations of state

$$(1 + \lambda_1 \frac{\partial}{\partial t}) \ddot{p}^{ik} = 2 \eta_0 (1 + \lambda_2 \frac{\partial}{\partial t}) e^{(1)ik}$$

is a special case of liquid B obtained by writing

$$N(\tau) = \eta_0 \left( \frac{\lambda_2}{\lambda_1} \right) \delta(\tau) + \eta_0 \left( \frac{\lambda_1 - \lambda_2}{\lambda_1} \right) \delta(\tau - \lambda_1)$$

in equations (1.2.32) and (1.2.33), where $\delta$ denotes a Dirac delta function, defined in such a way that

$$\delta(x) = 0 \ (x \neq 0), \ \int_{-\infty}^{\infty} \delta(x) dx = 0 \int_{-\infty}^{\infty} \delta(x) dx = 1 \cdots (1.2.36)$$

(In equation (1.2.34), $\frac{\partial}{\partial t}$ denotes convected differentiation of a tensor quantity in relation to the material in motion, as defined by Oldroyd (41). For a contravariant tensor $b^{ik}$,

$$\frac{\partial b^{ik}}{\partial t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{im},$$

where $v^i$ is the velocity vector). The Newtonian liquid of constant viscosity $\eta_0$ is given by

$$N(\tau) = \eta_0 \delta(\tau) \cdots (1.2.37)$$

We now consider the flow of liquid B near a stagnation point, using boundary-layer approximations. For mathematical
convenience, it is necessary to restrict the discussion to liquids with short memories (i.e. short relaxation times). The equations of state (1.2.32) can then be written in the simplified form, following Walters (42) as

\[ p^{ik} = 2 \eta_0 e^{(1)ik} - 2 \kappa_0 \frac{\delta}{\delta t} e^{(1)ik} \ldots \ldots \ (1.2.38) \]

where \( \eta_0 = \int_0^\infty \frac{N(\tau)}{\tau} d\tau \) is the limiting viscosity at small rates of shear, \( \kappa_0 = \int_0^\infty \frac{N(\tau)}{\tau} d\tau \) and terms involving \( \int_0^\infty \frac{N(\tau)}{\tau} d\tau \) have been neglected. The liquid with equations of state (1.2.31) and (1.2.38) are referred to as liquid B". In the case of Oldroyd's liquid B (equations (1.2.34) and (1.2.35)), \( \kappa_0 = \eta_0 (\lambda_1 - \lambda_2) \) and equation (1.2.38) becomes

\[ p^{ik} = 2 \eta_0 [ 1 - (\lambda_1 - \lambda_2) \frac{\delta}{\delta t} ] e^{(1)ik} \ldots \ldots \ (1.2.39) \]

In this case, the approximation is equivalent to neglecting second order terms in \( \lambda_1 \) and \( \lambda_2 \).

Using the equations of state (1.2.31) and (1.2.38), the equations of motion for the liquid in a cartesian frame of reference can be written in the form

\[ \rho \left[ \frac{\partial v^i}{\partial t} + v_k \frac{\partial v^i}{\partial x_k} \right] = -\frac{\partial p}{\partial x_i} + \eta_0 \frac{\partial^2 v^i}{\partial x_k \partial x_k} - \kappa_0 \left[ \frac{\partial}{\partial t} \left( \frac{\partial v^i}{\partial x_k} \right) \right] \]

\[ + v_m \frac{\partial^3 v^i}{\partial x_m \partial x_k \partial x_k} - \frac{\partial v^i}{\partial x_m} \frac{\partial^2 v_m}{\partial x_k \partial x_k} - 2 \frac{\partial^2 v^i}{\partial x_m \partial x_k} \frac{\partial v_m}{\partial x_k} \]

\ldots \ldots (1.2.40) .
where $\rho$ is the density, we now consider a steady two dimensional motion with velocity components

$$u = u(x,y), \quad v=v(x,y) \quad w = 0 \quad \ldots \ldots (1.2.41)$$

The relevant equations of motion become

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \kappa_0^* \left[ (u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}) \frac{\partial u}{\partial x} \right]$$

$$- \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} - (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \frac{\partial^2 u}{\partial x \partial y} \ldots \ldots (1.2.42)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial x^2} - \kappa_0^* \left[ (u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}) \frac{\partial v}{\partial x} \right]$$

$$- \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial y^2} - (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \frac{\partial^2 v}{\partial x \partial y} \ldots \ldots (1.2.43)$$

where

$$v = \eta_0/\rho, \quad \kappa_0^* = \frac{\kappa_0^*}{\rho} \quad \text{and} \quad \nu^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The equation of continuity reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots \ldots (1.2.44)$$

We now make the usual boundary-layer assumptions of viscous flow theory. Within the boundary-layer, $u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial p}{\partial x}$ are assumed to be $O(1)$ and $y$ to be $O(\delta)$, where $\delta$ is the thickness of the boundary-layer near a solid boundary at $y = 0$. From the
equation of continuity (1.2.44) we have \( v = 0 (\delta^2) \). In order that the viscous, elastico-viscous and inertial terms in the equations of motion shall be of the same order of magnitude, it is necessary that

\[
v = 0 (\delta^2), \quad \kappa_0^* = 0 (\delta^2)
\]

Under these conditions equations (1.2.42) and (1.2.43) reduce to

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} - \kappa_0^* \left[ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \cdot \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right] + O(\delta) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \ldots (1.2.45)
\]

\[
\frac{\partial p}{\partial y} = 0 (\delta) \quad \cdots \cdots \cdots \cdots \cdots \cdots (1.2.46)
\]

As in viscous flow theory, the change in pressure across the boundary layer is \( O(\delta^2) \) and the pressure gradient term in (1.2.45) can therefore be obtained from the flow just outside the boundary layer. In this region under steady flow conditions we have

\[
- \frac{1}{\rho} \frac{\partial p}{\partial x} = U \cdot \frac{dU}{dx} \quad \cdots \cdots \cdots \cdots \cdots \cdots (1.2.47)
\]

Where \( U \) is the mainstream velocity. Thus the elastico-viscous boundary layer equations are therefore
Following Saffman (26) let us derive the equations of motion governing the laminar flow of an unsteady, viscous and incompressible fluid. It will be supposed that the dust particles are uniform in size and shape, and that their velocity and number density can be described by fields \( \tilde{\nu} (x,t) \) and \( N (x,t) \). We also assume that the bulk concentration (i.e. concentration by volume) of the dust is very small so that the net effect of the dust on the gas is equivalent to an extra force \( K N (\tilde{\nu} - \bar{u}) \) per unit volume, where \( \bar{u} (x,t) \) is the velocity of the gas and \( K \) is a constant, where it is also supposed that the Reynolds number of the relative motion of dust and gas is small compared with unity so that the force between dust and gas is proportional to the relative velocity. Then with small bulk concentration, and the neglect of the compressibility of the gas, the equations of motion and continuity of the gas are

\[
\begin{align*}
\frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} &= U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \\
\kappa^* \left[ u \frac{\partial^3 u}{\partial x \partial y^2} + \nu \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \cdot \frac{\partial^2 u}{\partial y^2} \\
- \frac{\partial u}{\partial y} \cdot \frac{\partial u}{\partial x \partial y} \right] \quad (1.2.48)
\end{align*}
\]

and the equation of continuity (1.2.44)

(e) Equations governing the laminar flow of an unsteady viscous incompressible dusty gas:
\[ p \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \mu \nabla^2 \mathbf{u} + KN (\nabla \mathbf{u} - \mathbf{u}) \] (1.2.49)

\[ \text{div} \, \mathbf{u} = 0 \quad \ldots \quad (1.2.50) \]

where \( p \) is the pressure less the hydrostatic pressure, and \( p \) and \( \mu \) are the density and viscosity of the clean gas. If the dust particles are spheres of radius \( a \), \( K = 6\pi a \mu \) by the Stocks drag formula.

The effect of dust is measured by the mass concentration, \( f \) say. The bulk concentration is \( f \rho / \rho_1 \), where \( \rho_1 \) is the density of the material in the dust particles. For common materials, \( \frac{\rho_1}{\rho} \) will be of the order of several thousand or more. So that the mass concentration may be a significant fraction of unity while the bulk concentration is small. It is to be noted that for suspensions in liquids, the bulk and mass concentrations will be roughly the same, so that qualitative differences in the motion of dusty gases and suspensions in liquids may be expected. For spherical particles, the Einstein increase in viscosity is \( \frac{5}{2} \mu f \cdot \frac{\rho}{\rho_1} \), which is negligible for a dusty gas but may be significant for a liquid suspension.

The force exerted on the dust by the gas is equal and opposite to the force exerted on the gas by the dust, so that the equation of motion of the dust is
\[ m_N \left[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right] = m_N g + k_N (\vec{u} - \vec{v}) \ldots (1.2.51) \]

where \( m_N \) is the mass of dust per unit volume and \( g \) is the acceleration due to gravity. (The buoyancy force is neglected since \( \rho / \rho_1 \) is small). The equation of continuity for the dust is

\[ \frac{\partial N}{\partial t} + \text{div} (N\vec{v}) = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.2.52) \]

Equations (1.2.49) to (1.2.52) constitute the equations of motion of a dusty viscous fluid.

1.3  A brief review of the relevant literature of fluid flows in various geometric configurations:

(a) Hydrodynamic flows along plates:

Lighthill (43) initiated the study of unsteady two dimensional boundary-layer flow when external flow fluctuates about a steady mean. Sarma (44) has generalised Lighthill's work in the sense that the fluctuating unsteady part of the free stream velocity is replaced by an arbitrary function of time and that the on coming free stream is perturbed not only in magnitude but also in direction. Gill and Casal (45) have made a theoretical investigation of free convection effects in forced horizontal flows. They have obtained similarity
solutions of the boundary-layer equations for steady flow over a semi-infinite horizontal plate. Sparrow and Minkowycz (46) have also considered free convection effects on a horizontal plate by employing a series expansion of the stream function, which gives the perturbation of a basic forced convection flow due to buoyancy. Sharma and Nanda (47) have investigated the free convection effects on laminar boundary-layers in oscillatory flow. Gersten and Gross (48) have studied the effects of transverse sinusoidal suction velocity on the flow and heat transfer over a porous plane wall. Singh, Sharma and Misra (49) have extended this work to consider the free convection effects on skinfriction and heat transfer of the flow caused by periodic suction velocity perpendicular to the flow. Stuart (50) has found some interesting features for an oscillatory flow over an infinite plate with constant suction at the plate. Reddy (51) has extended Stuart's work to include a first order velocity slip and temperature jump conditions at the plate. Messiha (52) has further extended Stuart's problem to consider the effect of variable suction velocity upon the skinfriction and heat transfer in the boundary-layer flow along an infinite porous plate. Goldstein and Eckert (53) have studied the steady and transient free convection boundary-layers on a uniformly heated vertical plate.

Sparrow and Cess (54) have considered the steady convection heat transfer and laminar boundary-layer flow about
an isothermal vertical plate. Gebhert (55) has studied the free convection flow past a vertical plate. Soundalgekar (56) studied the effects of viscous dissipation on free convection steady motion past an infinite vertical porous plate. Soundalgekar and Gupta (57) have extended the work of former author (56) taking into account the steady motion of the porous plate. Vajravelu (58) extended the work of Soundalgekar and Gupta (57), taking into account the effect of the temperature dependent heat source/sink on the flow and heat transfer characteristics.

Katsuhisa and Ryuichi (59) have studied a three-dimensional analysis of the Navier-Stoke's equation for laminar natural convection around a vertical flat plate. Singh, Misra and Narayana (60) have made a systematic analysis of unsteady two-dimensional free convective flow through a porous medium bounded by an infinite vertical plate when the temperature of the plate is oscillating with time about a constant non-zero mean. Hudson and Dennis (61) have studied the flow of a viscous incompressible fluid past a normal flat plate at low and intermediate Reynolds numbers.

(b) MHD flow past an infinite plate:

Free and forced convective hydromagnetic flows past infinite plates have been studied widely because of their importance in technical fields.
Stoke's problem with transversely applied magnetic field was studied by Rossow (62). Soundalgekar, Gupta and Arnakae (63) studied the above case for a vertical impulsively started infinite plate. Gupta and Suryaprakash Rao (64) studied MHD free convective flow past a vertical porous plate subject to suction and injection. Soundalgekar (65) has studied the free convection effects on steady MHD flow past a vertical porous plate in presence of a uniform transverse magnetic field. Das (66) has studied the problem of a small unsteady perturbation of steady MHD boundary-layer flow past a semi infinite plate. Tokis (67) has investigated the unsteady free convection flow of an electrically conducting fluid near a vertical plate of infinite extent moving in its own plane in the presence of a uniform transverse magnetic field.

The MHD flow past a semi-infinite plate in the presence of a transverse magnetic field has been studied by Clauser (68), Dix (69), Sears (70) and Hector (71). Mittal (72) has considered the effect of plate temperature oscillations on the MHD thermal boundary-layer on a semi-infinite flat plate. Lewis (73) has considered the boundary-layer flow due to the uniform motion of a semi-infinite flat plate through an incompressible conducting fluid at rest, subject to a constant transverse magnetic field.
(c) Hydrodynamic channel flow

Berman (74), Sellars (75) and Yaun (76) have studied viscous flow in two dimensional porous channels. Tyagi (77) has discussed the laminar forced convection of a dissipative fluid in a channel. Terill (78) has studied forced convection heat transfer between two porous plates. Ostrach (79) has presented an analytical study of the laminar fully developed natural convection flow of viscous fluids with and without heat sources between two vertical plates. Lekoudis, Nayfeh and Saric (80) have made a linear analysis of compressible boundary-layer flows over a wavy wall. Lessen and Gangwani (81) have made a very interesting analysis of the effect of small amplitude wall waviness upon the stability of the laminar boundary-layer. Vajravelu and Sastri (82) have studied the free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. The same authors (83) have extended this work to consider both wall to be wavy. Rao (84) has further generalised the above work in the sense that the wave lengths of the two wavy walls are not equal and volumetric rate of heat generation is temperature dependent. Shankar and Sinha (85) have studied the Rayleigh problem for wavy wall.

(d) MHD channel flow

In recent years considerable work has been done in MHD channel flow because of its application in energy
conversion schemes, eg. power generators, electromagnetic pumps and flow meters.

Hartmann (86) investigated the two dimensional, steady Poiseuille flow of mercury between two parallel walls in presence of an applied transverse magnetic field. Shercliff (87, 87(a)) and Tanzawa (88) have obtained exact solutions for MHD flow in channels with rectangular and circular cross sections respectively. The problem of steady flow of an electrically conducting viscous fluid through uniformly porous parallel plates in the presence of transverse magnetic field has been investigated by Suryaprakas Rao (89) and Terril and Shrestha (90), Siegal (91), Perlmutter and Siegal (92) and Zimin (93) have studied the magnetic effects on the heat transfer in a fully developed flow between non-conducting parallel plates. Gupta (94) has investigated the flow of a conducting fluid under pressure gradient between two parallel plates subject to transverse magnetic field. Gershuni and Shukkhovitski (95) studied the free convection shear flow between parallel porous walls. Mori (96) studied the combined natural and forced convection flow with transverse magnetic field for a parallel plate channel. Rao and Sivaprasad (97) have studied the flow of an incompressible viscous, electrically conducting fluid in a horizontal channel bounded by a wavy wall and a flat wall in the presence of a constant
(e) Flow along stretching surfaces:

Boundary-layer behaviour over a moving continuous solid surface is an important type of flow occurring in several engineering processes. For example, materials manufactured by extrusion processes and heat treated materials travelling between a feed roll and a wind-up roll or on conveyor belts possess the characteristics of a moving continuous surface. Sakiadis (98) initiated the study of boundary-layer flow over a continuous solid surface moving with a constant speed. Due to entrainment of ambient fluid, this boundary-layer flow is quite different than boundary-layer flow over a semi-infinite flat plate. Erickson et al. (99) extended this problem to the case in which the transverse velocity at the moving surface is non zero, with heat and mass transfer in the boundary-layer being taken into account.

These investigations have a definite bearing on the problem of a polymer sheet extruded continuously from a dye. It is often tacitly assumed that the sheet is inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching plastic sheet, as pointed out by McCormack and Crane (100). Danberg and Fansler (101) investigated the nonsimilar solution for the flow in the
boundary-layer past a wall that is stretched with a velocity proportional to the distance along the wall. Gupta and Gupta (102) analyzed the heat and mass transfer corresponding to the similarity solution for the boundary-layer over a stretching sheet subject to suction or blowing. Recently, Chen and Char (103) investigated the effects of power-law surface temperature and power-law surface heat flux variation on the heat transfer characteristics of a continuous, linearly stretching sheet subject to suction or blowing.

All the above investigations are restricted to flows of Newtonian fluid. However, of late non-Newtonian fluids have become more important industrially. The laminar boundary-layer on an inextensible continuous flat surface moving with a constant velocity in its own plane in a non-Newtonian fluid characterised by a power-law model (Ostwald-de Waele fluid) is studied by Fox et al (104), using both exact and approximate methods. Rajagopal et al (105) studied the flow of a viscoelastic fluid over a stretching sheet and gave an approximate solution to the flow field. Troy et al (106) gave the exact solution for the problem of Rajagopal et al (105).

(f) Flow between two discs:

The rotating flows over a surface find applications in natural phenomena as well as in industry. The analogous
hydromagnetic case with magnetic field has got applications in cosmical and geophysical fluid dynamics. Also it has industrial applications such as in MHD generators, gaseous core nuclear reactors etc.

The steady flow of a viscous incompressible fluid due to an infinite rotating disc was first considered by Von Kármán (107). Stewartson (108) investigated, experimentally as well as theoretically, the steady motion of a viscous fluid between two co-axial rotating discs. It is found experimentally that when the discs rotate in the same sense the main body of the fluid rotates as well, but if they rotate in the opposite sense the main body of fluid is almost at rest. He has also given an adequate theory to explain this. Batchelor (109) extended Von Kármán's (107) solution, describing the flow over a single rotating disc, to the case of two discs. In addition, Batchelor gave a qualitative discussion of the nature of the flow field for various values of the ratio of the angular velocities of the two discs. Stuart (110) investigated the effects of suction on a single infinite rotating disc. He however omitted the case of blowing. The problem of the flow of a viscous liquid confined between two infinite rotating discs has been studied by Lance and Rogers (111), on the assumption that the similarity solution is valid. Using the method of numerical integration,
they drew velocity profiles for various values of the Reynolds number (defined in terms of the rotation of the discs) and for different values of the ratio of the angular velocities of the two discs. Narayana and Rudraiah (112) studied axisymmetric steady flow of a viscous incompressible fluid between two coaxial circular discs, one rotating and the other stationary, with uniform suction at the stationary disc. They obtained the solutions separately for small and large suction Reynolds Numbers.

Rao and Gupta (113) extended Stuart's problem (110) by bringing in the effect of the transverse magnetic field for large suction Reynolds number. Khare (114) extended the Narayana's problem (112) for an electrically conducting viscous fluid in presence of a transverse magnetic field. Again the unsteadiness in the flow field is caused by the rotation of the disc or fluid or both. Several authors including Chawla, (115, 115(a)), Debnath and Mukherjee (116, 116(a)), Debnath (117, 117(a), 117(b)), Loper and Benton (118) and Loper (119) have made investigations of the unsteady hydromagnetic rotating fluid flows in various geometric configurations.
1.4 Outline of the thesis

This thesis deals with a study of some flow and heat transfer problems in incompressible fluids. Excluding one particular problem (Chapter 8) all are steady. In all problems, excluding the chapter 2, the fluids are considered to be Newtonian.

In chapter 1, a comprehensive introduction of the subject at hand is presented. In chapter 2, we have considered the heat transfer in a viscoelastic fluid over a stretching plate with suction or blowing. The dimensionless heat transfer co-efficient and the temperature distribution of the stretching plate are determined for various values of the modified Prandtl number $P$. Finally the effects of suction or blowing parameter $m$ on the temperature distribution are discussed in detail.

In chapter 3, we have discussed the three dimensional free convection flow and heat transfer along a porous vertical plate with transverse magnetic field. The basic two dimensional flow along the plate has become three dimensional due to imposition of a small sinusoidal suction velocity in addition to the original suction at the wall. The governing equations of the problem have been solved by perturbation with respect to this small sinusoidal suction velocity. The wall shear stress in the direction of the main flow and the heat transfer at the wall are obtained for various values of the parameters involved in the problem.
In chapter 4, the free convective flow and heat transfer of an electrically conducting fluid along an infinite porous horizontal plate with sinusoidal suction at the wall and applied magnetic field parallel to the plate is investigated. Due to this type of suction velocity at the plate, the flow becomes three dimensional. For the asymptotic flow condition, the wall shear stress in the direction of the main flow and heat transfer at the wall for different values of $\alpha$ (suction parameter) and $M$ (Hartmann number) are discussed.

In chapter 5, we have discussed free convective steady flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. The rate of suction at the wavy wall is taken equal to the rate of injection at the flat wall. The governing equations of the problem are solved by perturbation method. The first order quantities of the flow and the temperature fields which are effected by the suction/injection parameter have been calculated numerically for various parameters entering the problem and the results are discussed. The skinfriction and the Nusselt number at the walls along with pressure drop are also calculated numerically for various parameters and the effects of suction/injection parameters on these quantities are discussed.
In chapter 6, we have considered free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall with suction at the wavy wall. The equations governing the fluid flow and heat transfer have been solved subject to appropriate boundary conditions by assuming that the solution consists of two parts: a mean part and a perturbed part. To obtain the solution of the perturbed part, long wave approximation has been applied and to solve the mean part, the well-known approximations used by Ostrach has been utilised. The zeroth order, the first order and the total solution of the problem have been evaluated numerically for several sets of parameters involved in the problem. Certain results indicating the influence of magnetic field and suction on the properties of the flow and heat transfer along with the pressure drop have also been investigated.

In chapter 7, we have analysed the combined free and forced convection MHD flow of an electrically conducting fluid in an inclined channel with permeable boundaries in presence of magnetic field. Expressions for the velocity distribution, mass flow rate and its fractional increases are obtained and the effect of magnetic field on these quantities are discussed.

In chapter 8, we have considered unsteady rotational MHD motion of an electrically conducting dusty viscous fluid
contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface. Initially, the liquid and dust particles are at rest. The effect of mass concentration of dust particles on the fluid flow in presence of magnetic field and the effect of the strength of magnetic field on velocity profiles at fixed time have been discussed.

In chapter 9, we have considered MHD steady flow and heat transfer of an electrically conducting viscous fluid between two rotating porous discs of different transpiration. The solution of the problem is obtained for small cross flow Reynolds number. The discs are rotating with different angular velocities and the rate of injection of the fluid at one disc is different from the rate of suction at the other disc. The governing equations have been solved using regular perturbation technique, taking cross flow Reynolds number as perturbation parameter. Expressions for radial, transverse and axial velocities and temperature have been obtained and these have been numerically evaluated for different values of the parameters involved in the solution. Two cases namely (i) both the discs having different angular velocities in the same direction (ii) both the discs rotating with different angular velocities in opposite direction are discussed for different values of the parameters $M$ (Hartmann number) and $n$ (the suction/injection parameters). The Nusselt number and the skin-friction coefficient for various cases have also been
calculated at both the discs for the two cases and the effect of magnetic field and suction on these quantities are also discussed for different values of $M$ and $n$.

In the thesis, equations are marked at the right-hand side. A decimal system is used to indicate the chapter, section and number of the equation; for example, equation (1.2.3) indicates the third equation in the second section of chapter 1. References are cited at the end of each chapter, giving the names of the authors followed by years of publication.
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